1.1Time value of money

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Interest rate

- The interest rate, *r,* is required rate of return; r is also called the discount rate or opportunity cost.
- The required rate of return on a security = real risk-free rate + expected inflation + default risk premium + liquidity premium + maturity risk premium.
- The interest rate, *r,* makes current and future currency amounts equivalent based on their time value.
- The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.
- The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.
- The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.

5-10 $$\text{Effective annual rate (EFF\%)} = \left(1 + \frac{I_{NOM}}{M}\right)^{M} - 1.0$$

- For non-annual time value of money problems, divide the stated annual interest rate by the number of compounding periods per year, *m, and multiply the number* of years by the number of compounding periods per year.

Annuity

- An annuity is a finite set of level sequential cash flows.
- There are two types of annuities, **the annuity due** and **the ordinary annuity**. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at *t = 1).*
- On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.
- Annuities may be handled in a similar fashion as single payments if we use annuity factors instead of single-payment factors.

Future value/Present value

5-1

$$\text{Future value} = FV_N = PV(1 + I)^N$$

5-2

$$\text{Present value} = PV = \frac{FV_N}{(1 + I)^N}$$

Future Value/Present Value of an Ordinary Annuity

$$
\begin{aligned}
FVA_N &= PMT(1+I)^{N-1} + PMT(1+I)^{N-2} \\
&\quad + PMT(1+I)^{N-3} + \ldots + PMT(1+I)^{0} \\
&= PMT\left[\frac{(1+I)^N - 1}{I}\right]
\end{aligned}
$$

5-3

$$PVA_N = PMT/(1+I)^1 + PMT/(1+I)^2 + \cdots + PMT/(1+I)^N$$

$$= PMT\left[\frac{1 - \frac{1}{(1+I)^N}}{I}\right]$$

5-5

$$\boxed{\begin{aligned} PV_{\text{Annuity Due}} &= PV_{\text{Ordinary Annuity}} \times (1+r) \\ FV_{\text{Annuity Due}} &= FV_{\text{Ordinary Annuity}} \times (1+r) \end{aligned}}$$

*FVAn- future value for an ordinary annuity
*PVAn- present value for an ordinary annuity

Perpetual annuities(PV annuities with infinite lives)

5-6 $$\text{PV of a perpetuity} = \frac{PMT}{I}$$

PV and FV of Uneven Cash Flow series

It is not uncommon to have applications in investments and corporate finance where it is necessary to evaluate a cash flow stream that is not equal from period to period.

$$\text{FV of cash flow stream} = \sum FV_{individual}$$

5-7 $$PV = \frac{CF_1}{(1+I)^1} + \frac{CF_2}{(1+I)^2} + \cdots + \frac{CF_N}{(1+I)^N} = \sum_{t=1}^{N} \frac{CF_t}{(1+I)^t}$$

SOLVING TIME VALUE OF MONEY PROBLEMS WHEN COMPOUNDING PERIODS ARE OTHER THAN ANNUAL

5-8 $$\text{Periodic rate}(I_{PER}) = \frac{\text{Stated annual rate}}{\text{Number of payments per year}} = I/M$$

5-9 $$\text{Number of periods} = (\text{Number of years})(\text{Periods per year}) = NM$$

$$\text{Periodic rate} = I_{PER} = 0.10/365 = 0.000273973 \text{ per day}$$

$$\text{Number of days} = (9/12)(365) = 0.75(365) = 273.75 \text{ rounded to } 274$$

$$\text{Ending amount} = \$100(1.000273973)^{274} = \$107.79$$