

Physics 1

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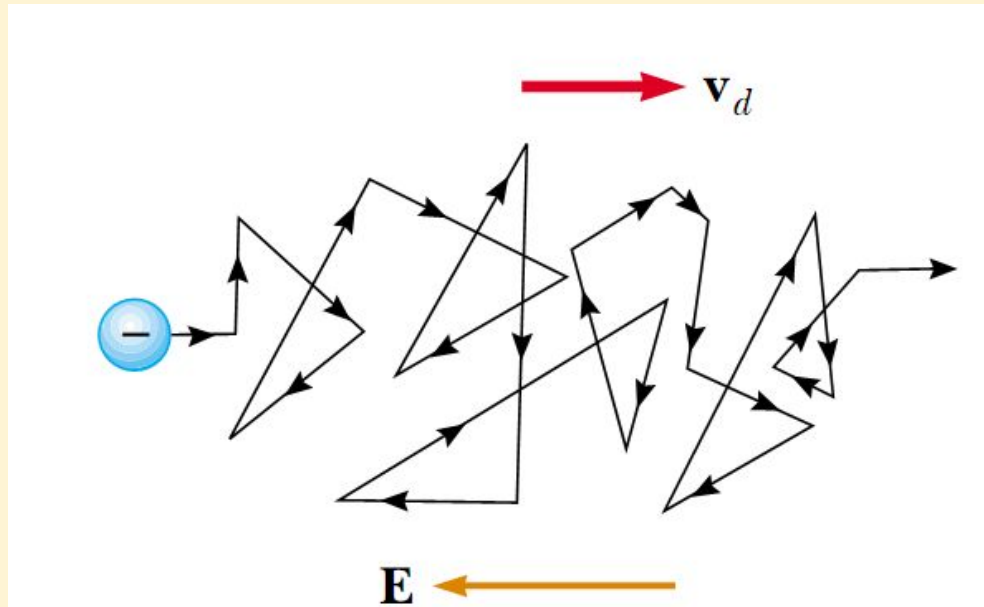
Lecture 11

- Currents in Metals
- The effects of magnetic fields.
- The production and properties of magnetic fields.

Types of Conductivity

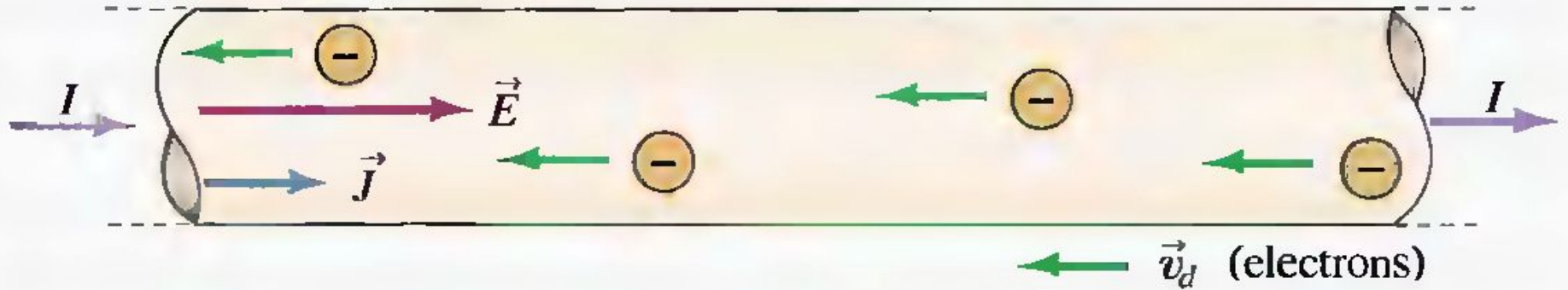
- ***Conductors*** are materials through which charge moves easily.
- ***Insulators*** are materials through which charge does not move easily.
- ***Semiconductors*** are materials intermediate to conductors and insulators.

Drift speed of electrons

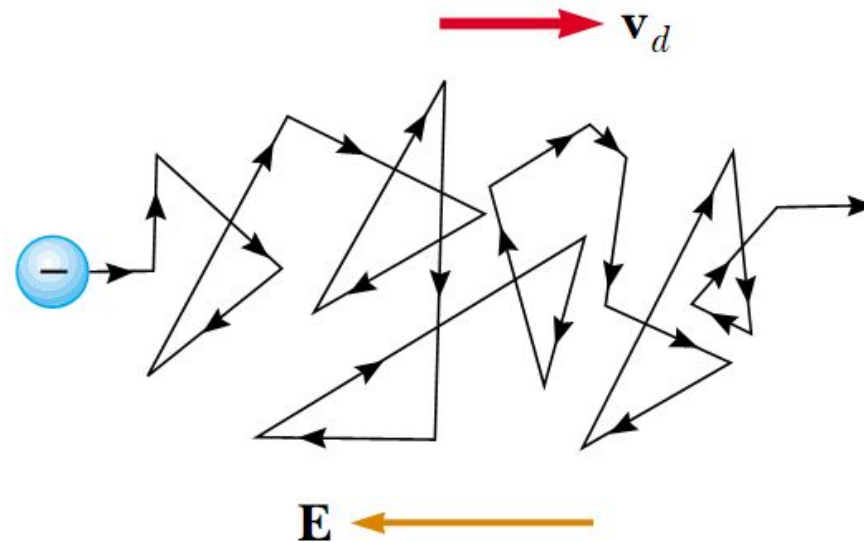


- There is a zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. The net motion – **drift speed** of the electron is opposite the direction of the electric field.

- So when we consider electric current as a flow of electrons:



In reality there happens zigzag motion of free electrons in the metal:



Current in metals

- Every atom in the metallic crystal gives up one or more of its outer electrons. These electrons are then free to move through the crystal, colliding at intervals with stationary positive ions, then the resistivity is:

$$\rho = m/(ne^2\tau)$$

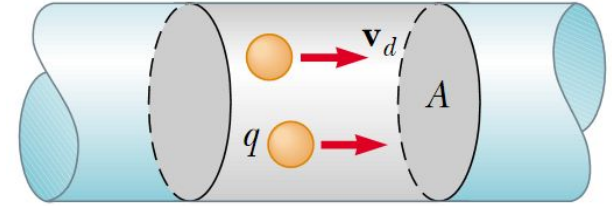
n - the number density of free electrons,

m and **e** – mass and charge of electron,

τ – average time between collisions.

Resistivity

- A conductor with current:



- Current density:

$$J \equiv \frac{I}{A} = nqv_d$$

$$\mathbf{J} = nq\mathbf{v}_d$$

- I – electric current
- A – the cross-sectional area of the conductor
- v_d – drift speed

$$\mathbf{E} = \rho\mathbf{J}$$

ρ - resistivity

Conductivity

- A current density \mathbf{J} and an electric field \mathbf{E} are established in a conductor whenever a potential difference is maintained across the conductor:

$$\mathbf{J} = \sigma \mathbf{E}$$

σ is conductivity:

$$\sigma = 1 / \rho.$$

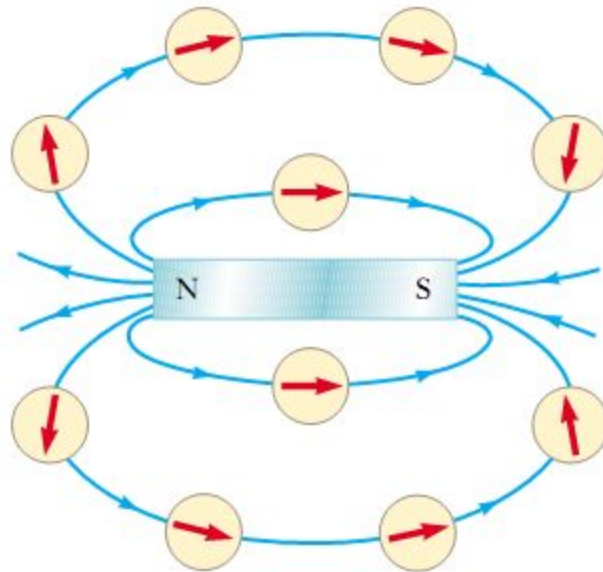
Ohm's law again

- For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current:

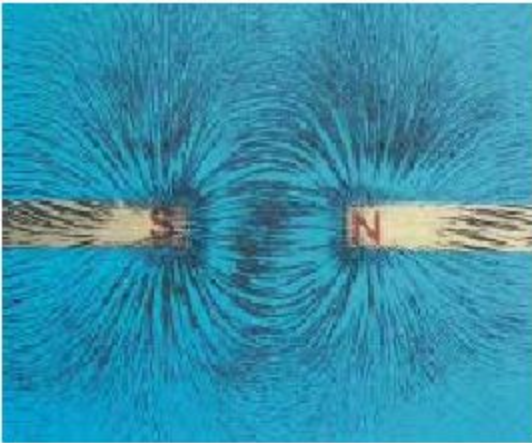
$$\mathbf{J} = \sigma \mathbf{E}$$

Magnets

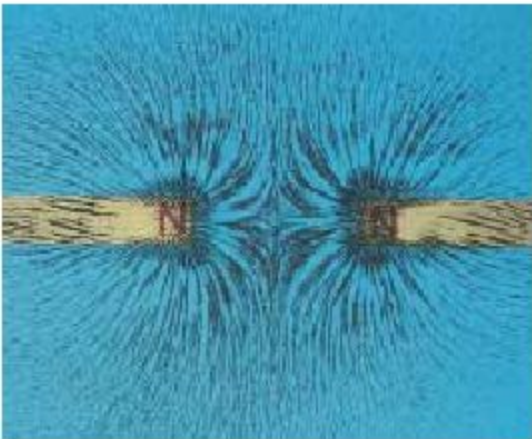
- A single magnetic pole has never been isolated. Magnetic poles are always found in pairs.
- The direction of magnetic field is from the North pole to the South pole of a magnet.



Magnet Poles



- Magnet field lines connect unlike poles.



- Magnet field lines repels from like poles.

Magnet Force

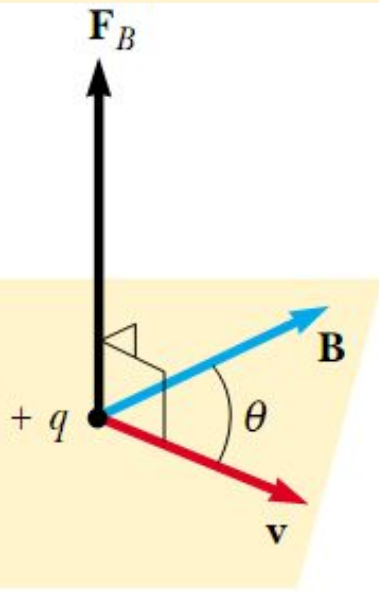
- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- The magnitude and direction of F_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field B .
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both v and B .
- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin \theta$, where θ is the angle the particle's velocity vector makes with the direction of B .

The text in the previous slide can be summarized as:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

So the units for B are:

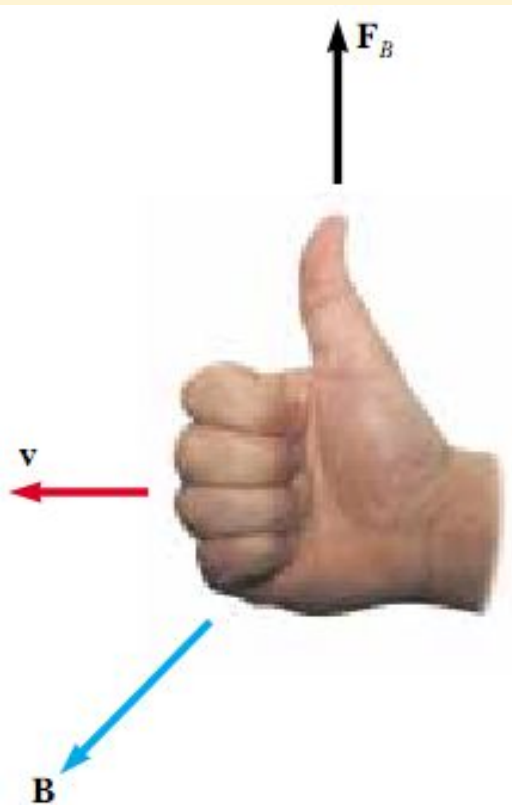
$$1 \text{ T} = 1 \frac{\text{N}}{\text{C} \cdot \text{m/s}}$$



magnetic force is
perpendicular to both \mathbf{v} and \mathbf{B} .

$$F_B = qvB \sin \theta$$

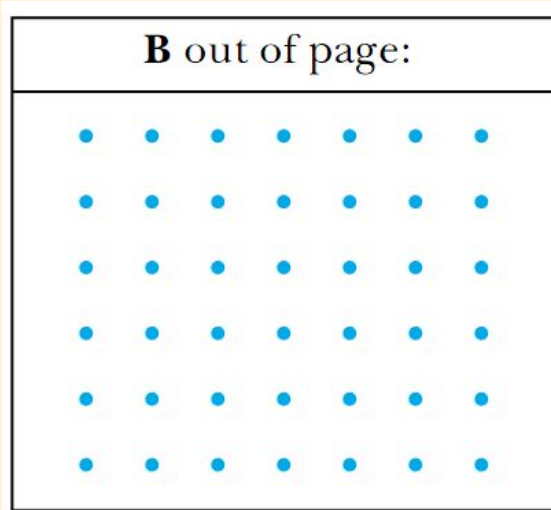
Direction of F_B



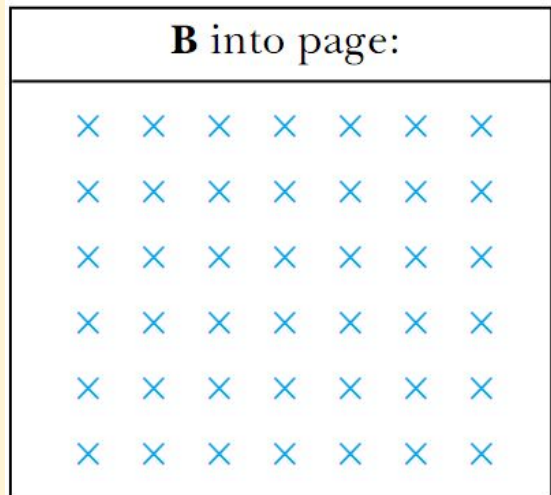
- Right hand rule:

The fingers point in the direction of \mathbf{v} , with \mathbf{B} coming out of your palm, so that you can curl your fingers in the direction of \mathbf{B} . The direction of $\mathbf{v} \times \mathbf{B}$, and the force on a positive charge, is the direction in which the thumb points.

Magnetic field direction



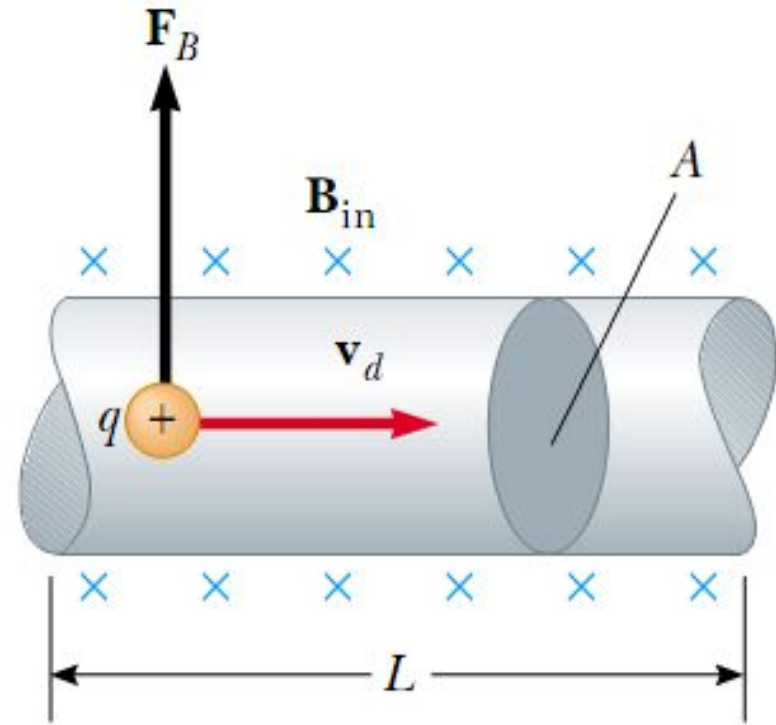
- Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward.



- Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

Magnetic Force on a Current

- Magnetic force is exerted on a single charge moving in a magnetic field. A current-carrying wire also experiences a force when placed in a magnetic field. This follows from the fact that the current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charges making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.



- n is the number density of charged particles q
- v_d is the drift speed of q
- A – area of the segment
- L – the length of the segment
- Then AL is the volume of the segment, and

• nAL is the number of charged particles q .

• Then the net force acting on all moving charges is:

$$\mathbf{F}_B = (q\mathbf{v}_d \times \mathbf{B}) nAL$$

Arbitrary shaped wire

- The force on a small segment of an arbitrary shaped wire is:

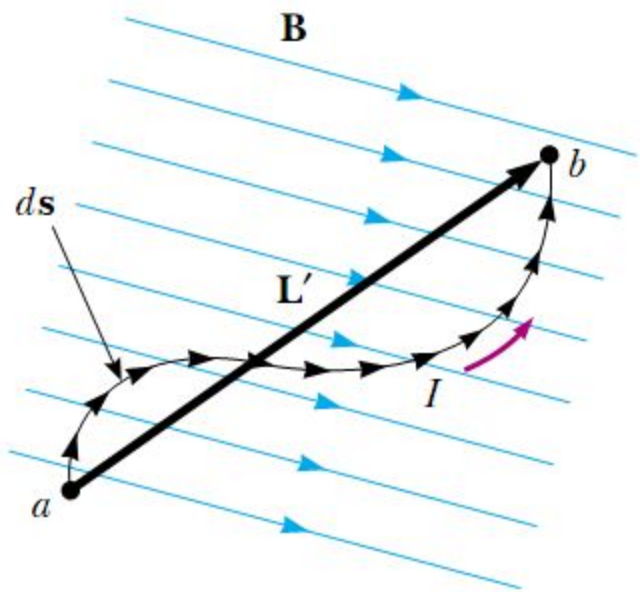
$$d\mathbf{F}_B = I d\mathbf{s} \times \mathbf{B}$$

- The total force is:

$$\mathbf{F}_B = I \int_a^b d\mathbf{s} \times \mathbf{B}$$

- a and b are the end points of the wire.

Curved Wire in a Uniform Magnetic field



s uniform:

$$\mathbf{F}_B = I \left(\int_a^b d\mathbf{s} \right) \times \mathbf{B}$$

$$\mathbf{F}_B = I \mathbf{L}' \times \mathbf{B}$$

- The magnetic force on a curved current-carrying wire in a uniform magnetic field is equal to that on a straight wire connecting the end points and carrying the same current.

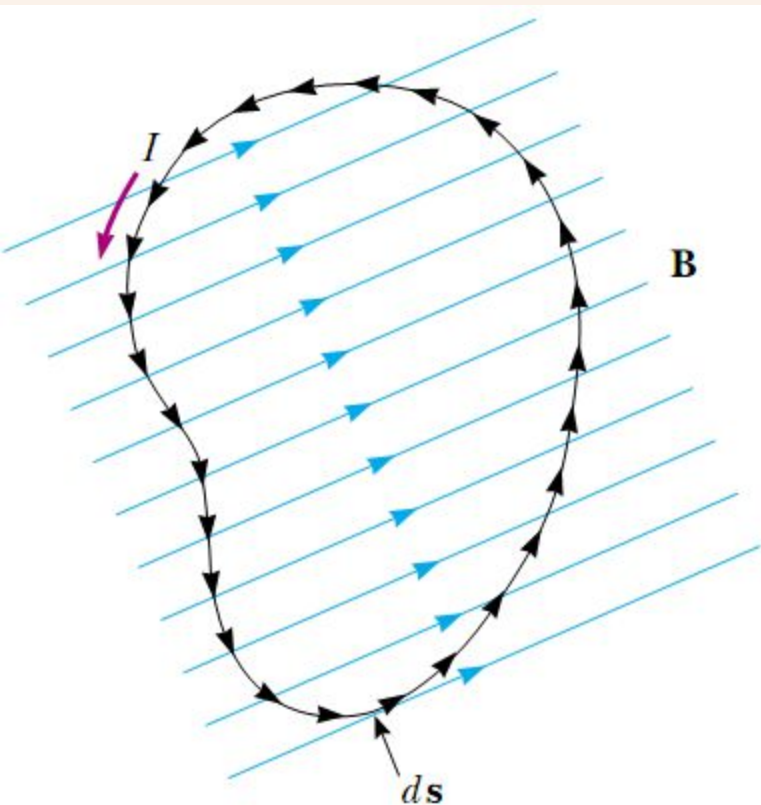
Magnetic force on a straight wire

So, the force on a straight wire in a uniform magnetic field is:

$$\mathbf{F}_B = I \mathbf{L} \times \mathbf{B}$$

- $\mathbf{L} \times \mathbf{B}$ is a vector multiplication.
- Where \mathbf{L} is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment. This expression applies only to a straight segment of wire in a uniform magnetic field.

Loop Wire in a Uniform Magnetic field



- The net magnetic force acting on any closed current loop in a uniform magnetic field is zero:

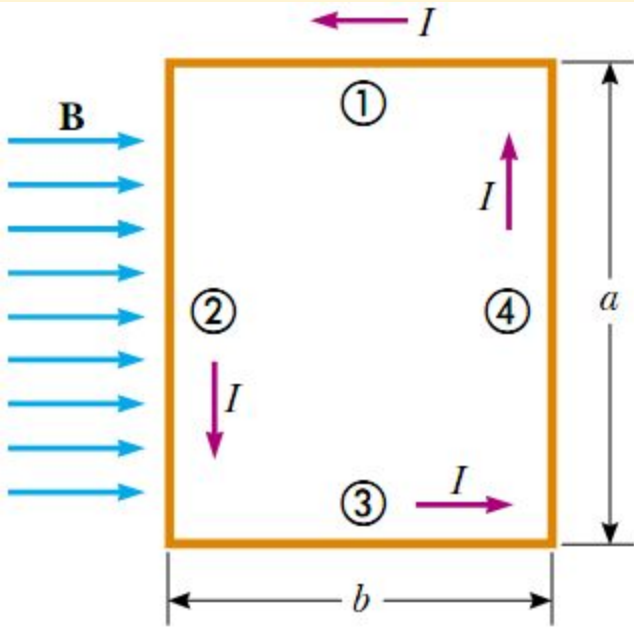
$$\mathbf{F}_B = I \left(\oint d\mathbf{s} \right) \times \mathbf{B}$$

$$\oint d\mathbf{s} = 0$$

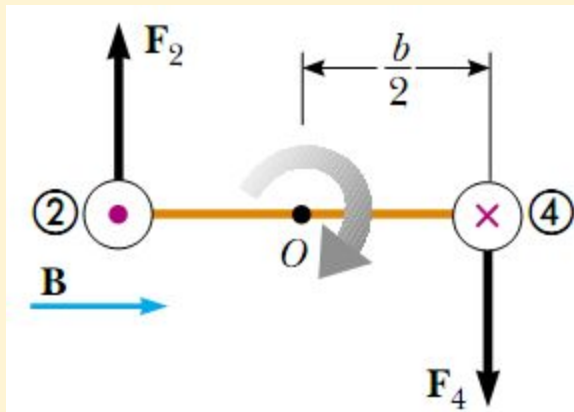
- Then the net force is zero:

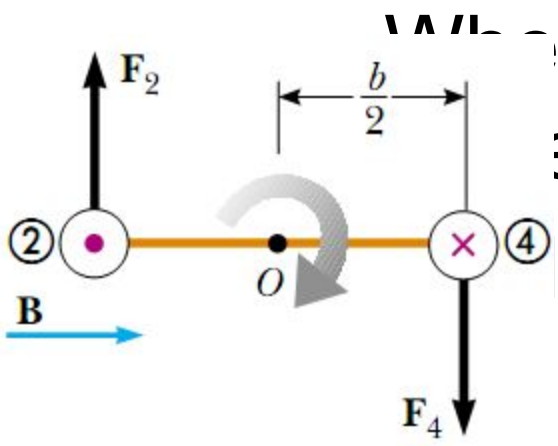
$$\mathbf{F}_B = 0$$

Current Loop Torque in a Uniform Magnetic Field



- Overhead view of a rectangular loop in a uniform magnetic field.
- Sides 1 and 3 are parallel to magnetic field, so only sides 2 and 4 experience magnetic forces.
- Magnet forces, acting on sides 2 and 4 create a torque on the loop.

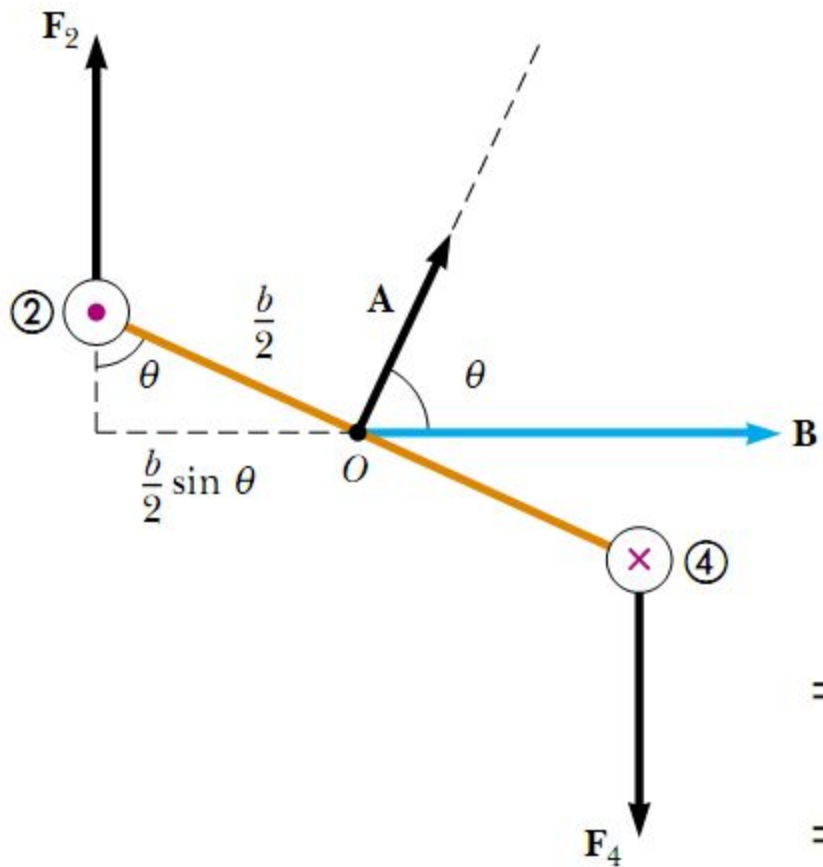




When the magnetic field is in the plane of the loop, the torque on the loop is:

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB) \frac{b}{2} + (IaB) \frac{b}{2} = IabB$$

$$\tau_{\max} = IAB$$



The loop is not
 magnetic field, i.e. the
 between **A** and **B** is

$$\tau = F_2 \frac{b}{2} \sin \theta + F_4 \frac{b}{2} \sin \theta$$

$$= IaB \left(\frac{b}{2} \sin \theta \right) + IaB \left(\frac{b}{2} \sin \theta \right)$$

$$= IabB \sin \theta = IAB \sin \theta$$

This formula is correct for a rectangular loop,
 but for a planar loop of any shape.

$$\boldsymbol{\tau} = I \mathbf{A} \times \mathbf{B}$$

Area Vector

In formula for torque

we have vector **A**: $\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$

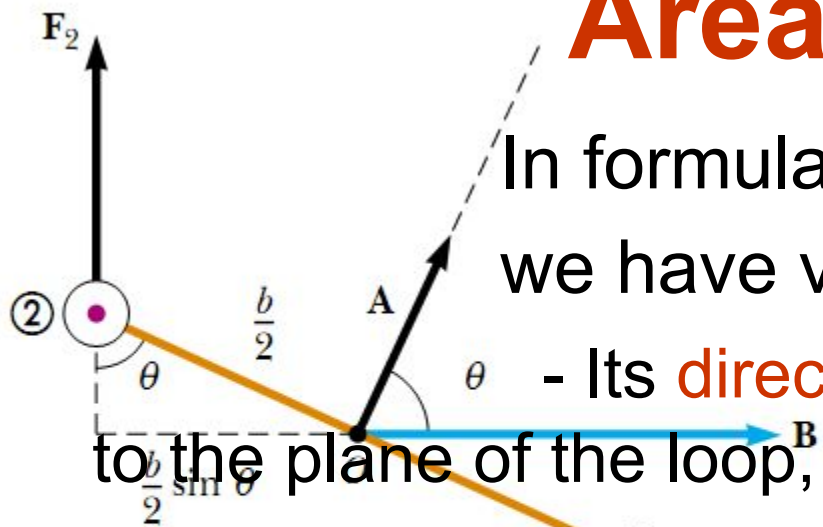
- Its **direction** is perpendicular

to the plane of the loop,

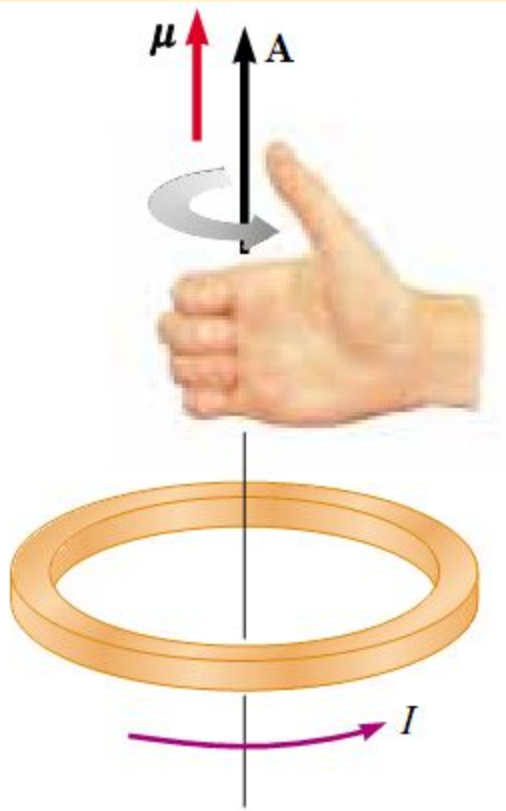
- Its **magnitude** is equal to the

area of the loop.

- We determine the **direction** of **A** using the **right-hand rule**. When you curl the fingers of your right hand in the direction of the current in the loop, your thumb points in the direction of **A**.



Right – hand rule for loop



The direction of the
moment is the same as
A.

the

Magnetic Moment

- The vector product $I\mathbf{A}$ is defined to be the magnetic dipole moment $\boldsymbol{\mu}$ (often simply called the “magnetic moment”) of the current loop:

$$\boldsymbol{\mu} = I\mathbf{A}$$

- Then the torque on a current-carrying loop is:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

Potential Energy of a Magnetic Moment

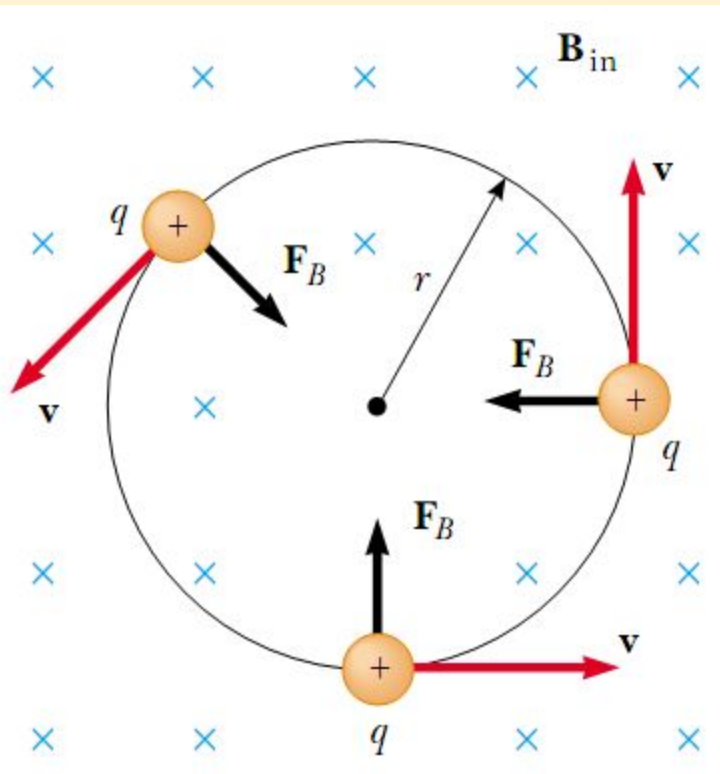
- The potential energy of a system having magnetic dipole μ in the magnetic field \mathbf{B} is:

$$U = -\mu \cdot \mathbf{B}$$

- Here we have **scalar** product $\mu \mathbf{B}$. Then the lowest energy is when μ points as \mathbf{B} , the highest energy is when μ points opposite \mathbf{B} :

$$U_{\min} = -\mu B$$
$$U_{\max} = +\mu B$$

Motion of a Charged Particle in a Uniform Magnetic Field



$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

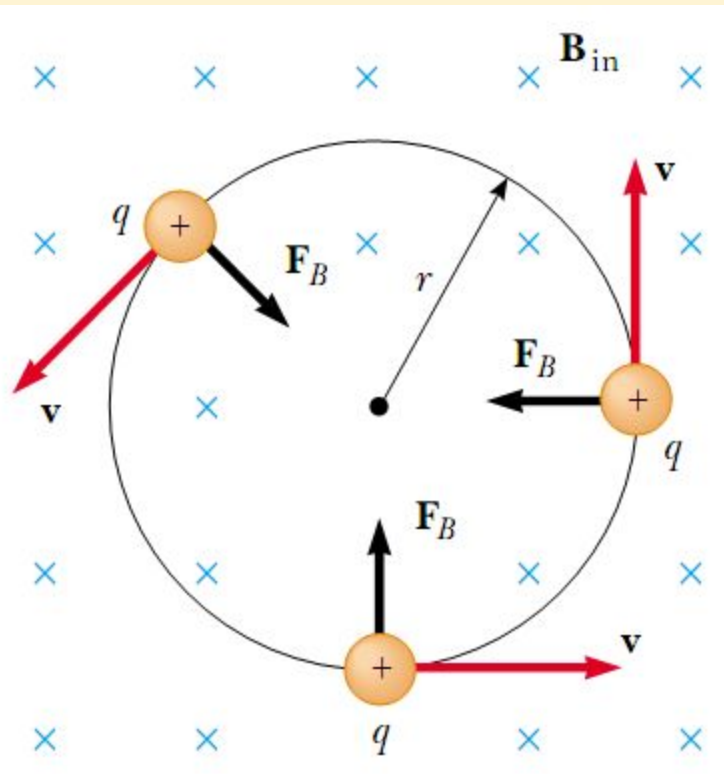
the velocity of a

is perpendicular

magnetic field, the particle moves

in a circular path in a plane perpendicular to \mathbf{B} .

The magnetic force \mathbf{F}_B acting on the charge is always directed toward the center of the circle.



the obtained formula

$$r = \frac{mv}{qB}$$

the angular velocity

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

here \mathbf{v} is perpendicular to \mathbf{B} .

Lorentz Force

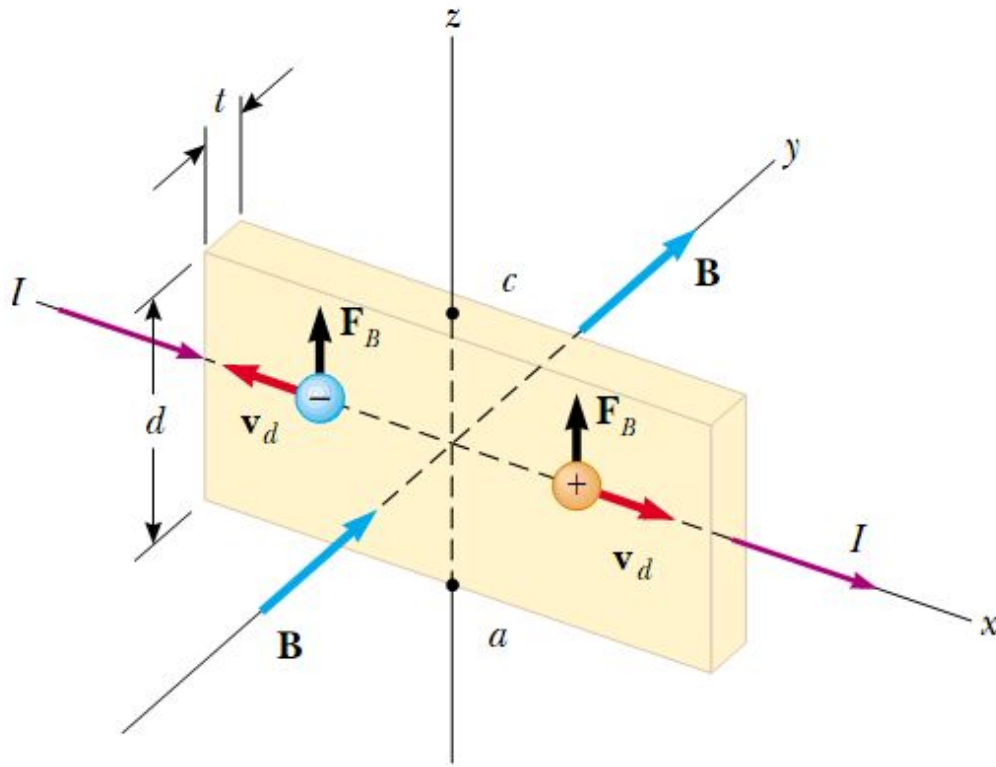
- In the presence of \mathbf{E} and \mathbf{B} , the force acting on a charged particle is:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

- here q is the charge of the particle,
- v – the speed of the particle,
- \mathbf{E} – electric field vector
- \mathbf{B} – magnetic field vector

The Hall Effect

- When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.



force

has magnitude

balanced

$$qv_d B = qE_H$$

$$E_H = v_d B$$

of the conductor:

$$\Delta V_H = E_H d = v_d B d$$

n – charge density: $v_d = \frac{I}{nqA}$ charge carrier drift speed.

then we obtain the Hall voltage:

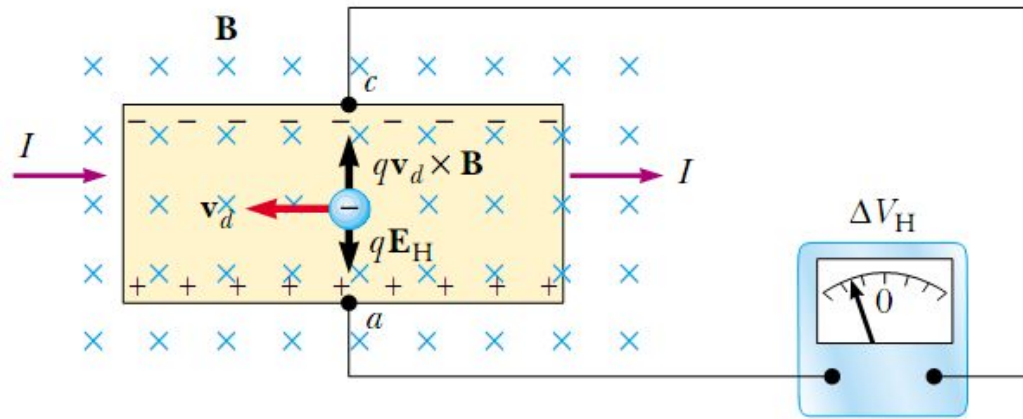
$$\Delta V_H = \frac{IBd}{nqA}$$

Using that $A=td$ – cross sectional area of the conductor,
 t – thickness of the conductor we can obtain:

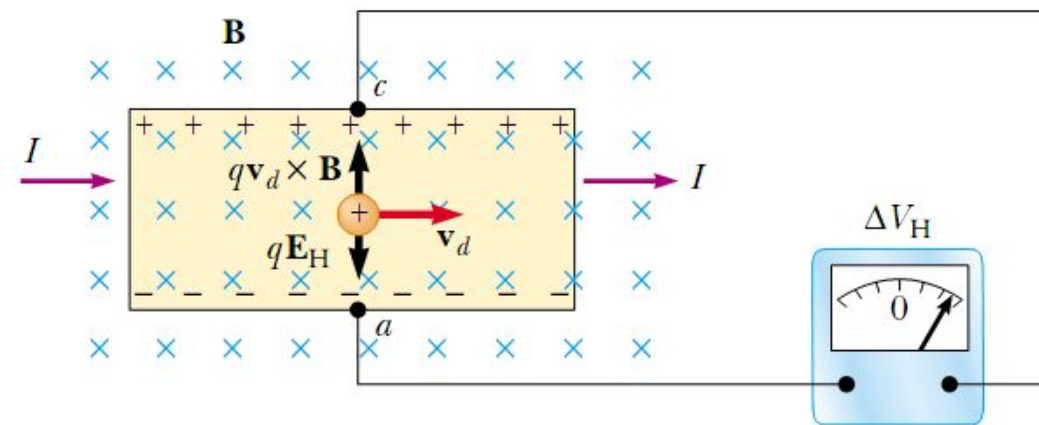
$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

R_H is the Hall coefficient:

$$R_H = 1/(nq)$$



When the charge carriers in a Hall-effect apparatus are negative, the upper edge of the conductor becomes negatively charged, and **c** is at a lower electric potential than **a**.



When the charge carriers are positive, the upper edge becomes positively charged, and **c** is at a higher potential than **a**.

Units in Si

- Magnetic field B $T = N \cdot s / (C \cdot m)$
 $T = N / (A \cdot m)$
- Electric Field E $V/m = N/C$
- Number density n $1/m^3$
- Torque τ $N \cdot m$