Discounted cash flow applications

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Net present value (NPV)

- **Net present value** is the sum of the present values of all the positive cash flows minus the sum of the present values of all the negative cash flows.
- **Decision rule**: Accept positive NPV projects they increase shareholder wealth

Steps in calculating NPV

1. Identify all the incremental cash flows associated with the project.
2. Determine the appropriate discount rate.
3. Using that discount rate, calculate the present value of all of the inflows (positive sign) and outflows (negative sign).
4. Sum the present values together → the result is the project's NPV.
5. Apply the NPV decision rule.
   - If you have mutually exclusive projects → accept the one with the highest NPV.

$$\mathrm{NPV} = \sum_{t=0}^{N} \frac{\mathrm{CF}_t}{(1+r)^t}$$

Internal rate of return (IRR)

- The **internal rate of return** is the discount rate that sets the sum of the present value of the positive cash flows equal to the sum of the present value of the negative cash flows.
  - The discount rate at which NPV = 0
- **Interpretation**: IRR is the expected compound return when all intervening cash flows in the project can be reinvested at the IRR and the investment will be held until maturity.
- **Calculation**: In practice, use a spreadsheet, financial software, or financial calculator to determine IRR.
- **Decision**: Accept projects for which IRR > hurdle rate → increases shareholder wealth.

$$\text{NPV} = \sum_{t=0}^{N} \frac{CF_t}{(1+\text{IRR})^t} = 0$$

*The IRR rule can be affected by problems of scale and timing of cash flows.

NPV vs. IRR

- If projects are **independent**, the decision to invest in one does not preclude investment in the other.
  - NPV and IRR will yield the same investment decisions.
- Projects are **mutually exclusive** if the selection of one project precludes the selection of another project project selection is determined by rank.
  - NPV and IRR may give different ranks when
    - The projects have different scales (sizes)
    - The timing of the cash flows differs
  - If projects have different ranks use NPV

Portfolio return measurement

- **Performance measurement** is the calculation of the return on an asset or a portfolio of assets.
  - This is particularly challenging when funds flow into or out of the portfolio or when the composition of the assets changes over time.
  - Holding period returns are one fundamental building block of performance measurement.

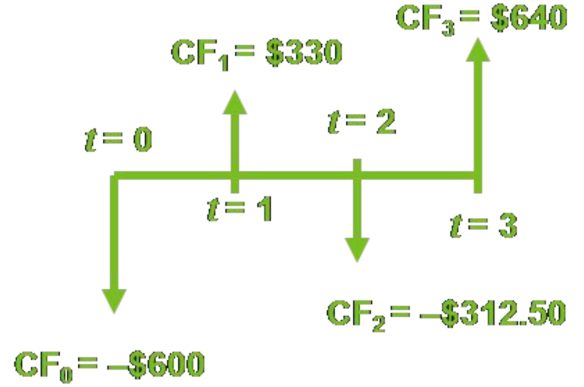
$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0}$$

- **Performance appraisal** is the comparison of the rate of return on an asset or a portfolio of assets to an appropriate required (desired) rate of return.
  - The rate against which the performance is being assessed is often known as a benchmark rate and may be a rate on a reference set of assets, such as an index.

Money-weighted Rate of return

- The money-weighted rate of return is the internal rate of return on a portfolio, taking account of all cash flows.
- Money-weighted rate of return can be appropriate if the investor exercises control over additions and withdrawals to the portfolio.

- Recall: Money-weighted return (MWR below) is inherently an IRR.



$$0 = -\$600 + \frac{\$330}{(1+\mathrm{MWR})^1} + \frac{-\$312.50}{(1+\mathrm{MWR})^2} + \frac{\$640}{(1+\mathrm{MWR})^3}$$

MWR = ?

Time-Weighted Rate of return

- Time-weighted rate of return is the standard in the investment management industry.
- The time-weighted rate of return removes the effects of timing and amount of withdrawals and additions to the portfolio and reflects the compound rate of growth of one unit of currency invested over a stated measurement period.

	Time			
	0	1	2	3
Share Price	\$300	\$325	\$315	\$320
Activity	Buy 2 shr	Div = \$2.50/shr Sell 1 shr after	Div = \$2.50/shr Buy 1 shr	Sell 2 shrs
Change in Value		\$650 + \$5.00 – \$600	\$315 + \$2.50 – \$325	\$640 – 630
Initial Value		\$600	\$325	\$630
HPR		0.09167	–0.02307	0.01587

$$\text{TWR} = \sqrt[3]{(1.09167)(0.97693)(1.01587)} - 1$$

TWR vs. MWR

- Money-weighted returns place greater weight on those periods in which investment is higher and, therefore, give a “better” picture of the actual investor experience.
- Time-weighted returns remove the effect of inflows and outflows to the portfolio and are, therefore, a better indicator of managerial skill.

Differing Money market Yields

- Instruments that mature in less than a year a known as money market instruments
- There are a number of different conventions for calculating yields on money market instruments.
  - Bank discount yield
  - Holding period yield
  - Effective annual yield
  - Money market yield

Bank discount yield

- By convention, Treasury instruments of less than a year in original maturity (T-bills) have yield quoted on a bank discount basis.

- **Bank discount yield** is a percentage of face value instead of price and assumes 360 days in a year.

$$r_{BD} = \frac{D}{F}\left(\frac{360}{t}\right) = \frac{\text{Par} - \text{Price}}{\text{Par}}\left(\frac{360}{t}\right)$$

  - For a \$10,000 par value T-bill trading at \$9,945.70 with 67 days to maturity, the bank discount yield is

$$r_{BD} = \frac{\$10{,}000 - \$9{,}945.70}{\$10{,}000}\left(\frac{360}{67}\right) = 0.0292 = 2.92\%$$

  - Problems: ignores compounding, is a percentage of face value instead of price, and uses 360 days instead of 365.

Holding period yield (HPY)

- HPY measures the (unannualized) return over the remaining life of the investment when P_1 is the return of face value.

$$\text{HPY} = \frac{P_1 - P_0 + D_1}{P_0}$$

- For a $10,000 par value T-bill trading at $9,945.70 with 67 days to maturity, the HPY is

$$\text{HPY} = \frac{\$10{,}000 - \$9{,}945.70}{\$9{,}945.70} = 0.00546 = 0.546\%$$

- Important aside: Make sure the prices used include any accrued interest.

*Treasury bill, HPY = *D/P0*

Effective annual yield (EAY)

- The annualized expression of the holding period yield is

$$\text{EAY} = (1 + \text{HPY})^{365/t} - 1$$

- For a $10,000 par value T-bill trading at $9,945.70 with 67 days to maturity, the EAY is

$$\text{EAY} = (1 + 0.00546)^{365/67} - 1 = 0.03011 = 3.011\%$$

Money market yield

- This convention makes the quoted yield on a T-bill comparable to yield quotations on interest-bearing money market instruments that pay interest on a 360-day basis.
- Also known as the CD equivalent yield.
- Generally, the money market yield will be equivalent to the holding period yield adjusted to a 360-day basis.

$$r_{MM} = \frac{360(r_{BD})}{360 - t(r_{BD})}$$

- For a \$10,000 par value T-bill trading at \$9,945.70 with 67 days to maturity, the money market yield is

$$r_{MM} = \frac{360(0.0292)}{360 - 67(0.0292)} = 0.02936 - 2.936\%$$

*For a Treasury bill, money market yield can be obtained from the bank discount yield using *rMM = (360 x rBD)/(360 − t xrBD).*

Yield conversions

- We can convert back and forth using holding period yield, which is common to all of the calculations.
- Bank discount yield is always less than effective annual yield or money market yield.
- Given any one of HPY, r_{MM}, or r_{BD}, we can calculate the others.

$$\text{HPY} = \left(\frac{360}{t}\right) = r_{MM} = r_{BD}\frac{F}{P_0}$$

- Recall for our bond that $r_{MM} = 0.02936.$
- If HPY = 0.00546, then $r_{MM} = 0.00546\left(\frac{360}{67}\right) = 0.02934.$
- If $r_{BD} = 0.0292$, then $r_{MM} = 0.0292\frac{\$10{,}000}{\$9{,}945.70} = 0.02936.$

Bond-equivalent yield (BEY)

- An annualized periodic yield wherein the process of annualizing is accomplished by multiplying the periodic yield by the number of periods in a year.
- By convention, the yield on most bonds is expressed as a bond-equivalent yield instead of as an equivalent annual yield.
- Problem: This process ignores intraperiod compounding.
- Calculating a BEY:
  - Compute the period yield to maturity (YTM).
  - Multiply the periodic YTM by the number of periods in a year.
    - For example: YTM x 2 for semiannual, YTM x 4 for quarterly