1.1Time value of money

By Dias Kulzhanov

Interest rate

- The interest rate, r, is required rate of return; r is also called the discount rate or opportunity cost.
- The required rate of return on a security = real risk-free rate + expected inflation + default risk premium + liquidity premium + maturity risk premium.
- The interest rate, r, makes current and future currency amounts equivalent based on their time value.
- The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.
- The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.
- The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.

Effective annual rate (EFF%) =
$$\left(1 + \frac{I_{NOM}}{M}\right)^{M} - 1.0$$

 For non-annual time value of money problems, divide the stated annual interest rate by the number of compounding periods per year, *m*, and multiply the number of years by the number of compounding periods per year.

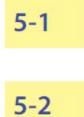
Annuity

- An annuity is a finite set of level sequential cash flows.
- There are two types of annuities, the annuity due and the ordinary annuity. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at *t* = 1).
- On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.
- Annuities may be handled in a similar fashion as single payments if we use annuity factors instead of single-payment factors.

Future value/Present value

Future value = $FV_N = PV(1 + I)^N$

 $\label{eq:Present_value} Present \ value = PV = \ \frac{FV_N}{\left(1+I\right)^N}$



Future Value/Present Value of an Ordinary Annuity

$$\begin{split} FVA_N &= PMT(1+I)^{N-1} + PMT(1+I)^{N-2} \\ &+ PMT(1+I)^{N-3} + \ldots + PMT(1+I)^0 \\ &= PMT \Bigg[\frac{(1+I)^N - 1}{I} \Bigg] \end{split}$$

$$PVA_{N} = PMT/(1 + I)^{1} + PMT/(1 + I)^{2} + \dots + PMT/(1 + I)^{N}$$

$$= \mathsf{PMT} \left[\frac{1 - \frac{1}{(1+I)^{\mathsf{N}}}}{\mathsf{I}} \right]$$

$$PV_{Annuity Due} = PV_{Ordinary Annuity} \times (1+r)$$

$$FV_{Annuity Due} = FV_{Ordinary Annuity} \times (1+r)$$

*FVAn- future value for an ordinary annuity *PVAn- present value for an ordinary annuity

5-5



Perpetual annuities (PV annuities with infinite lives)

PV of a perpetuity = $\frac{PMT}{I}$

PV and FV of Uneven Cash Flow series

It is not uncommon to have applications in investments and corporate finance where it is necessary to evaluate a cash flow stream that is not equal from period to period.

FV of cash flow stream =
$$\sum FV_{individual}$$

5-7
$$PV = \frac{CF_1}{(1+I)^1} + \frac{CF_2}{(1+I)^2} + \dots + \frac{CF_N}{(1+I)^N} = \sum_{t=1}^N \frac{CF_t}{(1+I)^t}$$

5-6

SOLVING TIME VALUE OF MONEY PROBLEMS WHEN COMPOUNDING PERIODS ARE OTHER THAN ANNUAL

5-8Periodic rate(IPER) = $\frac{\text{Stated annual rate}}{\text{Number of payments per year}} = I/M$ 5-9Number of periods = (Number of years)(Periods per year) = NMPeriodic rate = IPER = 0.10/365 = 0.000273973 per dayNumber of days = (9/12)(365) = 0.75(365) = 273.75 rounded to 274Ending amount = \$100(1.000273973)^{274} = \$107.79