Made by ALEX

P1 Chapter 3

«Functions and combining functions»

A function is a **rule**, which calculates values of $f(x)$ for a set of values of x.

e.g. $f(x) = 2x - 1$ and $g(x) = \sin x$ are functions.

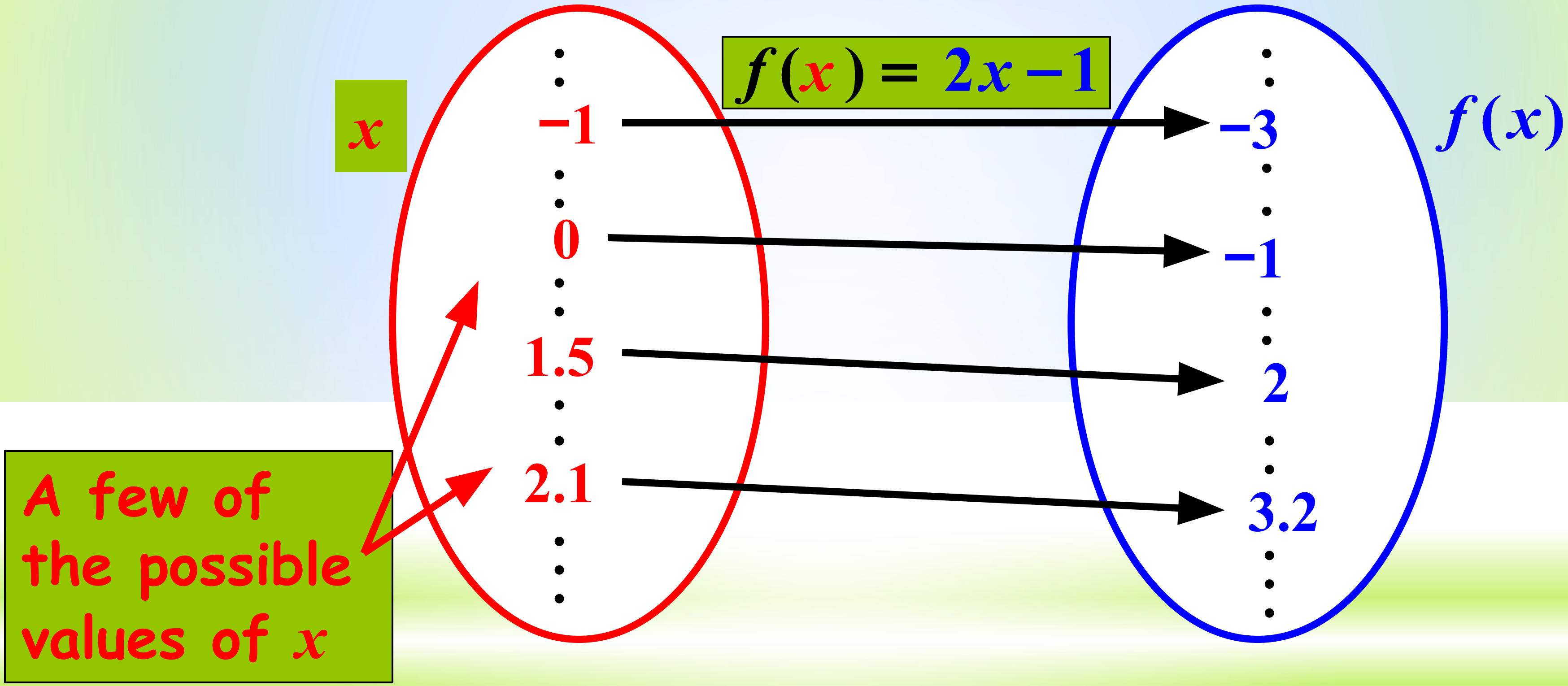
$f(x)$ is called the **image** of x

Another Notation

$f : x \mapsto 2x - 1$ means $f(x) = 2x - 1$

$f(x)$ is often replaced by y.

We can illustrate a function with a diagram



The rule is sometimes called a mapping.

A bit more jargon

To define a function fully, we need to know the values of x that can be used.

The set of values of x for which the function is defined is called the **domain**.

In the function $f(x) = x^2$ **any** value can be substituted for x, so the domain consists of

all real values of x

We say "real" values because there is a branch of mathematics which deals with numbers that are not real.

We write $x \in \mathbb{R}$

$\mathbb{R}$ stands for the set of all real numbers.

$\in$ means "belongs to."

So, $x \in \mathbb{R}$ means **x is any real number**

The range of a function $f(x)$ is the set of values given by $f(x)$.

e.g. Any value of x substituted into $f(x) = x^2$ gives a positive (or zero) value.

So the range of $f(x) = x^2$ is $\boxed{f(x) \geq 0}$

If $y = f(x)$, the range consists of the set of y-values, so

domain: x-values

range: y-values

Tip: To help remember which is the domain and which the range, notice that d comes before r in the alphabet and x comes before y.

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The set of values of x for which the function is defined is called the **domain**.

The **range** of a function is the set of values given by the rule.

domain: x-values	**r**ange: y-values

e.g. 1 Sketch the function $y = f(x)$ where $f(x) = x^2 + 4x - 1$ and write down its domain and range.

Solution: The quickest way to sketch this quadratic function is to find its vertex by completing the square.

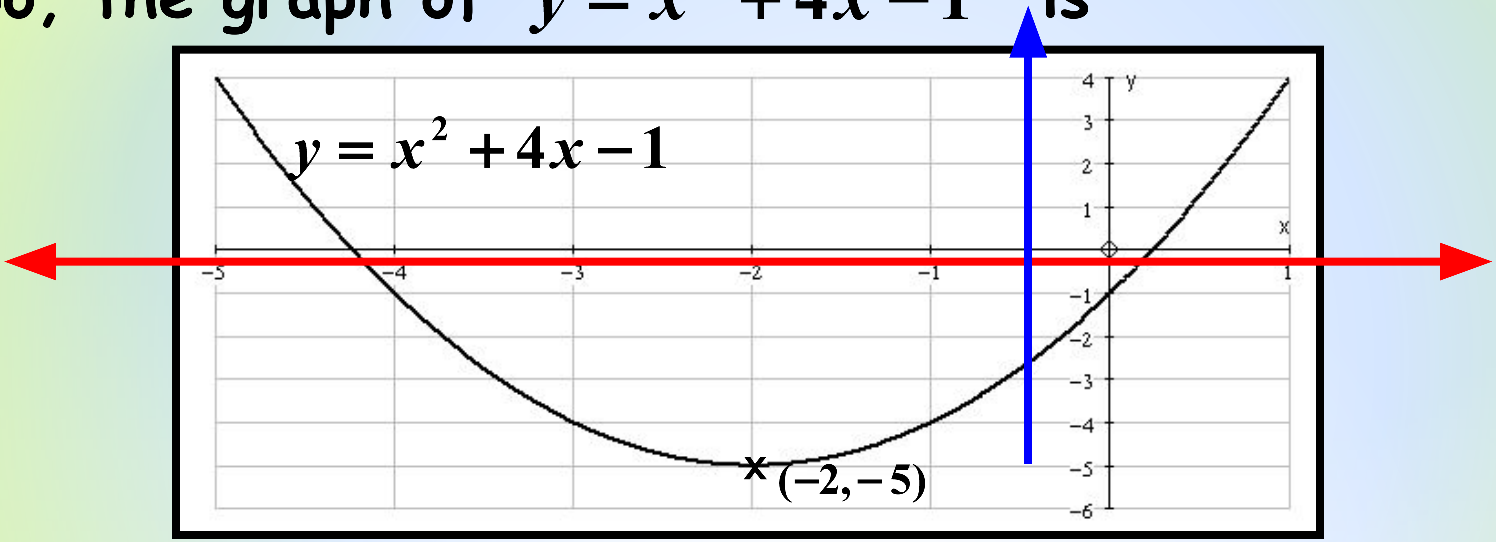
$$y = x^2 + 4x - 1 \quad \Rightarrow \quad y = (x+2)^2 - 4 \; - 1$$

$$\Rightarrow \quad y = (x+2)^2 - 5$$

This is a translation from $y = x^2$ of $\begin{bmatrix} -2 \\ -5 \end{bmatrix}$

so the vertex is $(-2, -5)$.

So, the graph of $y = x^2 + 4x - 1$ is



domain: The x-values on the part of the graph we've sketched go from -5 to $+1$ . . . BUT we could have drawn the sketch for <u>any</u> values of x.

So, we get $x \in \mathbb{R}$ (x is any real number)

BUT there are no y-values less than -5, . . .

so the range is $y \geq -5$

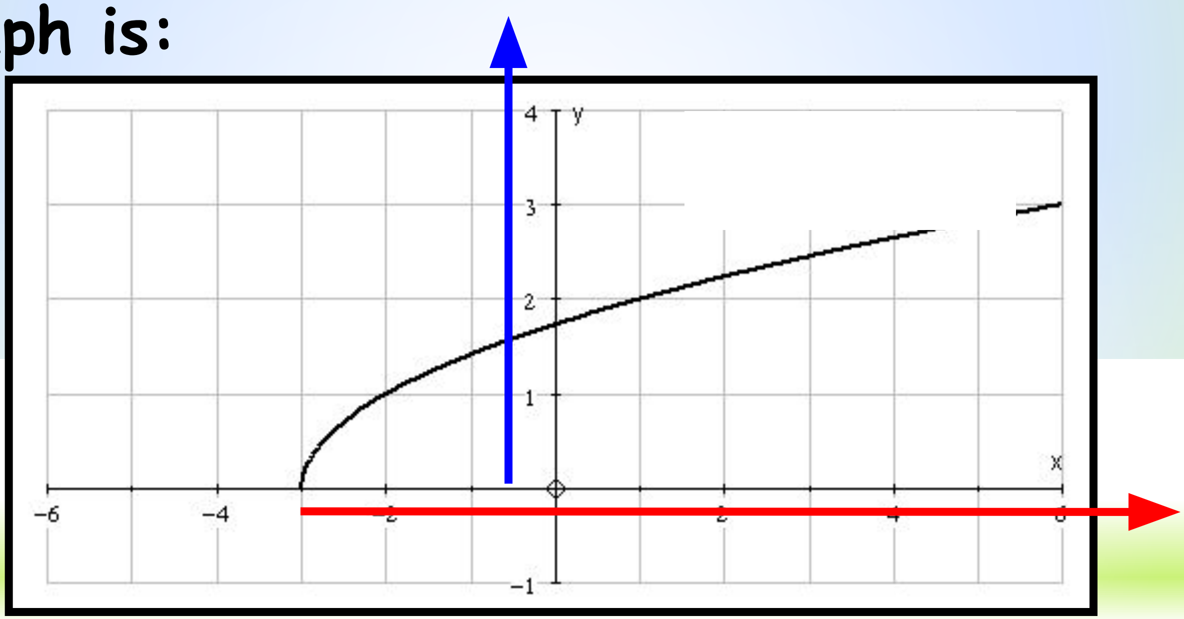
(y is any real number greater than, or equal to, -5)

e.g.2 Sketch the function where .

Hence find the domain and range of

Solution: $y = f(x)$ is a translation from of

so the graph is:



domain: x-values

$x \geq -3$

range: y-values

(We could write instead of y)

SUMMARY

- To define a function we need
  a rule and a set of values.
- Notation:

  means

- For ,

  the x-values form the **domain**

  the or y-values form the **range**

  e.g. For

  the domain is

  the range is

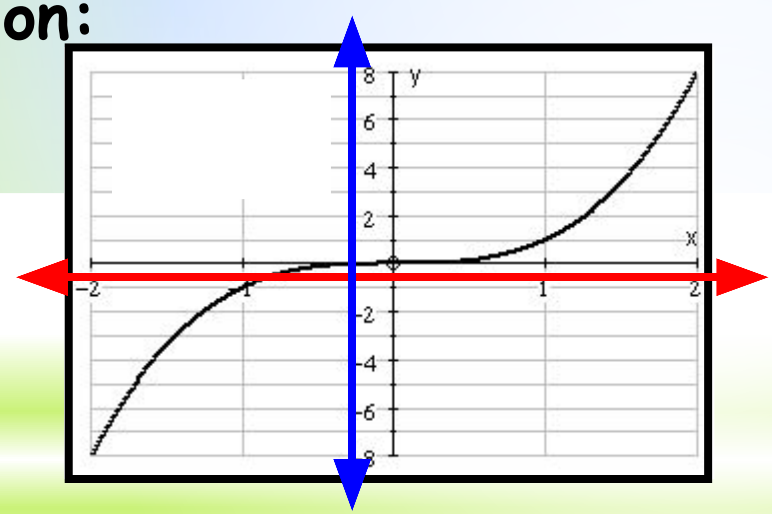
Exercise

1. Sketch the functions where

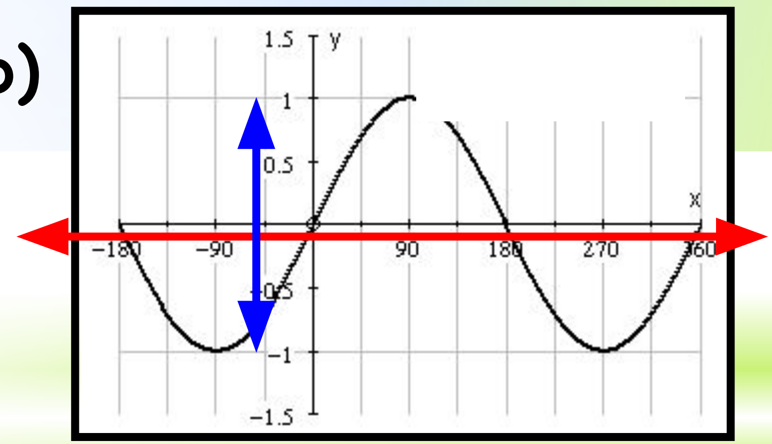
For each function write down the domain and range

Solution:

(a)



(b)



domain: □

range: □

domain: □

range:

We can sometimes spot the domain and range of a function without a sketch.

e.g. For we notice that we can't square root a negative number (at least not if we want a real number answer) so,

$x + 3$ must be greater than or equal to zero.

So, the domain is

The smallest value of is zero.

Other values are greater than zero.

So, the range is

Functions of a Function

Suppose and

then,

x is replaced by 3

Functions of a Function

Suppose and

then,

and

x is replaced by -1
x is replaced by

Functions of a Function

Suppose and

then,

and

We read as "f of g of x"

is "a function of a function" or compound function.

is the inner function and the outer.

x is "operated" on by the inner function first.
So, in we do g first.

Notation for a Function of a Function

is often written as .

does NOT mean multiply g by f.

When we meet this notation it is a good idea to change it to the full notation.

I'm going to write always !

e.g. 1 Given that and find

Solution:

e.g. 1 Given that and find

Solution:

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N.B. is not the same as

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Exercise

1. The functions f and g are defined as follows:

R

(a) What is the range of f ?

(b) Find (i) and (ii)

Solution: (a) The range of f is

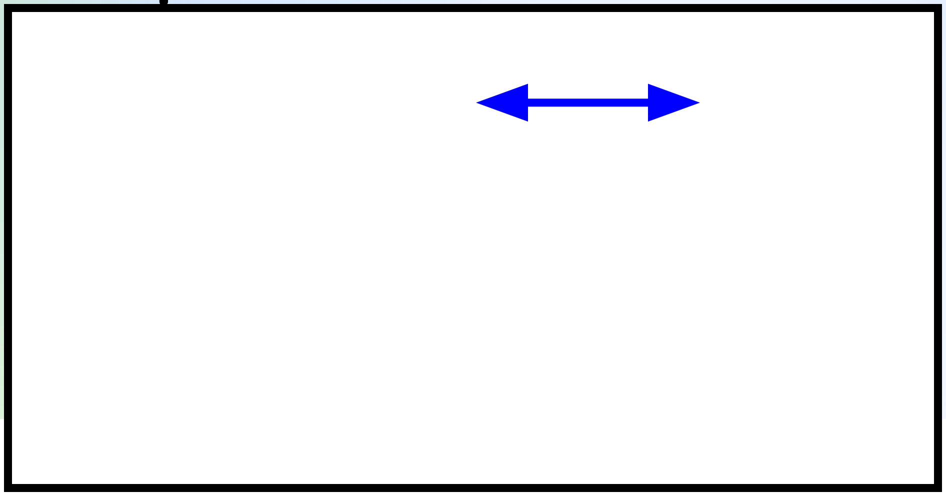
(b) (i)

(ii)

Periodic Functions

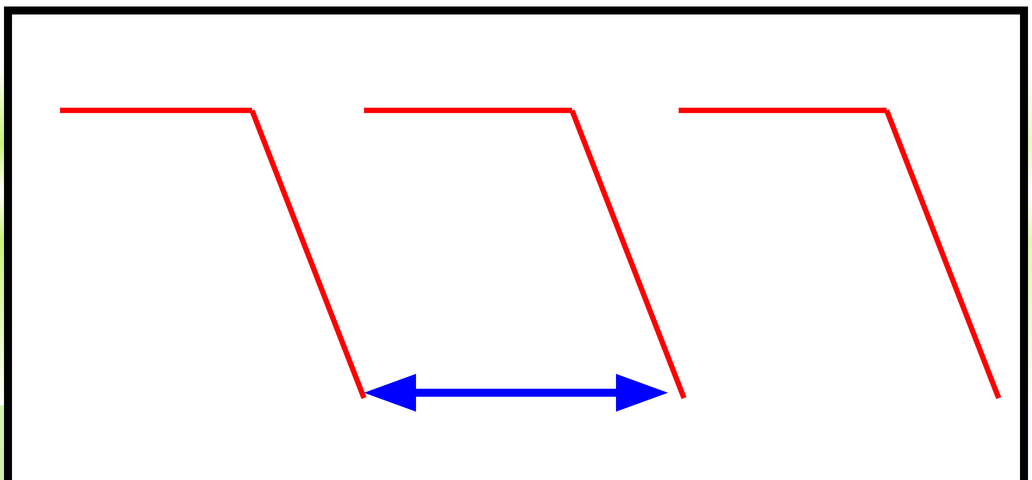
Functions whose graphs have sections which repeat are called periodic functions.

e.g.



peats every radian

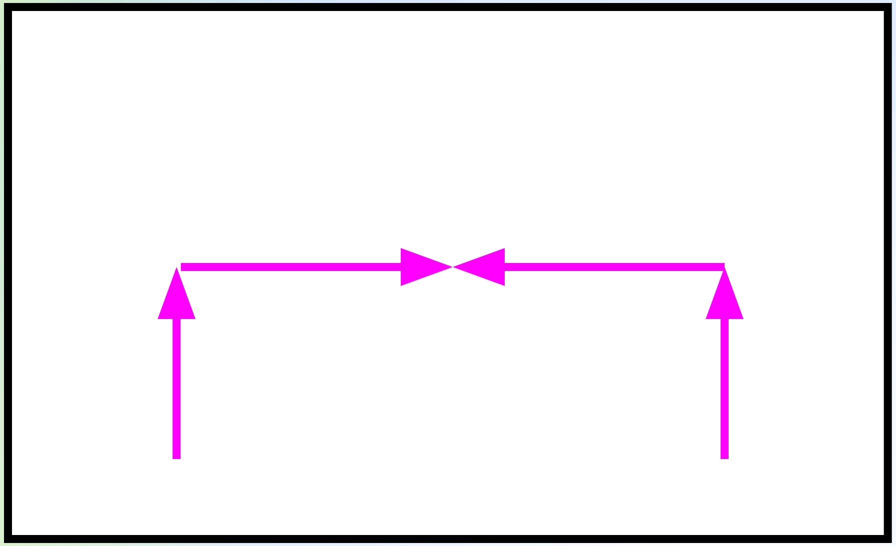
It has a period of



This has a period of 3.

Some functions are $\boxed{\text{even}}$

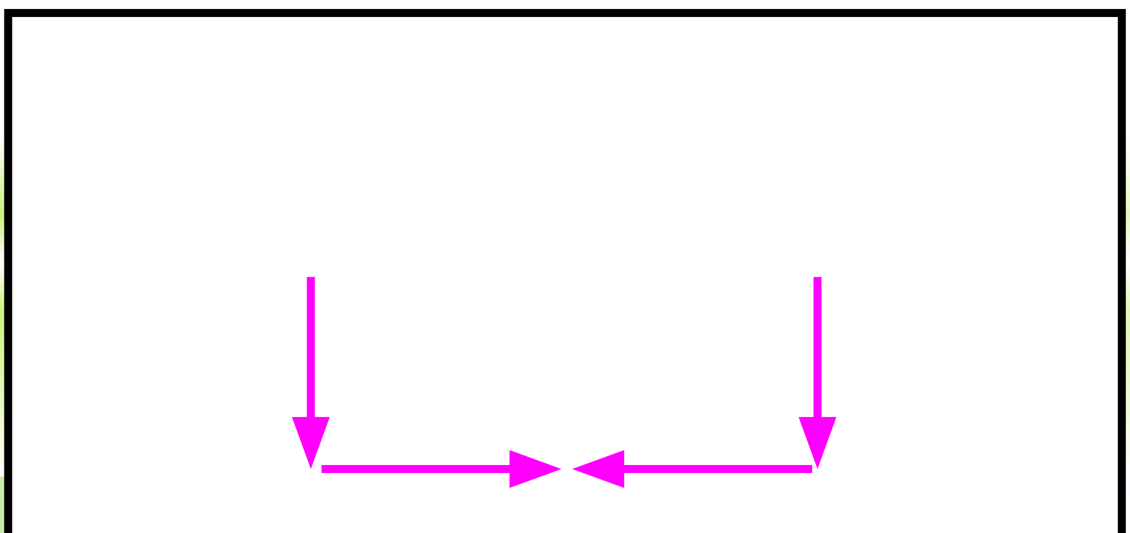
e.g.



Even functions are symmetrical about the y - axis

So,

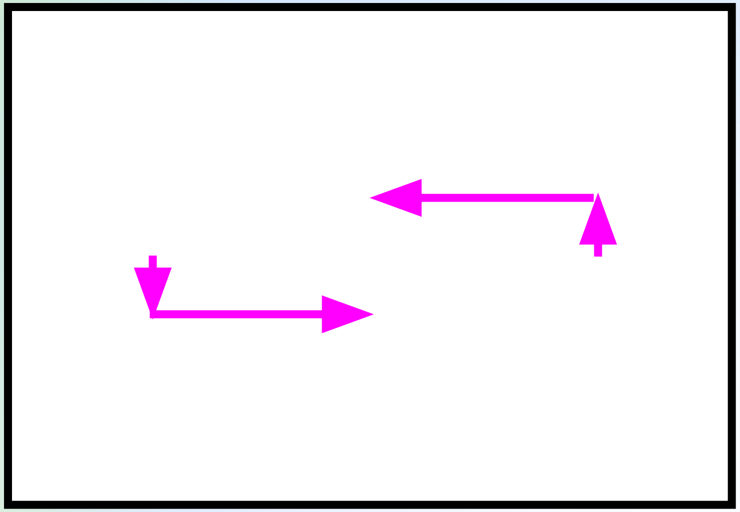
e.g.



e.g.

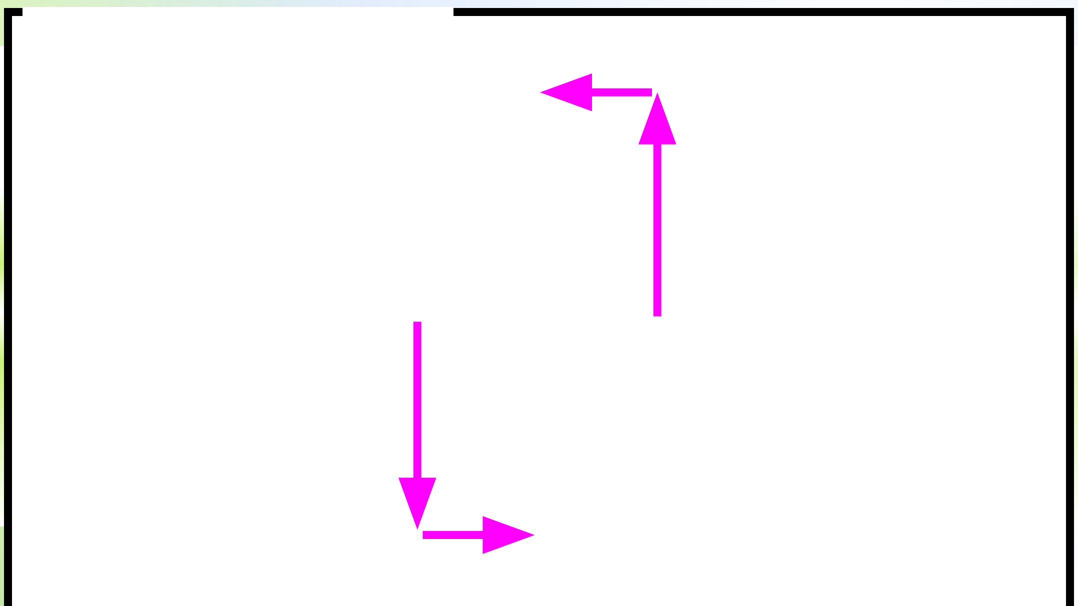
Others are odd

e.g.



Odd functions have 180° rotational symmetry about the origin

e.g.



e.g.

Many functions are neither **even** nor **odd**

e.g.

Try to sketch one even function, one odd and one that is neither. Ask your partner to check.

SUMMARY

- A compound function is a function of a function.
- It can be written as which means
- The inner function is
- is not usually the same as
- is read as "f of g of x".