*Made by ALEX*

*P1 Chapter 3*

# «Functions and combining functions»

A function is a **rule** , which calculates values of $f(x)$ for a set of values of $x$.

e.g. $f(x) = 2x - 1$ and $g(x) = \sin x$ are functions.
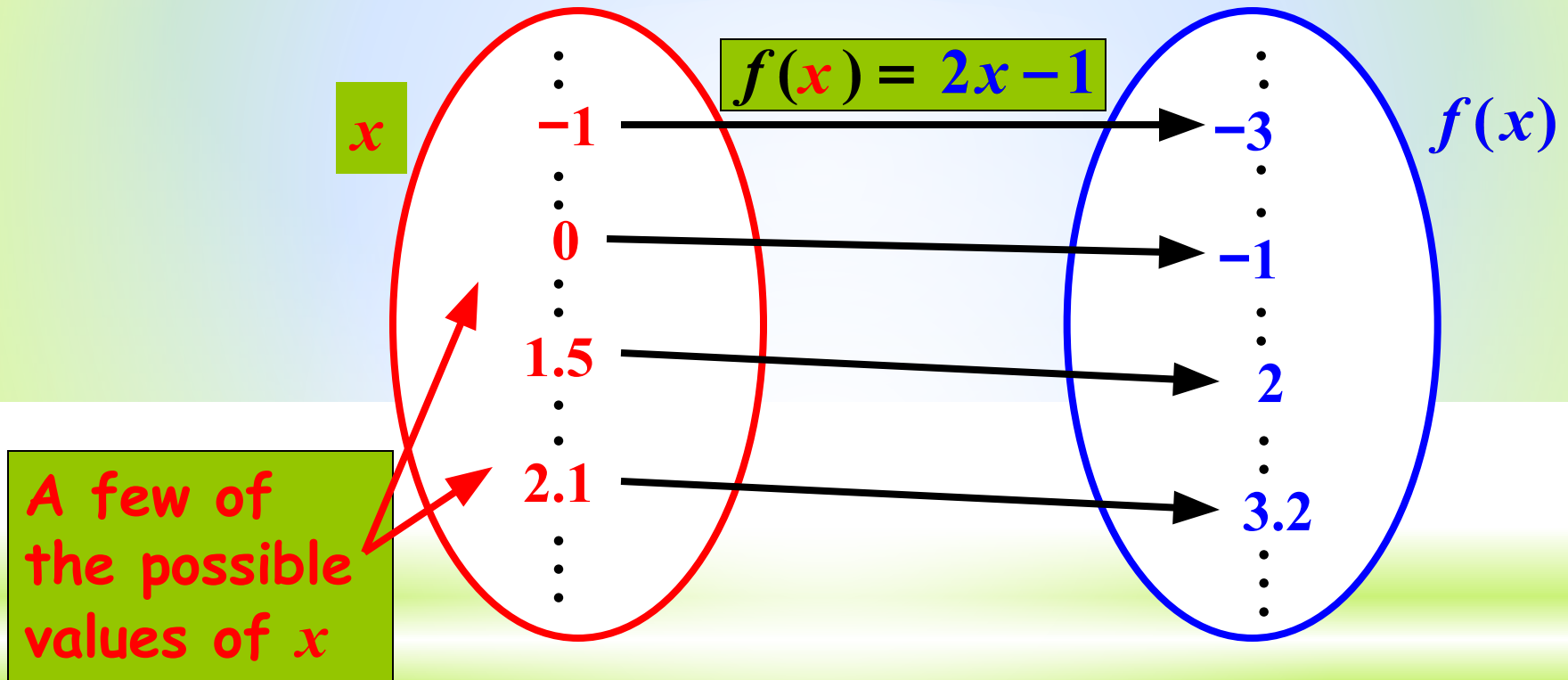
$f(x)$ is called the **image** of $x$

$$f : x \quad \rightarrow \quad 2x - 1 \quad \text{means} \quad f(x) = 2x - 1$$

$f(x)$ is often replaced by $y$.

# We can illustrate a function with a diagram

$$f(x) = 2x - 1$$

$x$

$-1 \longrightarrow -3$

$f(x)$

$0 \longrightarrow -1$

$1.5 \longrightarrow 2$

$2.1 \longrightarrow 3.2$

A few of the possible values of $x$

The rule is sometimes called a **mapping**.

To define a function fully, we need to know the values of $x$ that can be used.

The set of values of $x$ for which the function is defined is called the domain.

In the function $f(x) = x^2$ any value can be substituted for $x$, so the domain consists of

all real values of $x$

We write $x \in \mathbb{R}$

We say "real" values because there is a branch of mathematics which deals with numbers that are not real

$\mathbb{R}$ stands for the set of real numbers

∈ means "belongs to."

So, $x \in \mathbb{R}$ means $x$ is any real number

The $\boxed{\text{range}}$ of a function $f(x)$ is the set of values given by $f(x)$.

e.g. Any value of $x$ substituted into $f(x) = x^2$ gives a positive ( or zero ) value.

So the range of $f(x) = x^2$ is $\boxed{f(x) \geq 0}$

If $y = f(x)$, the range consists of the set of $y$-values, so

| domain: $x$-values | range: $y$-values |
|---|---|

Tip: To help remember which is the domain and which the range, notice that $d$ comes before $r$ in the alphabet and $x$ comes before $y$.

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The set of values of $x$ for which the function is defined is called the **domain**.

The **range** of a function is the set of values given by the rule.

**d**omain: **x**-values

**r**ange: **y**-values

e.g. 1 Sketch the function $y = f(x)$ where $f(x) = x^2 + 4x - 1$ and write down its domain and range.

Solution: The quickest way to sketch this quadratic function is to find its vertex by completing the square.
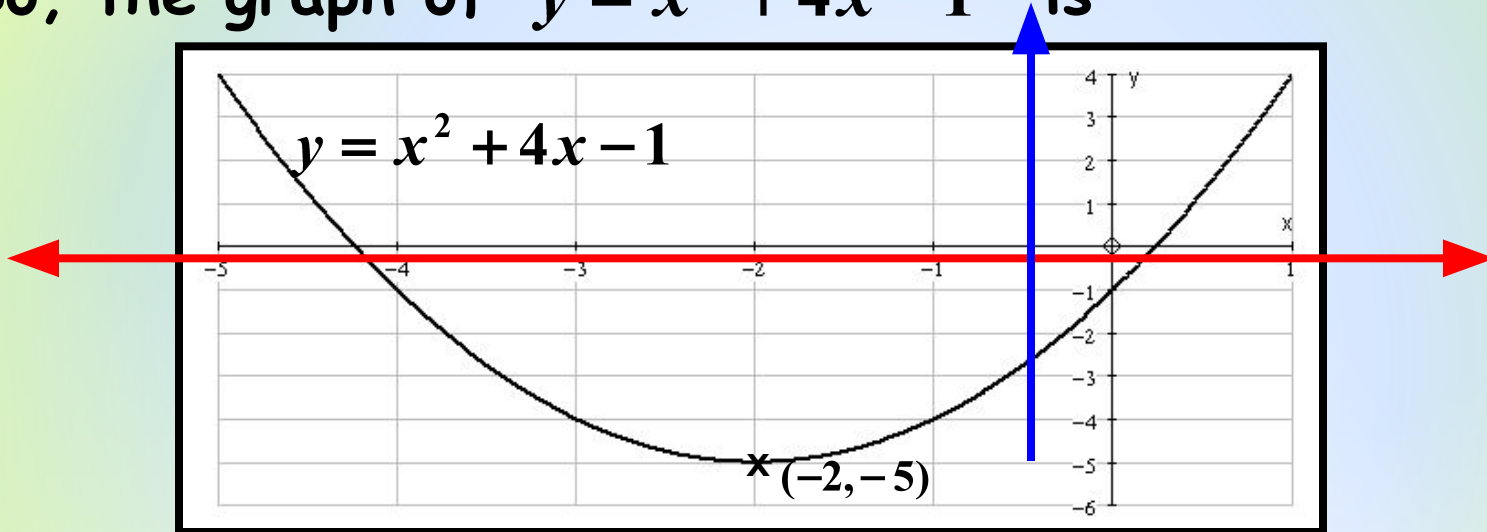
$$y = x^2 + 4x - 1 \quad \Rightarrow \quad y = (x+2)^2 - 4 - 1$$

$$\Rightarrow \quad y = (x+2)^2 - 5$$

This is a translation from $y = x^2$ of $\begin{bmatrix} -2 \\ -5 \end{bmatrix}$

so the vertex is $(-2, -5)$ .

So, the graph of $y = x^2 + 4x - 1$ is



$y = x^2 + 4x - 1$

$(-2, -5)$

**domain:** The $x$-values on the part of the graph we've sketched go from $-5$ to $+1$ . . . BUT we could have drawn the sketch for **any** values of $x$. So, we get $x \in \square$ ( $x$ is any real number )
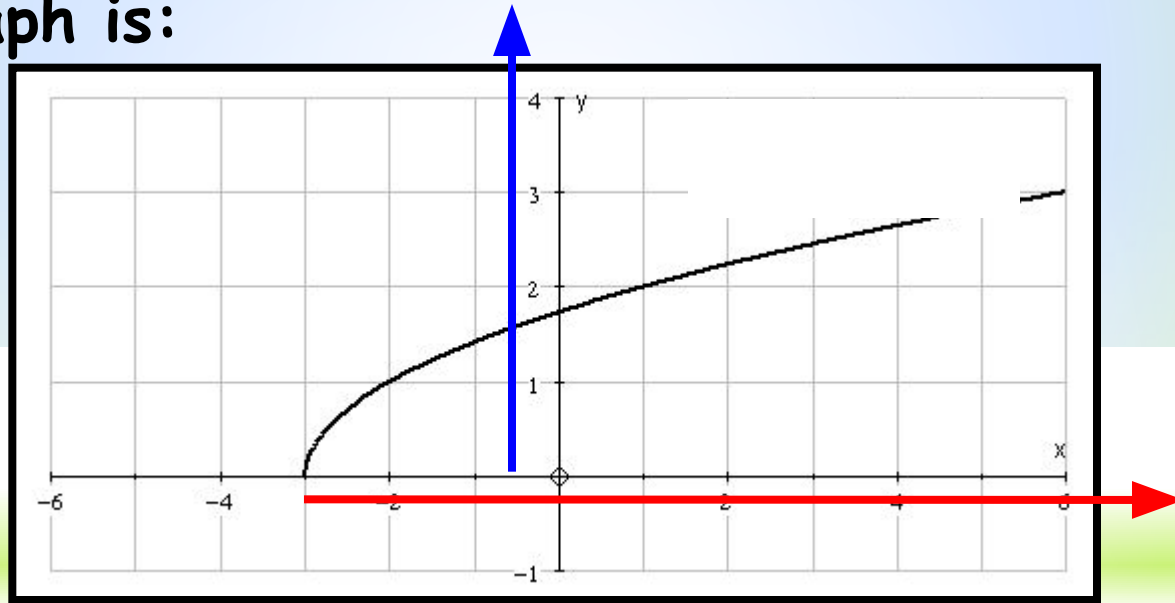
BUT there are no $y$-values less than $-5$, . . .

so the range is $y \geq -5$

( $y$ is any real number greater than, or equal to, $-5$ )

**e.g.2** Sketch the function ⬜ where ⬜
 Hence find the domain and range of ⬜

Solution: $y = f(x)$ is a translation from ⬜ of ⬜

so the graph is:



| domain: $x$-values | range: $y$-values |
|---|---|
| $x \geq -3$ | ⬜ |

( We could write ⬜ instead of $y$ )

# SUMMARY

- To define a function we need a **rule** and a **set of values**.
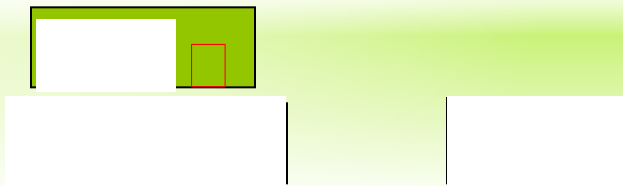
- Notation:

  means

- For           ,

  the $x$-values form the domain

  the        or $y$-values form the range

  e.g. For

  the domain is

  the range is

# Exercise

1. Sketch the functions ___ where ___

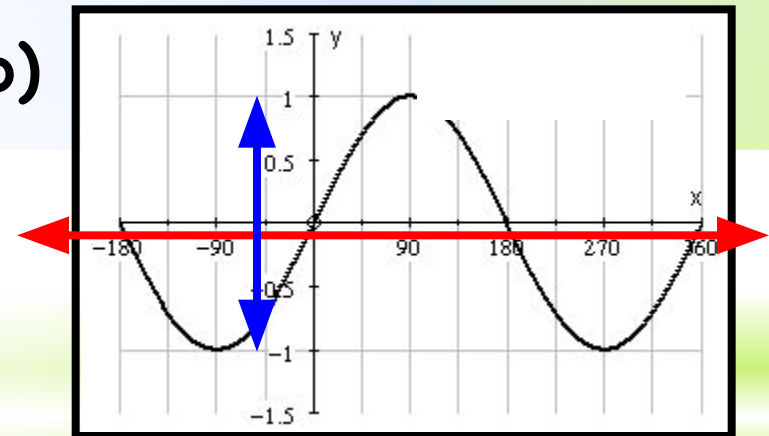   For each function write down the domain and range

Solution:

(a)



(b)
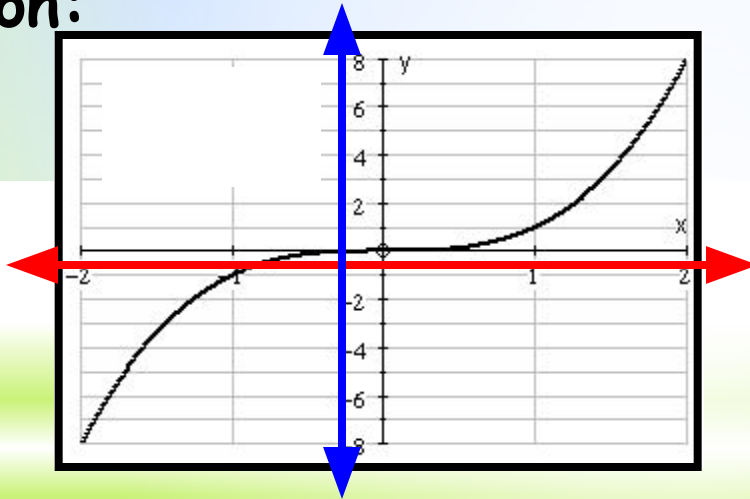


domain: ___ 

range: ___ 

domain: ___ 

range: ___

We can sometimes spot the domain and range of a function without a sketch.

e.g. For ⬚ we notice that we can't square root a negative number ( at least not if we want a real number answer ) so,

$x + 3$ must be greater than or equal to zero.

So, the domain is ⬚

The smallest value of ⬚ is zero.

Other values are greater than zero.

So, the range is ⬚

## Functions of a Function

Suppose [    ] and [    ]

then, [    ] $x$ is replaced by 3

## Functions of a Function

Suppose ☐ and ☐

then, ☐ ☐

and ☐

| $x$ is replaced by $-1$ |
| $x$ is replaced by ☐ |

# Functions of a Function

Suppose ⬚ and ⬚

then,

and

We read ⬚ as "*f* of *g* of *x*"

⬚ is "a function of a function" or compound function.

⬚ is the inner function and ⬚ the outer.

*x* is "operated" on by the inner function first.

So, in ⬚ we do *g* first.

★

[          ] is often written as [          ].

[          ] does NOT mean multiply $g$ by $f$.

When we meet this notation it is a good idea to change it to the full notation.

I'm going to write [          ] always !

e.g. 1   Given that                          and                         find

Solution:

**e.g. 1** Given that ⬜ and ⬜ find

⬜

Solution:

⬜

e.g. 1   Given that ⬜ and ⬜ find

⬜

Solution:

⬜ ⬜ ⬜

e.g. 1   Given that                    and                    find

Solution:

e.g. 1   Given that                        and                    find

Solution:

e.g. 1   Given that [          ] and [          ]   find

[                              ]

Solution:

N.B. [        ] is not the same as [        ]

e.g. 1  Given that _____ and _____  find

_____

Solution:

N.B. _____ is not the same as _____

e.g. 1   Given that                    and                    find

Solution:

e.g. 1  Given that                        and                        find

Solution:

e.g. 1   Given that ⬚ and ⬚ find

⬚

Solution:

e.g. 1   Given that                    and                    find

Solution:

e.g. 1   Given that                    and                    find

Solution:

e.g. 1   Given that                 and                 find

Solution:

1.  The functions $f$ and $g$ are defined as follows:

$$R$$

(a)  What is the range of $f$ ?

(b) Find (i)           and (ii)

Solution:  (a) The range of $f$ is

(b) (i)

(ii)

**Periodic Functions**

Functions whose graphs have sections which repeat are called periodic functions.

e.g.
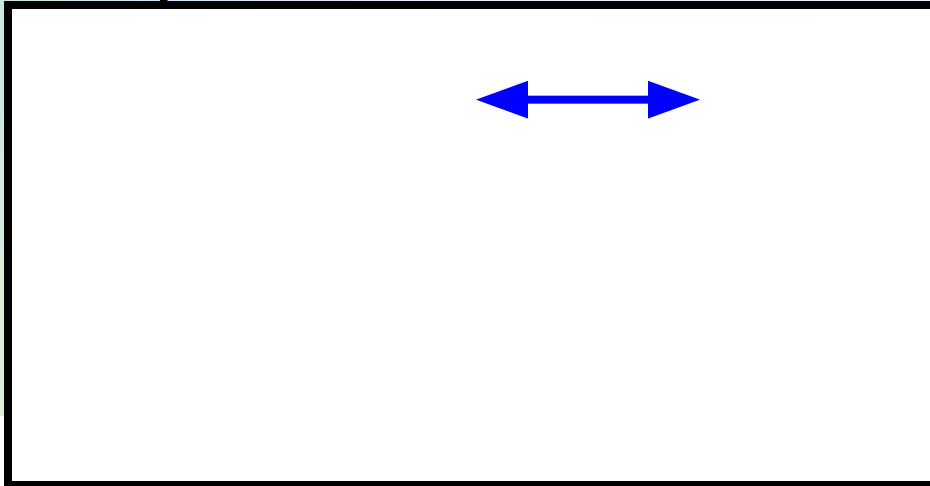
...peats every radian

It has a period of
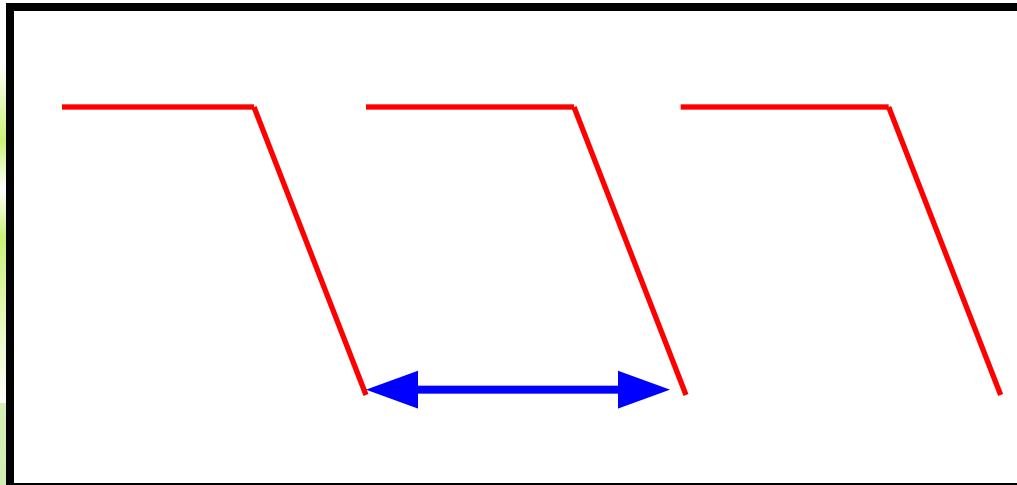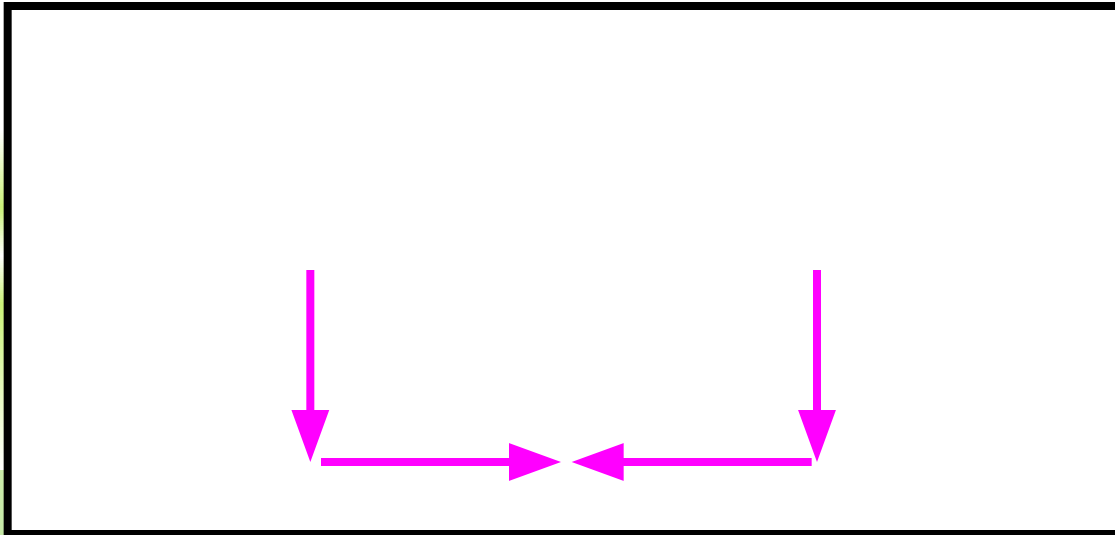
This has a period of 3.

Some functions are  **even**

e.g.
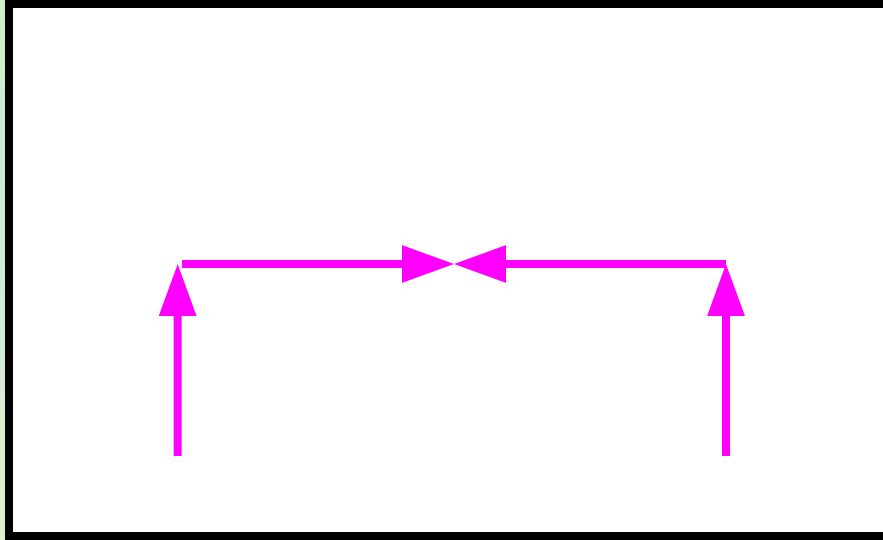


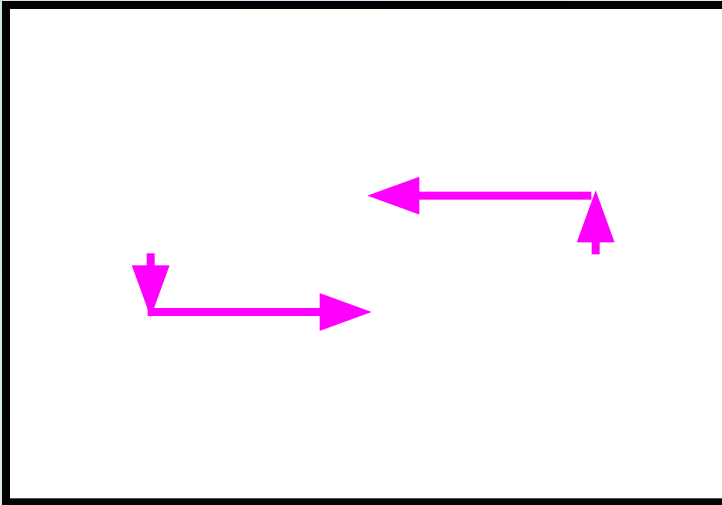**Even functions are symmetrical about the $y$ - axis**
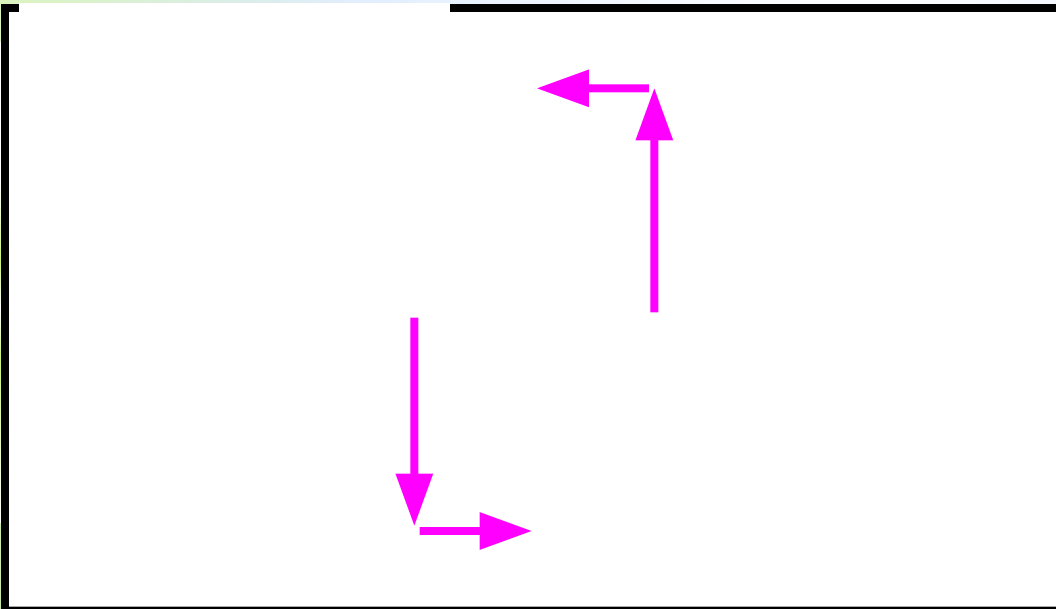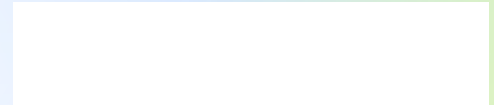
So,

e.g.



e.g.
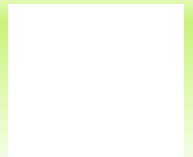
**Others are** odd

**e.g.**

Odd functions have 180° rotational symmetry about the origin

**e.g.**

**e.g.**

Many functions are neither **even** nor **odd**

e.g.

Try to sketch one even function, one odd and one that is neither.  Ask your partner to check.

## SUMMARY

- A compound function is a function of a function.

- It can be written as [blank] which means [blank]

- The inner function is [blank]

- [blank] is not usually the same as [blank]

- [blank] is read as "*f* of *g* of *x*".