1.1Time value of money

By Dias Kulzhanov

Interest rate

- The interest rate, *r,* is required rate of return; r is also called the discount rate or opportunity cost.
- The required rate of return on a security = real risk-free rate + expected inflation + default risk premium + liquidity premium + maturity risk premium.
- The interest rate, *r,* makes current and future currency amounts equivalent based on their time value.
- The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.
- The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.
- The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.

5-10 $$\text{Effective annual rate (EFF\%)} = \left(1 + \frac{I_{NOM}}{M}\right)^{M} - 1.0$$

- For non-annual time value of money problems, divide the stated annual interest rate by the number of compounding periods per year, *m, and multiply the number* of years by the number of compounding periods per year.

Annuity

- An annuity is a finite set of level sequential cash flows.
- There are two types of annuities, **the annuity due** and **the ordinary annuity**. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at $t = 1$).
- On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.
- Annuities may be handled in a similar fashion as single payments if we use annuity factors instead of single-payment factors.

Future value/Present value

5-1 $$\text{Future value} = FV_N = PV(1 + I)^N$$

5-2 $$\text{Present value} = PV = \frac{FV_N}{(1 + I)^N}$$

Future Value/Present Value of an Ordinary Annuity

$$FVA_N = PMT(1+I)^{N-1} + PMT(1+I)^{N-2} + PMT(1+I)^{N-3} + \ldots + PMT(1+I)^0 = PMT\left[\frac{(1+I)^N - 1}{I}\right]$$

5-3

$$PVA_N = PMT/(1+I)^1 + PMT/(1+I)^2 + \cdots + PMT/(1+I)^N$$

$$= PMT\left[\frac{1 - \frac{1}{(1+I)^N}}{I}\right]$$

5-5

$$PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1+r)$$
$$FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1+r)$$

*FVAn- future value for an ordinary annuity
*PVAn- present value for an ordinary annuity

Perpetual annuities(PV annuities with infinite lives)

5-6

$$\text{PV of a perpetuity} = \frac{\text{PMT}}{\text{I}}$$

PV and FV of Uneven Cash Flow series

It is not uncommon to have applications in investments and corporate finance where it is necessary to evaluate a cash flow stream that is not equal from period to period.

$$\text{FV of cash flow stream} = \sum \text{FV}_{\text{individual}}$$

5-7

$$\text{PV} = \frac{\text{CF}_1}{(1+\text{I})^1} + \frac{\text{CF}_2}{(1+\text{I})^2} + \cdots + \frac{\text{CF}_N}{(1+\text{I})^N} = \sum_{t=1}^{N} \frac{\text{CF}_t}{(1+\text{I})^t}$$

SOLVING TIME VALUE OF MONEY PROBLEMS WHEN COMPOUNDING PERIODS ARE OTHER THAN ANNUAL

5-8 $$\text{Periodic rate}(I_{PER}) = \frac{\text{Stated annual rate}}{\text{Number of payments per year}} = I/M$$

5-9 $$\text{Number of periods} = (\text{Number of years})(\text{Periods per year}) = NM$$

$$\text{Periodic rate} = I_{PER} = 0.10/365 = 0.000273973 \text{ per day}$$

$$\text{Number of days} = (9/12)(365) = 0.75(365) = 273.75 \text{ rounded to } 274$$

$$\text{Ending amount} = \$100(1.000273973)^{274} = \$107.79$$