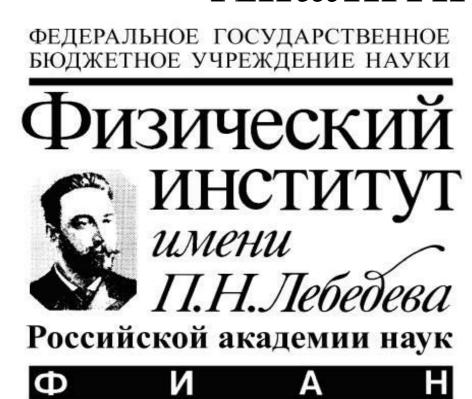# Аналитическая модель асимптотически радиальной структуры пульсарного ветра

Л.И.Арзамасский, В.С.Бескин, В.В.Прокофьев

Физический институт им. П.Н. Лебедева Российской академии наук

Московский Физико-Технический Институт

ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ УЧРЕЖДЕНИЕ НАУКИ

Физический ИНСТИТУТ имени П.Н.Лебедева Российской академии наук

Ф И А Н

МФТИ

научно-образовательный центр
фундаментальные частицы и астрофизика

SAPERE AUDE

Абстракт

Within the force-free approximation we obtain simple asymptotic solutions of the Grad-Shafranov equation for quasi-spherical pulsar wind. We show that the shape of current sheet does not depend on the radial structure of magnetic field.

For the internal region of the current sheet in the pulsar wind we use two-fluid approximation. Passing into the comoving reference frame we determine electric and magnetic field structure as well as the velocity component perpendicular to the sheet. It allows us to estimate the efficiency of particle acceleration. Finally, investigating the motion of individual particles in the time-dependent current sheet we find self-consistently the width of the sheet and its time evolution.

1. Введение

Core element of pulsar wind is believed to be current sheet separated opposite directed magnetic flux [1, 2, 3]. According to numerical simulation[4], Lateral distribution of pulsar wind luminosity per unit solid angle differs from the simplest "split-monopole" model. In cases of large angles between magnetization and rotation axis radial magnetic field and energy flux concentrates near equatorial plane. This has necessitated the construction of a more realistic model allowed us to obtain main parameters of pulsar wind for arbitrary geometry.

Another important issue is an internal structure of current sheet. Existing models do not solve it either analytically [3] or numerically [4]. We will describe internal structure of pulsar wind in comoving frame in order to eliminate dominant components of the electromagnetic fields, prevented us from distinguishing electromagnetic field of current sheet.

2. Бессиловое приближение

To obtain configuration of magnetosphere in axisymmetric case we need to solve one scalar equation. It is so-called Grad-Shafranov equation [6].

$$, -\left(1 - \frac{\Omega_F^2 \varpi^2}{c^2}\right)\nabla^2\Psi + 2\frac{1}{\varpi}\frac{\partial\Psi}{\partial\varpi} + \frac{\varpi^2\Omega_F}{c^2}(\nabla\Psi)^2\frac{d\Omega_F}{d\Psi} - \frac{16\pi^2}{c^2}I\frac{dI}{d\Psi} = 0$$ totic

series:
$$\Psi(r,\theta) = \sum_{n=0}^{\infty}\Psi_n(\theta)\left(\frac{R_L}{r}\right)^{2n}$$

If we put first term of this series as *Ψ0(r; θ)* we will obtain following equation for next term:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Psi_1}{d\theta}\right) - \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\frac{\Psi_0'}{\sin\theta}\right) = \Psi_1\left(\cot^2\theta + 3\frac{\Psi_0''}{\Psi_0'}\cot\theta + \frac{\Psi_0'''}{\Psi_0'} - 3\right)$$

According to data from numerical simulation [4] we may put $\frac{d\Psi_0}{d\theta} = \sin^m\theta$ m=m(\alpha), where \alpha is angle between magnetization and rotation axis. In case of large \alpha from[4] we obtain m=2 thus $B_p(\theta)$ proportional to sin θ.

In oblique case we verified that configuration:

$$B_r = \frac{1}{2\pi r^2}\frac{\partial\Psi}{\partial\theta}\sin^{m-1}\theta\,\text{sign}(\Phi) \quad B_\phi = E_\theta = -\frac{\Omega}{2\pi cr}\sin^m\theta\,\text{sign}(\Phi)$$

$$\Phi = \sin\alpha\sin\theta\sin(\varphi - \Omega t + \Omega r/c) + \cos\theta\cos\alpha$$

satisfied Maxwell equation and force-free assumption. The shape of current sheet is similar to Bogovalov solution [3]. In more realistic solution with smoother function than sign(*Φ*) Maxwell equations remain true though force free assumption is corrupt. It means that inside the current sheet we need to analyze more precisely without the force-free assumption.

3. Поля в движущейся системе отсчета.

To study mechanism of particle acceleration inside current sheet we will considerate following solution od Maxwell equation within force-free approximation for arbitrary function *f(r − ct)* [5]:

$$B_\phi = E_\theta = -\frac{B_s\Omega R_s^2}{cr}\sin^m\theta f(r-ct)$$

We can use this solution in orthogonal case because shame of the current sheet in this case is similar to spherical wave. We modify this solution in order to work with a current sheet moving with a speed βc. We also neglect B_r as we are interested only in pulsar wind structure. In this case configuration changes to

$$\beta B_\phi = E_\theta = -\frac{B_s\Omega R_s^2}{cr}\sin^m\theta f(r-\beta ct)$$

In next step we write field configuration in reference frame moving with a velocity *βc* at the angle *Θ* to equatorial plane. If we put *f(ξ) = tanh(ξ/Δ)*, we obtain

$$E_x = \frac{zB_s\Omega R_s^2\sin^m\Theta}{t^2\beta^2\gamma^2}\tanh\left(\frac{x}{\gamma\Delta}\right) \qquad B_y = \frac{B_s\Omega R_s^2\sin^m\Theta}{t\beta^2\gamma^2}\tanh\left(\frac{x}{\gamma\Delta}\right)$$

In this case two of Maxwell equations without currents and charges will satisfy.

4. Оценка ускорения частиц

In comoving frame we put :

$$B_y = \frac{A}{ct}f(x,t) \quad E_x = \frac{A}{c^2t^2}zf(x,t) \quad A = \frac{B_s\Omega R_s^2}{c\gamma^2\beta^2}$$

Since density proportional to $1/t^2$, f(x,t) is proportional to $1/t$. We put $f(x,t) = x/kct$ To satisfy Maxwell equations we need to add z-component of electric field. Electric field in center of sheet $E_z = kA/2ct$ In ultrarelativistic case we find $k = 1/4\lambda\gamma$ and thickness of current sheet
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As estimation of particles acceleration we obtain expression for derivative of gamma factor of particles at the initial time

$$\left.\frac{d\gamma}{dt/t_0}\right|_{t=t_0} = \frac{\sigma}{2\gamma}$$

Where $\sigma = \gamma^3$ outside of fast magnetosonic surface[6]. This estimate shows that acceleration of particles is large enough so our assumption of ultrarelativistic velocities are correct.

After formation of current sheet when pressure inside it small it began to collapse thus density start increase. Natural to assume that the final thickness of sheet will be comparable to skin depth c/ω_p. In this case particle concentration inside sheet will be $\sigma^{1/3}$ times larger than outside the sheet..

5. Заключение

Within two fluid MHD assumption we investigate particle motion inside current sheet. In particular
• obtain fields in commoving frame,
• shown existence of accelerating electric component inside sheet,
• shown that electric field accelerate particle to ultrarelativistic velocities.
• given the estimations of width of current sheet and density inside it. нем.

In force-free assumption we build the model consist with numerical simulations:
•We obtain the equation for second term of asymptotic series.
• We show that for oblique case the shape of current sheet is similar to shape obtaining in [3] and remains universal.

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