Двухконтактный квантовый интерферометр и процессы в нем.

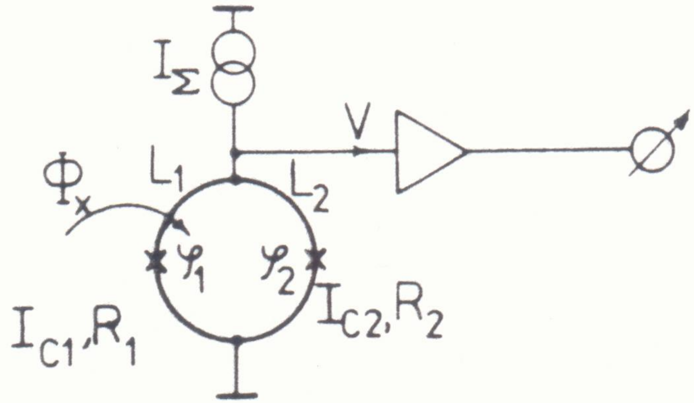


Fig. 7 Basic circuit of a DC SQUID. We assume the junction critical currents I_{C1}, I_{C2} and normal resistances R_1, R_2 to be coupled with a relation $I_{C1}R_1 = I_{C2}R_2 = V_C$.

RSJ -модель

$$\overline{V}_1 = \overline{V}_2 = \overline{V}$$

$$\varphi_1 - \varphi_2 = \frac{2\pi}{\Phi_0}\Phi$$

$$\Phi = \Phi_x - L_1 I_1 + L_2 I_2$$

$$I_\Sigma = I_1 + I_2$$

$$I_1 = I_{C1}\sin\varphi_1 + \frac{\Phi_0}{2\pi R_1}\dot{\varphi}_1 + C_1\frac{\Phi_0}{2\pi}\ddot{\varphi}_1 + \widetilde{I}_{f1}$$

$$I_2 = I_{C2}\sin\varphi_2 + \frac{\Phi_0}{2\pi R_2}\dot{\varphi}_2 + C_2\frac{\Phi_0}{2\pi}\ddot{\varphi}_2 + \widetilde{I}_{f2}$$

Двухконтактный квантовый интерферометр S- состояние

$$\Psi = \varphi_1 + \eta = \varphi_2 - \varphi_e + \eta$$

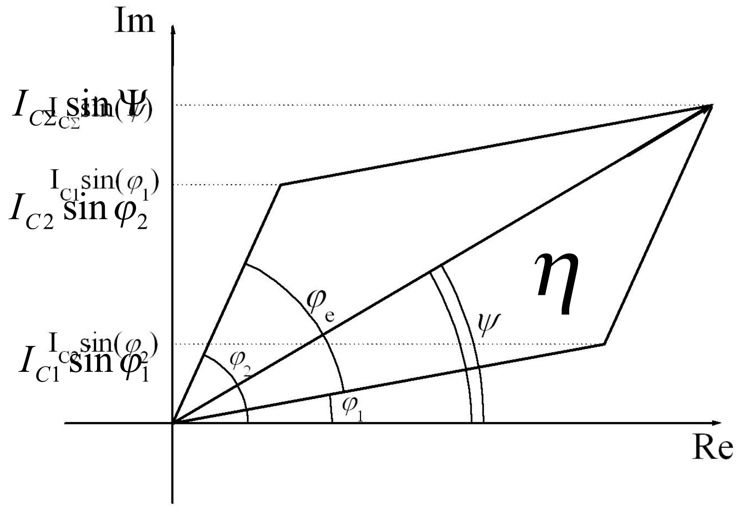
$$tg\eta = \frac{I_{C2}\sin\varphi_e}{I_{C1} + I_{C2}\cos\varphi_e}$$

$$\bar{\Phi}_{1,2} = \bar{\varphi}_{1,2} = 0, \quad \bar{V} = 0, \quad I_1 < I_{C1}, \quad I_2 < I_{C2}$$

$$\varphi = \varphi_1 - \varphi_2 = \varphi_e - \frac{2\pi}{\Phi_0}(L_1 + L_2)(L_1 I_1 - L_2 I_2)/(L_1 + L_2)$$

$$\varphi = \varphi_x - L_+ I_L \frac{2\pi}{\Phi_0}$$

$$I_L = (L_1 I_1 - L_2 I_2)/(L_1 + L_2)$$



$$L_1 I_{C1} << \Phi_0, \quad L_2 I_{C2} << \Phi_0$$

$$\varphi = \varphi_1 - \varphi_2 \approx \varphi_e$$

$$I_\Sigma = I_1 + I_2 = I_{C1}\sin\varphi_1 + I_{C2}\sin\varphi_2$$

$$I_1 = I_\Sigma \frac{L_2}{L_+} + I_L$$

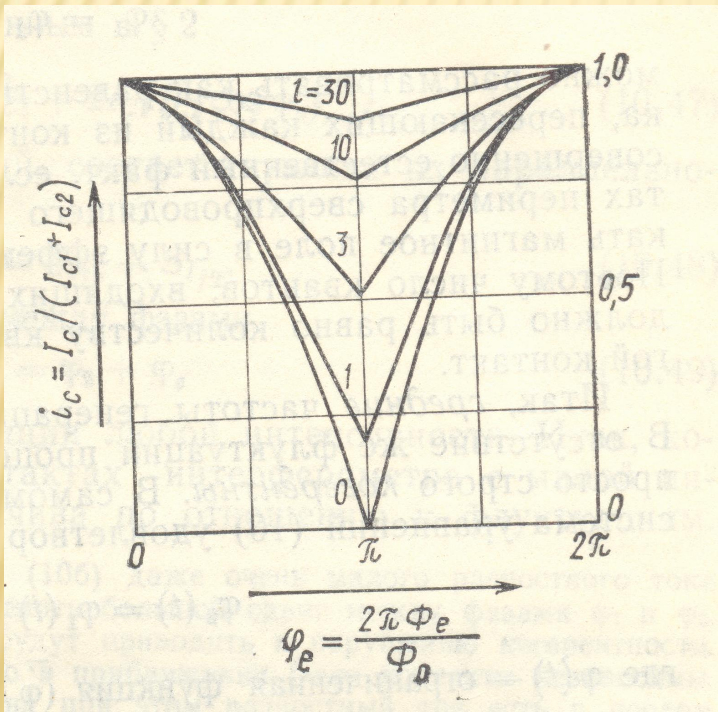
$$I_2 = I_\Sigma \frac{L_1}{L_+} - I_L$$

Двухконтактный квантовый интерферометр S- состояние

$$2I_L = I_1 - I_2 + I_\Sigma (L_1 - L_2)/(L_1 + L_2)$$

$$I_L = \frac{I_{c1}^2 - I_{c2}^2}{2I_{C\Sigma}} \sin\psi + \frac{I_{c1} I_{c2}}{I_{C\Sigma}} \cos\psi \sin\varphi_X + \frac{L_1 - L_2}{2L_+} I_\Sigma$$

$$I_{C\Sigma} = I_{C\Sigma}(I_{c1}/I_{c+}, \varphi_X) = (I_{C1}^2 + I_{C2}^2 + 2I_{C1} I_{C2} \cos\varphi_X)^{1/2}$$



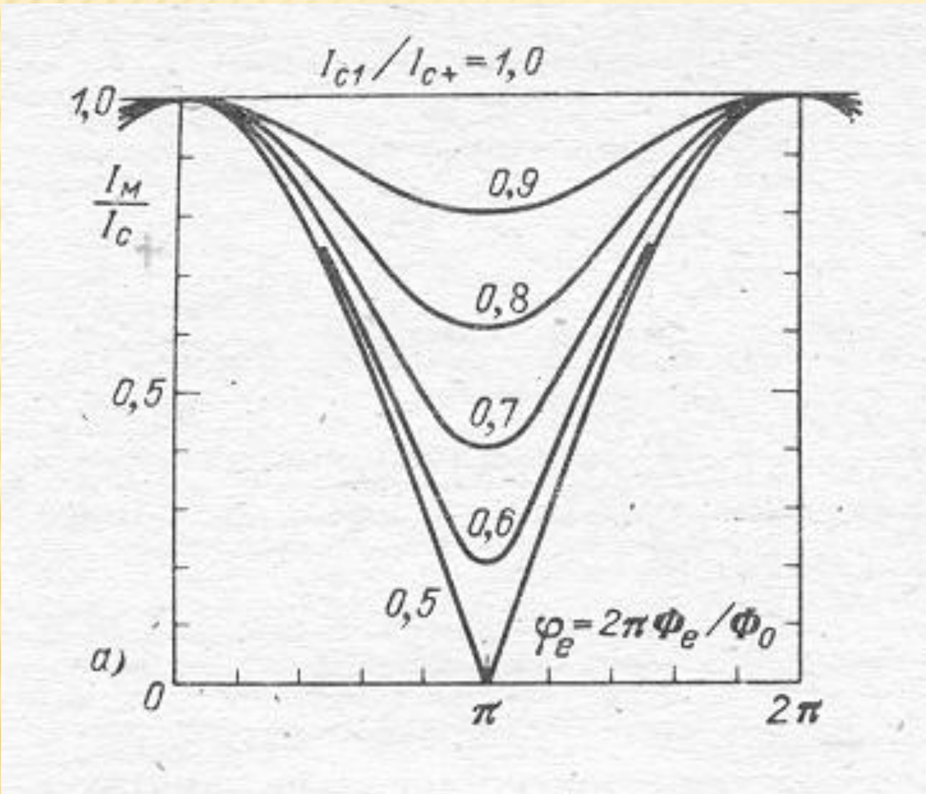
$$(I_{CM})_{\max} = I_{c1} + I_{c2}$$

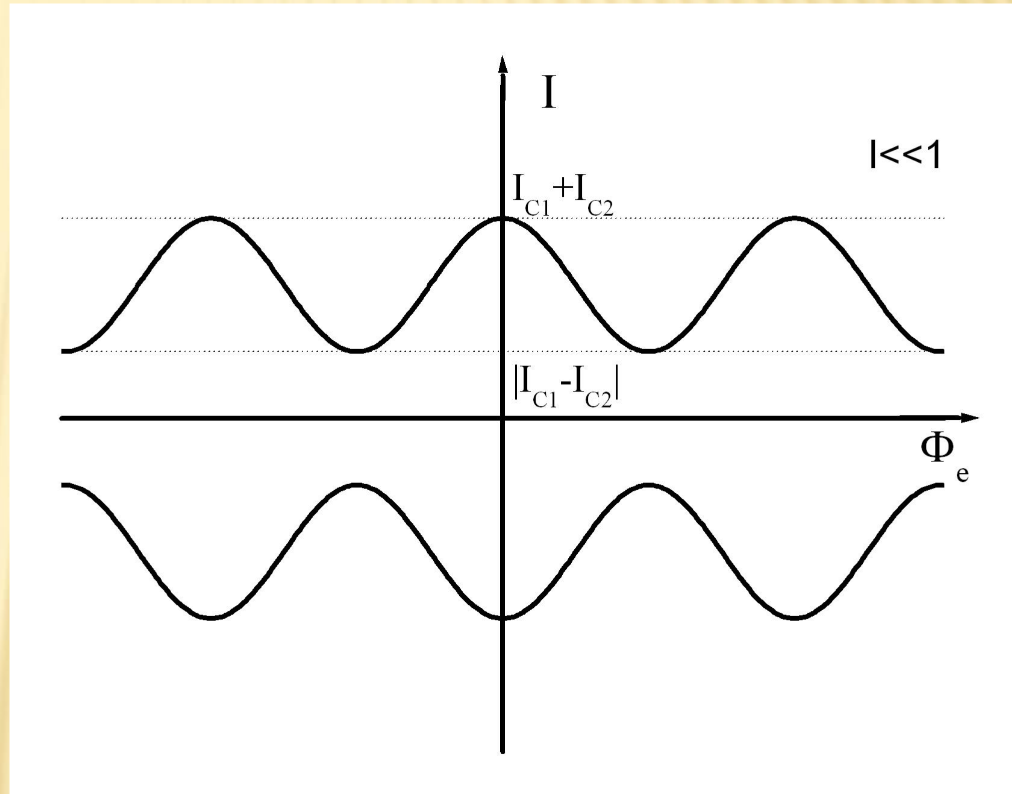
$$(\varphi_e = 0,\ 2\pi n)$$

$$(I_{CM})_{\min} = | I_{c1} - I_{c2} |$$

$$(\varphi_e = \pi, \ldots)$$

Двухконтактный квантовый интерферометр S- состояние





Двухконтактный квантовый интерферометр S- состояние

S-Состояния, $L_1 I_{C1} >> 1$, $L_2 I_{C2} >> 1$

$$\varphi_1 - \varphi_2 = \varphi \approx 2\pi n + \phi, \quad \phi << \pi$$

$$\phi_e = \frac{2\pi}{\Phi_0}(\Phi_e - n\Phi_0) = 2\pi \frac{\Phi_e}{\Phi_0}$$

$$\phi = \phi_e - \frac{2\pi}{\Phi_0} L_+ I_L \Rightarrow I_L = \frac{\Phi'_e}{L_+}$$

$$I_1 = \frac{L_2}{L_+} I_\Sigma + \frac{\Phi'_e}{L_+}$$

Граница S-состояния

$$I_1 = \pm I_{C1}$$

$$I_2 = \frac{L_1}{L_+} I_\Sigma - \frac{\Phi'_e}{L_+}$$

$$I_2 = \pm I_{C2}$$

Допустим $n = 0, \Phi'_e = \Phi_e$

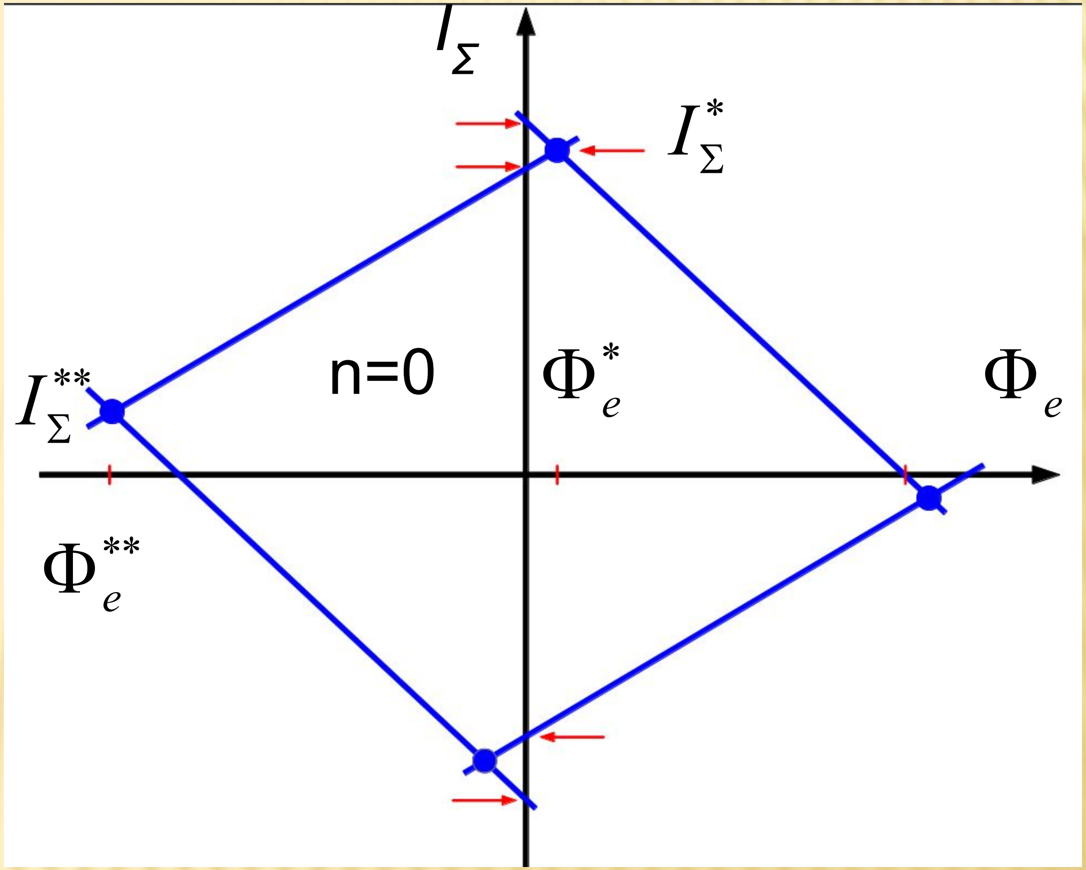
Двухконтактный квантовый интерферометр S- состояние

$$I_{\Sigma 1} = \frac{L_+}{L_2} I_{C1} - \frac{\Phi'_e}{L_2}$$

$$I_{\Sigma 2} = -\frac{L_+}{L_2} I_{C1} - \frac{\Phi'_e}{L_2}$$

$$I_{\Sigma 3} = \frac{L_+}{L_1} I_{C2} + \frac{\Phi'_e}{L_1}$$

$$I_{\Sigma 4} = -\frac{L_+}{L_1} I_{c2} + \frac{\Phi'_e}{L_1}$$



Характерные точки: 1) I_{CMAX} определяется из условия $I_{\Sigma 1} = I_{\Sigma 3}$

$$I^*_\Sigma = \frac{L_+}{L_2} I_{C1} - \frac{\Phi_e}{L_2} = \frac{L_+}{L_1} I_{C2} + \frac{\Phi_e}{L_1} \Rightarrow I^*_\Sigma = I_{C1} + I_{C2} \quad \Phi^*_e = L_2 I_{C2} - L_1 I_{C1}$$

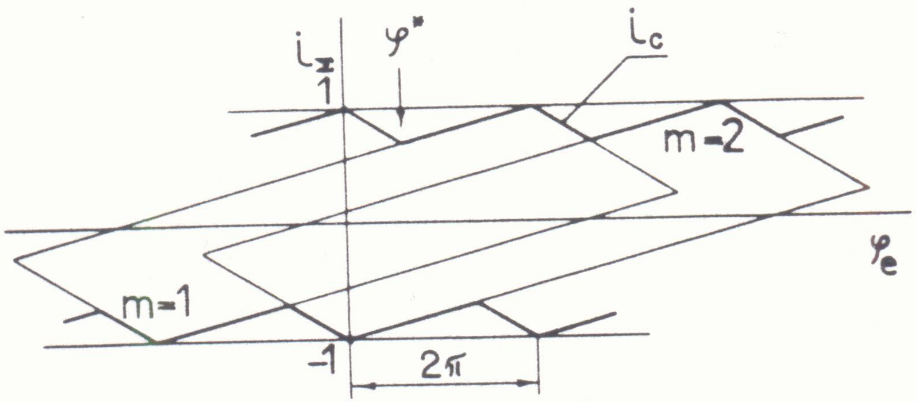
Двухконтактный квантовый интерферометр S- состояние

$$I_{\Sigma}^{**} = I_{C2} - I_{C1} \qquad \Phi_e^{**} = -(L_2 I_{C2} + L_1 I_{C1}) >> \Phi_0$$

$$I_{C\Sigma} \approx I_{C+} - \min[\tilde{\Phi}_e / L_2; (\Phi_0 - \tilde{\Phi}_e) / L_1)$$

$$I_{C\Sigma} \approx I_{C+} - \min[\tilde{\Phi}_e / L_2; (\Phi_0 - \tilde{\Phi}_e) / L_1)$$

$$\tilde{\Phi}_e = \Phi_e - (L_1 I_{C1} - L_2 I_{C2}) - n\Phi_0$$



SQUID. cur- stan- h a

Fig. 8 Regions corresponding to various numbers of magnetic flux quanta stored in the double-junction interferometer, at $\ell \gg 1$. For large but finite ℓ, the tops of the parallelograms are rounded - see 24 for the analytical description.

$$\Delta I_{C\Sigma} \approx \Phi_0 / L_+$$

Двухконтактный квантовый интерферометр. Потенциальная энергия. Зависимость $\Phi(\Phi_e)$ в S-состоянии.

$$I_\Sigma = 0$$

$$U(\varphi_1,\varphi_2) = \frac{I_{C1}\Phi_0}{2\pi}(1-\cos\varphi_1) + \frac{I_{C2}\Phi_0}{2\pi}(1-\cos\varphi_2) + \frac{1}{2L_+}\left[\frac{\Phi_0}{2\pi}(\varphi_1-\varphi_2) - \Phi_e - n\Phi_0\right]^2$$

Стационарное состояние:

$$\frac{\partial U}{\partial \varphi_1} = \frac{\partial U}{\partial \varphi_2} = 0$$

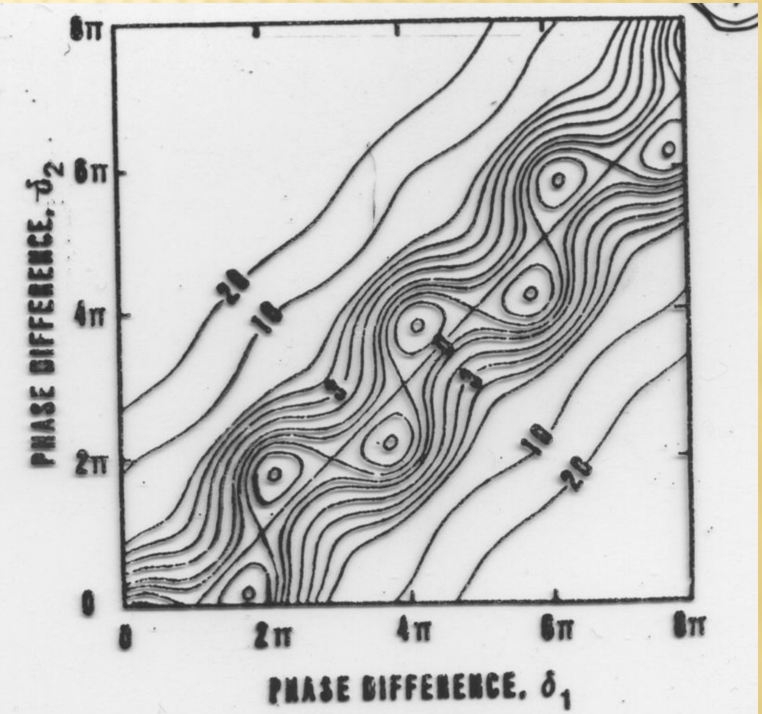


Fig. 1: Contour plot of potential energy as a function of δ_1 and δ_2 for $i = 0$, $\psi = 0.5$ and $\beta = 1$.

5

$$I_M \simeq I_{c+} - \min\left[\tilde{\Phi}_e/L_2 \,;\, (\Phi_0 - \tilde{\Phi}_e)/L_1\right]$$

$$\tilde{\Phi}_e = \Phi_e - (L_1 I_{c1} - L_2 I_{c2}) - \cancel{2\pi}\,\Phi_0$$

3 Потенциальная энергия и зависимость $\Phi(\Phi_e)$ в S-состоянии.

$\boxed{\text{при } I_e = 0}$!

$$U(\varphi_1, \varphi_2) = \frac{I_{c1}\Phi_0}{2\pi}(1 - \cos\varphi_1) + \frac{I_{c2}\Phi_0}{2\pi}(1 - \cos\varphi_2) +$$

$$+ \frac{1}{2L}\left[\frac{\Phi_0}{2\pi}(\varphi_1 - \varphi_2) - \Phi_e - n\Phi_0\right]^2$$

Стац. состояние : $\dfrac{\partial U}{\partial \varphi_1} = \dfrac{\partial U}{\partial \varphi_2} = 0$

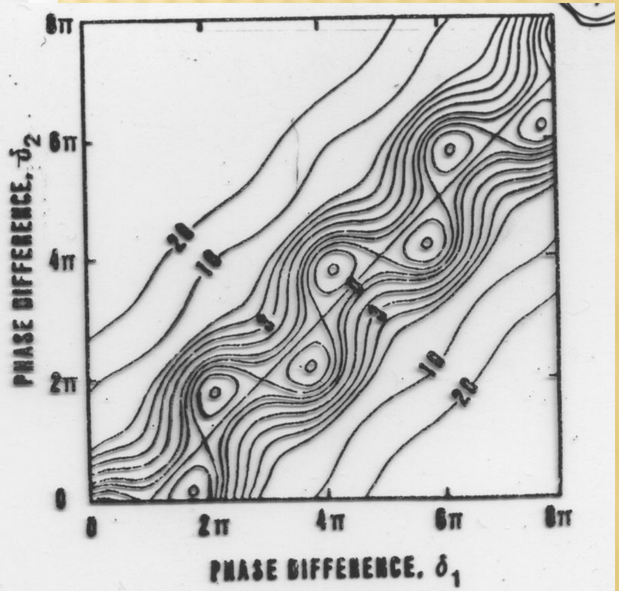


Fig. 1: Contour plot of potential energy as a function of δ_1 and δ_2 for $i = 0$, $\psi = 0.5$ and $\beta = 1$.

(1) $$\frac{\Phi_0}{2\pi}(\varphi_1-\varphi_2) = \Phi_e - L I_{c1}\sin\varphi_1 + n\Phi_0 = \Phi_e + L I_{c2}\sin\varphi_2 + n\Phi_0$$

Состояние при $I_e = 0$ дает также:

(2) $$I_{c1}\sin\varphi_1 + I_{c2}\sin\varphi_2 = 0$$

а) $I_{c2} \gg I_{c1}$, $\varphi_2 \simeq k\pi$ $\qquad \varphi \cong \varphi_1 - k\pi$

$$\varphi = \varphi_e - \frac{2\pi L I_{c1}}{\Phi_0}\sin\varphi\,\cancel{(-1)^k}$$

б) $I_{c1} \simeq I_{c2}$; Условия минимума $U(\varphi_1,\varphi_2)$:

(3) $$\frac{\partial^2 U(\varphi_1,\varphi_2)}{\partial\varphi_1^2} > 0; \quad \frac{\partial^2 U}{\partial\varphi_2^2} > 0; \quad \left(\frac{\partial^2 U}{\partial\varphi_1^2}\right)\left(\frac{\partial^2 U}{\partial\varphi_2^2}\right) - \left(\frac{\partial^2 U}{\partial\varphi_1\partial\varphi_2}\right)^2 > 0$$

Пусть $I_{c1} = I_{c2} = I_c$ (6)

Из (2) : $\varphi_1 = -\varphi_2 + k\cdot 2\pi$

Из (3) $\cos\varphi_1 > 0.$

$$\boxed{\frac{\Phi_0}{2\pi}(2\varphi_1) = \Phi_e - L I_c \sin\varphi_1 + n'\Phi_0}$$ или

$$\Phi = \Phi_e - L I_c \sin\pi\left(\frac{\Phi}{\Phi_0} + n'\right) = \Phi_e \mp L I_c \sin\left(\pi\frac{\Phi}{\Phi_0}\right)$$

Период $\Delta\Phi = 2\Phi_0$ $(\Delta\varphi = 4\pi)$;

n' – четное знак $-$

n' – нечетное —″ $+$

$\cos\varphi_1 > 0$ при $2k - \frac{1}{2} < \frac{\Phi}{\Phi_0} + n' < 2k + \frac{1}{2}$

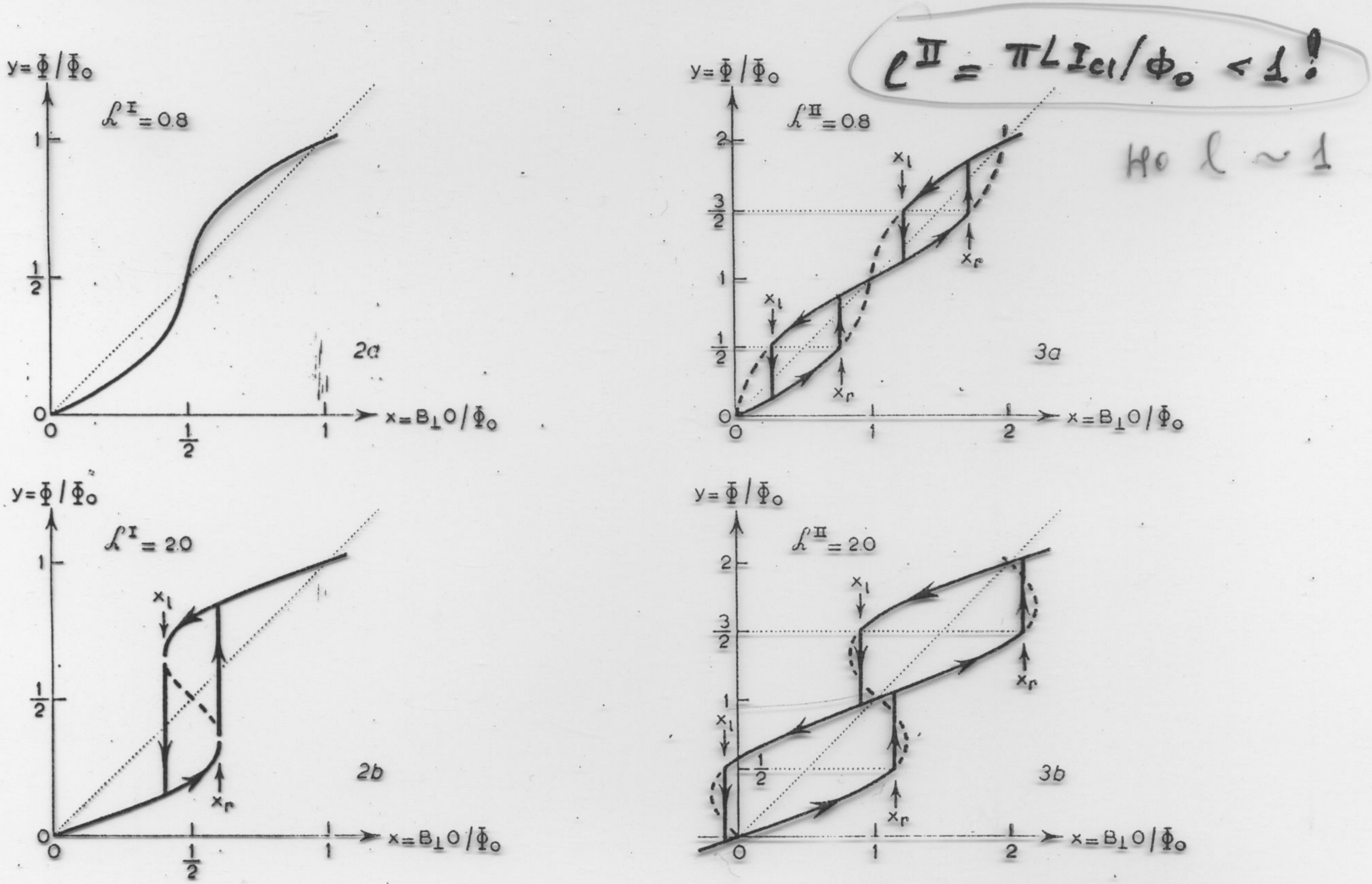


Fig. 2. a) The reduced total embraced magnetic flux $y \equiv \Phi/\Phi_0$ inside a ring, containing one weak link, is plotted as a function of the reduced applied magnetic flux $x \equiv B_\perp O/\Phi_0$ for $\mathcal{L}^{I} = 0.8$. b) $y(x)$ dependence for $\mathcal{L}^{I} = 2$. Flux jumps will occur where the function $y(x)$ is reentrant as is indicated by arrows at $y = 1/2\pi \arccos(\mp 1/\mathcal{L}^{I})$.

Fig. 3. a) The reduced total embraced magnetic flux $y \equiv \Phi/\Phi_0$ inside a ring, containing two symmetrical weak links, is plotted as a function of the reduced applied magnetic flux $x \equiv B_\perp O/\Phi_0$ for $\mathcal{L}^{II} = 0.8$. b) $y(x)$ dependence for $\mathcal{L}^{II} = 2$. Flux jumps will occur for both cases $\mathcal{L}^{II} > 1$ and $\mathcal{L}^{II} < 1$ at $y = m + \frac{1}{2}$ when $x = x_R \equiv (m + \frac{1}{2}) + \mathcal{L}^{II}/\pi$ for increasing applied magnetic field; and when $x = x_L \equiv (m + \frac{1}{2}) - \mathcal{L}^{II}/\pi$ for decreasing applied magnetic field.

Двухконтактный квантовый интерферометр и процессы в нем. Резистивное состояние, β <<1.

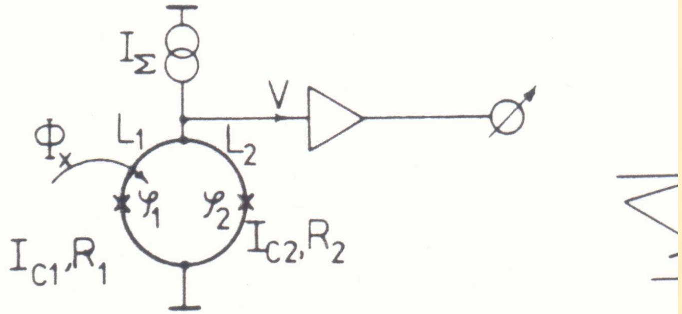


Fig. 7 Basic circuit of a DC SQUID. We assume the junction critical currents I_{C1}, I_{C2} and normal resistances R_1, R_2 to be coupled with a relation $I_{C1}R_1 = I_{C2}R_2 = V_C$.

$$i_1 + i_2 = i_\Sigma$$

$$l_1 i_1 - l_2 i_2 = l i_\Sigma > I_M, \quad \overline{V} \neq 0$$

$$l_{1,2} = 2\pi L_{1,2} I_{C+} / \Phi_0; \quad l = l_1 + l_2$$

$$\varphi_X = \overline{\varphi}_X + \widetilde{\varphi}_X$$

$$i = \{\varphi_X - (\varphi_1 - \varphi_2)\} / l$$

$$i_{1,2} = i_{C1,2}(\omega_C^{-1} \dot{\varphi}_{1,2} + \sin\varphi_{1,2} + \widetilde{i}_{f1,2})$$

$$s_{i_{f1,2}} = \gamma / (\pi i_{C1,2} \omega_c); \quad \gamma = 2\pi k_B T / I_C \Phi_0$$

$$I_\Sigma > I_M, \quad \overline{V} \neq 0$$

$$l << 1; \quad \varphi_1 - \varphi_2 = \varphi_X = \overline{\varphi}_X + \widetilde{\varphi}_X$$

$$\omega_C^{-1} \dot{\Psi} + i_{C\Sigma} \sin\Psi + \widetilde{i}_{f+} = i_\Sigma^{\otimes}$$

$$i_{C\Sigma}^2 = i_{C1}^2 + i_{C2}^2 + 2 i_{C1} i_{C2} \cos\overline{\varphi}_X$$

Двухконтактный квантовый интерферометр и процессы в нем. Резистивное состояние, β <<1.

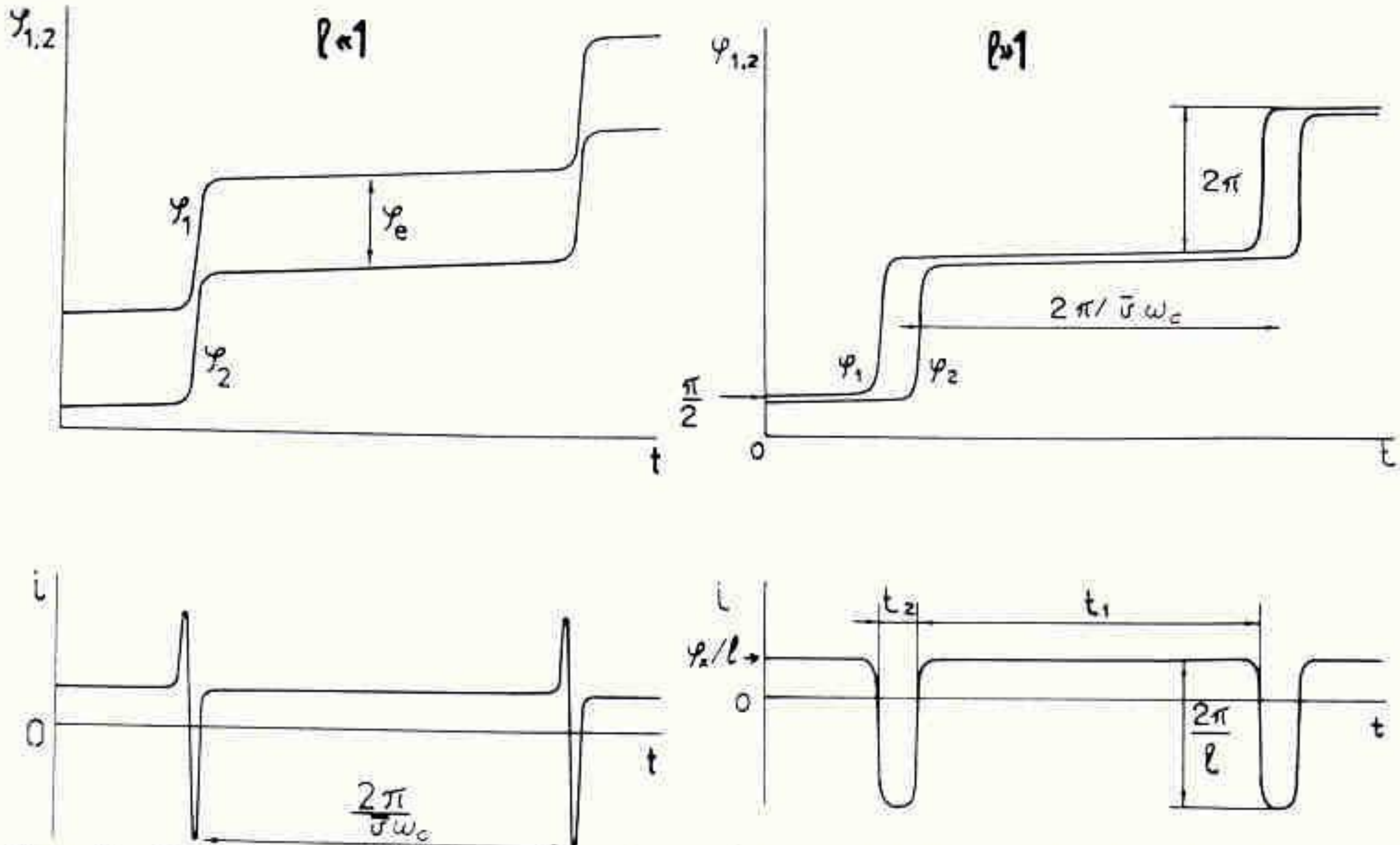
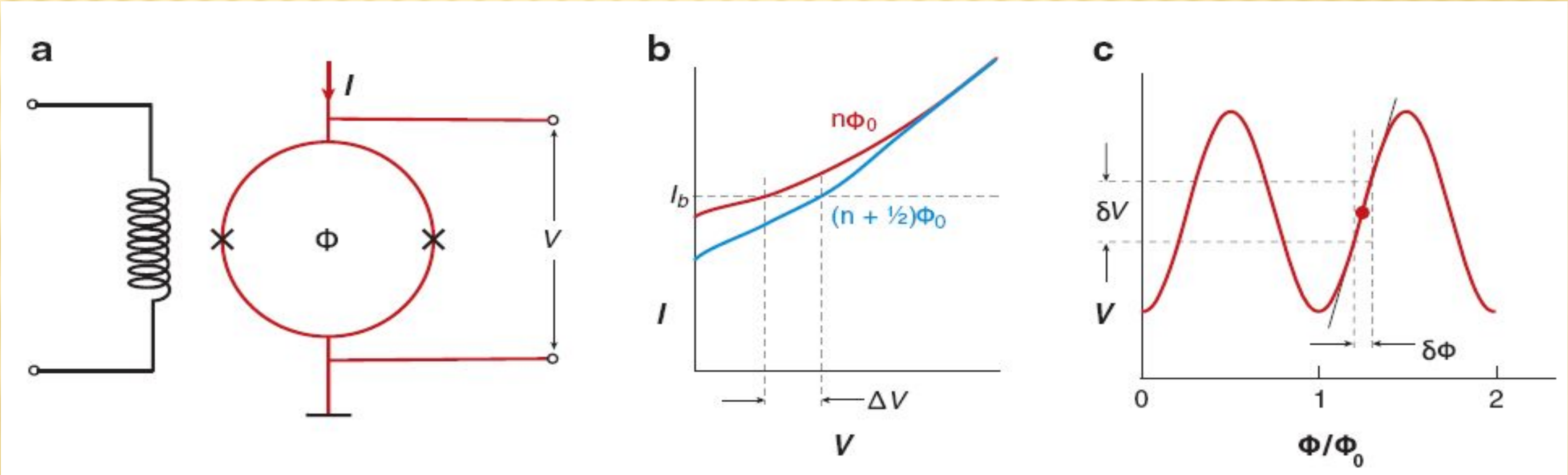


Fig. 9 Dynamics of the Josephson junction phases $\varphi_{1,2}$ and the circulating current i in the double-junction interferometer at small (a, $l \ll 1$) and large (b, $l \gg 1$, but $l\bar{v}^2 \ll 1$) values of inductance – schematically.

Двухконтактный квантовый интерферометр и процессы в нем.
I. Сигнал и шум отсутствуют. Процессы в рабочей точке

$$i_{\Sigma}^{\otimes} = i_{\Sigma} + \tilde{\varphi}_X \frac{2 i_{C1} i_{C2} (i_{C1} - i_{C2})}{\omega_C i_{C\Sigma}^2 (\overline{\varphi}_X)}$$

$$\omega_C^{-1} \Psi = \frac{\overline{\upsilon}^{\,2}}{i_{\Sigma} - i_{C\Sigma} \cos\Theta}; \quad \omega_C^{-1} \Theta \equiv \overline{\upsilon} = [i_{\Sigma}^2 - i_{C\Sigma}^2 (\overline{\varphi}_X)]^{1/2}$$



Двухконтактный квантовый интерферометр и процессы в нем.
II. Сигнальные и шумовые характеристики интерферометра

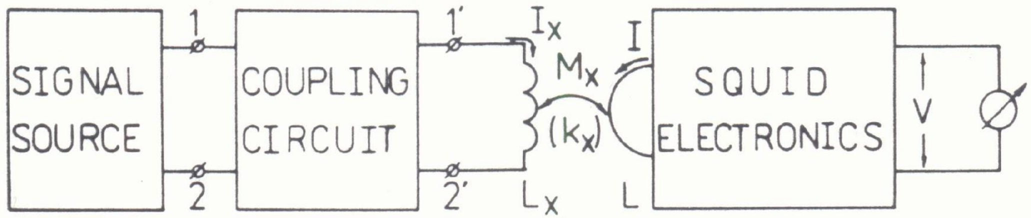


Fig. 1 Basic circuit of signal measurement with a SQUID. Current I_x applies magnetic flux $\Phi_x = M_x I_x$ to the superconducting quantum interferometer, which consists of a superconducting ring with inductance L, closed with one (for RF SQUID) or two (for DC SQUID) Josephson junctions. Current I flowing along the interferometer ring induces the back reaction e.m.f. $E_x = M_x \dot{I}$ in the signal coil L_x, thus acting upon the signal source.

$$\tilde{\Phi}_X = M_X \tilde{I}_X;$$

$$V = V_N + H\tilde{\Phi}_X$$

$$\varepsilon_I = \frac{<\Phi_{NI}^2>}{2L\Delta f}$$

$$\hat{I} - \bar{I} = I_N + j\omega Y(\omega)\tilde{\Phi}_X;$$

$$H = \partial\bar{V} / \partial\Phi_X$$

$$\frac{1}{\Im_i} = \frac{\partial I}{\partial \Phi_e};$$

$$Y(\omega) = \frac{1}{j\omega\Im_i} + G_i;$$

$$\Phi_{NV} = V_N / H$$

$$G_i = \frac{\partial I}{\partial \Phi_X} = \frac{1}{\Re_i}$$

$$\Phi_{NI} = LI_N$$

$$\varepsilon_V = \frac{<\Phi_{NV}^2>}{2L\Delta f}$$

Двухконтактный квантовый интерферометр и процессы в нем.
II. Сигнальные и шумовые характеристики интерферометра

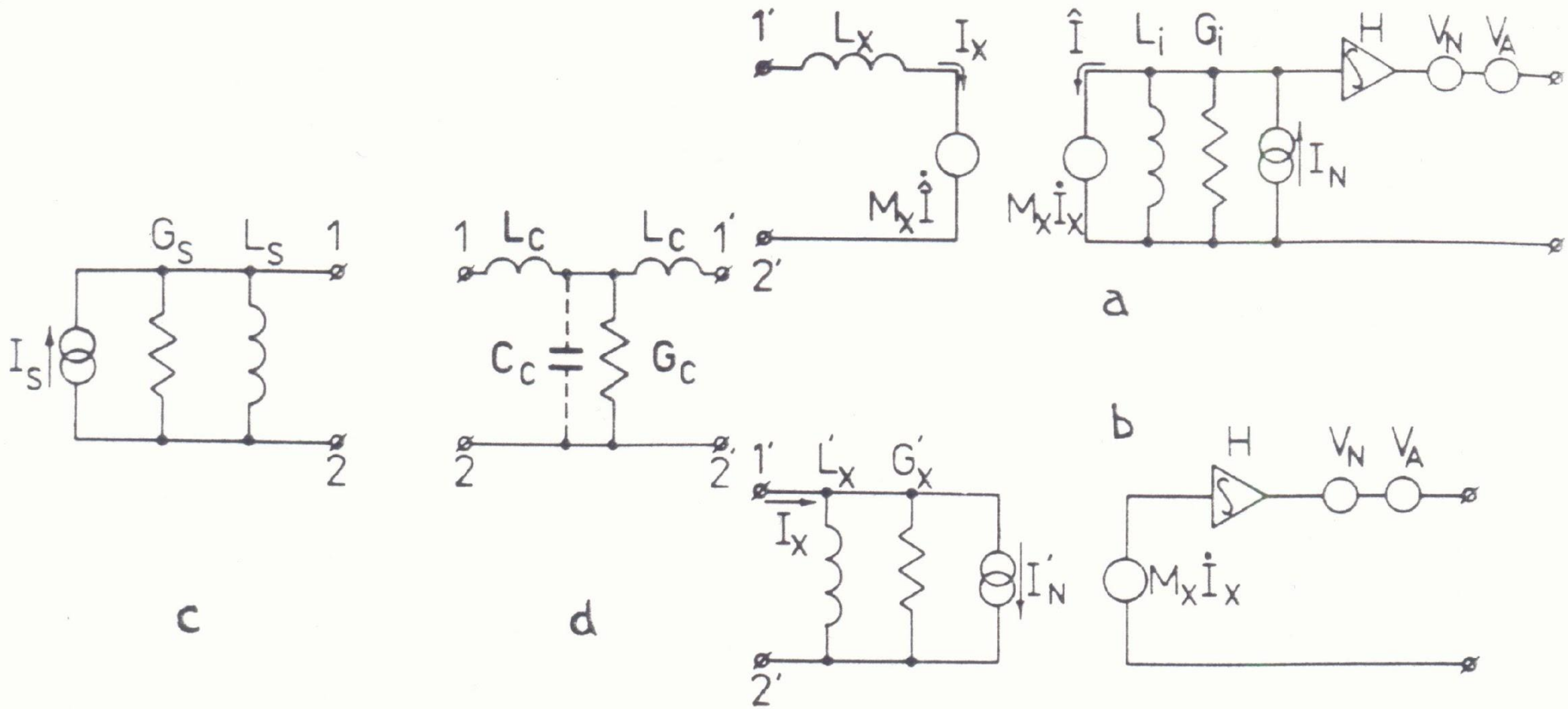


Fig. 10 Equivalent circuits of a SQUID (a,b), a signal source (c) and a coupling circuit (d). Voltage generators V_N and V_A describe the SQUID and the amplifier output noise . Integrators reflect the fact that the SQUID output signal $V = H\Phi_x$ is proportional to $\Phi_x = \int M_x \dot{I}_x dt$.

Двухконтактный квантовый интерферометр и процессы в нем.
II. Сигнальные и шумовые характеристики интерферометра

$$H=\frac{\partial\overline{V}}{\partial\overline{\Phi}_X}=\frac{2\pi}{\Phi_0}V_C\frac{\partial\overline{\upsilon}}{\partial\varphi_X}=\omega_C\frac{1}{\overline{\upsilon}}i_{C1}i_{C2}\sin\overline{\varphi}_X$$

Отклик на шумовое воздействие

$$\dot{\Psi}+\sin\Psi=\bar{i}+\tilde{i}\,;\quad \textit{нормировки}:\bar{i}=i_\Sigma/i_{C\Sigma};\tilde{i}=i_{f+}/i_{C\Sigma}$$

$$\tilde{i}=\int i_\omega e^{j\omega t}d\omega;\qquad \tilde{\dot{\Psi}}\equiv\tilde{\upsilon}=\int\upsilon_\omega e^{i\omega t}d\omega$$

Задача: найти

$S_{\tilde{\upsilon}}(\omega)$ в простейшем случае

$$i_\omega=a\cos\omega t;\quad a<<1$$

$$\Psi=\Psi_0+\Psi_1+\Psi_2+\ldots.$$

$$\Psi_k\propto a^k;\qquad \left|\Psi_1\right|<<1$$

$$\hat{\dot{\Psi}}_k=0,\quad k>0$$

Двухконтактный квантовый интерферометр и процессы в нем.
II. Сигнальные и шумовые характеристики интерферометра

$$\sin\Psi = \sin\Psi_0 + \Psi_1\cos\Psi_0 + \Psi_2\cos\Psi_0 - \frac{\Psi_1^2}{2}\sin\Psi_0$$

$$\bar{i} = \hat{i}_0 + \hat{i}_1 + \hat{i}_2 + \ldots.; \qquad \left|\hat{i}_k\right| \propto a^k$$

$$\dot{\Psi}_0 + \sin\Psi_0 = \hat{i}_0$$

$$\dot{\Psi}_1 + \Psi_1\cos\Psi_0 = \hat{i}_1 + \tilde{i}$$

$$\dot{\Psi}_2 + \Psi_2\cos\Psi_0 = \hat{i}_2 + \sin\Psi_0\frac{\Psi_1^2}{2}$$

Решение уравнения для Ψ_0

$$\hat{i}_0 = \bar{i} - \hat{i}_1 - \hat{i}_2 - \ldots$$

$$\dot{\Psi}_0 = \frac{\bar{\upsilon}^2}{\hat{i}_0 - \cos\Theta}; \qquad \dot{\Theta} \equiv \hat{\upsilon} \approx \bar{\upsilon}$$

$$\hat{\upsilon} = \bar{\upsilon}^A - \frac{d\bar{\upsilon}^A}{d\bar{i}}\hat{i}_1 - \frac{d\bar{\upsilon}^A}{d\bar{i}}\hat{i}_2 - \ldots.$$

Решение уравнения для Ψ_1 это уравнение типа

$$\dot{\tilde{\varphi}} + \tilde{\varphi}\cos\varphi = \tilde{i}\,; \quad \textit{решение}: \quad \tilde{\varphi}(t) = \int_0^t \exp\{\int_t^{t'} \cos\varphi\, dt\}\,\tilde{i}\,(t')\,dt'$$

$$\cos\varphi = -\frac{\ddot{\varphi}}{\dot{\varphi}} = -\ln\dot{(\dot{\varphi})}; \quad \exp\{\pm\int\cos\varphi\, dt\} = const \times \dot{\varphi}^{\mp 1}$$

Поэтому получаем:

$$\tilde{\varphi} = \dot{\varphi}\int^t \tilde{i}\,(t')[\dot{\varphi}(t')]^{-1}\,dt'; \quad \tilde{\upsilon} \equiv \dot{\tilde{\varphi}} = \tilde{i} + \ddot{\varphi}\int^t \frac{\tilde{i}\,(t')}{\dot{\tilde{\varphi}}(t')}\,dt'$$

Возвращаясь к Ψ_k видим, что $\dot{\hat{\Psi}}_k = 0$ если $<(i_k\varphi_0^{-1})>=0$

$$<[i_k(i_0 - \cos\Theta)]> \quad \approx \quad \hat{i}\,\bar{i}_1 - \tilde{i}\,\cos\Theta + \ldots = 0$$

$$\hat{\upsilon} = \Theta = \overline{\upsilon}^{A} + r_d^{A}\hat{i}_1 \frac{<(\tilde{i}\cos\Theta)>}{\bar{i}} = \frac{<(\tilde{i}\cos\Theta)>}{\bar{i}}$$

$$\hat{\upsilon}_N = \Theta = \overline{\upsilon}^{A} + r_d^{A}\Big[\hat{i}_{f+} - \frac{<(\tilde{i}_{f+}\cos\Theta)>}{\bar{i}}\Big]$$

$$\Phi_{NV} = \frac{\upsilon_N}{h}\frac{\Phi_0}{2\pi} = \frac{\Phi_0}{2\pi}\frac{\bar{i}}{i_{C1} i_{C2}\sin\varphi_X}\Big[\hat{i}_{f+} - \frac{<(\tilde{i}_{f+}\cos\Theta)>}{\bar{i}}\Big]$$

$$S_{i_{f+}} = \frac{\gamma}{\pi i_{C1}\omega_C} + \frac{\gamma}{\pi i_{C2}\omega_C}; \quad \gamma = \frac{2\pi k_B T}{I_{C\Sigma}\Phi_0}$$

$$\varepsilon_V = \frac{<\Phi_{NV}^2>}{2L\Delta f} \approx \frac{12 k_B T}{l\omega_C \sin^2(\varphi_X/2)}$$

Сквид со слабосвязанной катушкой! (15)

$$k_x^2 \equiv M_x^2/L_sL_x \ll 1$$

$$\boxed{L_s^r = (1-k_x^2)L_s \simeq L_s}$$

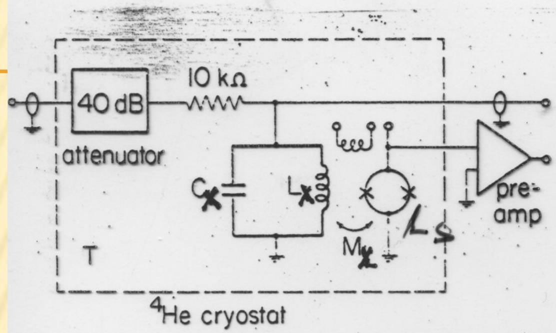
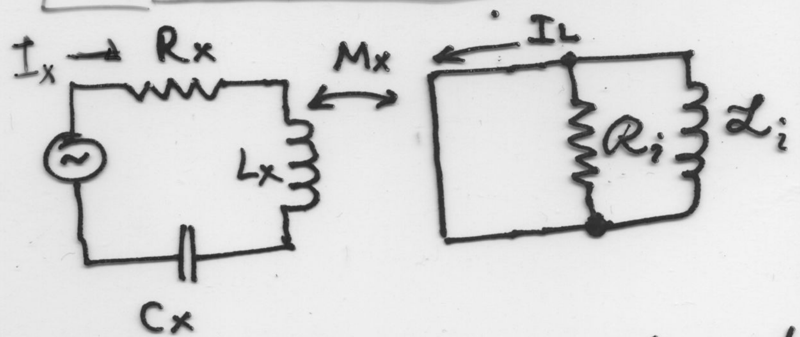


Fig. 9. Experimental configuration for the measurement of the dynamic input impedance of a dc SQUID amplifier.



$$Z_T = R_x\left(1+\frac{L_s k_x^2}{\mathcal{L}_i}\right) + \frac{k_x^2 L_s}{R_i C_x} + j\omega\left[L_x + k_x^2 L_s\left(\frac{R_x}{R_i} - \frac{1}{\omega_s^2 \mathcal{L}_i C_x}\right)\right] + \frac{1}{j\omega C_x}$$

$$\Delta L_x = k_x^2 L_s\left(\frac{R_x}{R_i} - \frac{1}{\omega^2 \mathcal{L}_i C_x}\right) = \frac{1}{(2\pi)^2 C_x}\left[\frac{1}{f_0^2} - \frac{1}{(f_0^0)^2}\right]$$

$$\Delta R_x = k_x^2 L_s\left(\frac{R_x}{\mathcal{L}_i} + \frac{1}{R_i C_x}\right) \simeq \frac{1}{2\pi C_x}\left[\left(1+k_x^2\frac{L_s}{\mathcal{L}_i}\right)\frac{\Delta f}{f_0^2} - \frac{\Delta f^0}{(f_0^0)^2}\right]$$

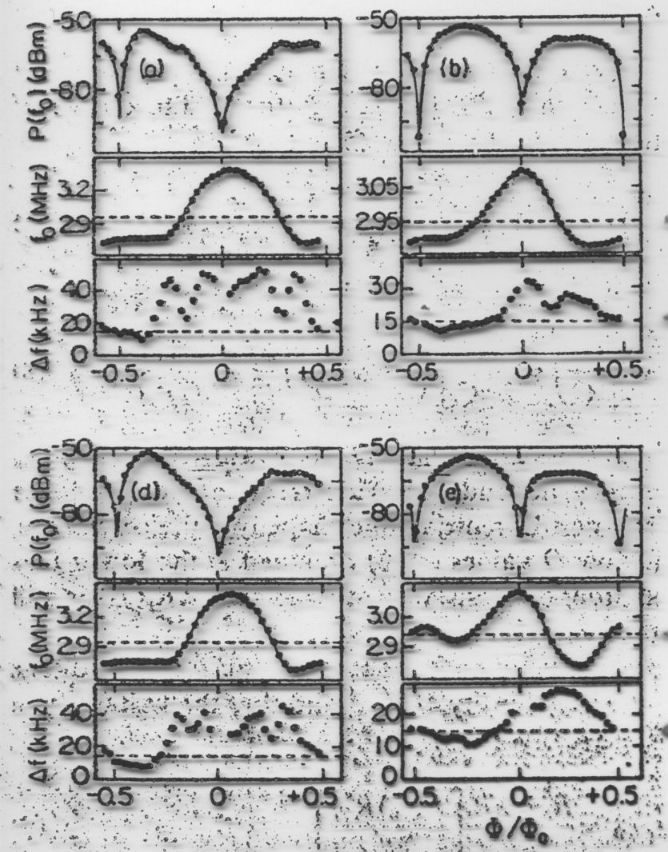
$$H = \frac{R_x + \Delta R_x}{M_x}\left[\frac{P(f_0) R_d}{k_B T \Delta f \cdot \eta}\right]^{1/2}$$

η — mismatch factor

$r_{dsa} \Leftrightarrow 50\Omega$ cable

$\eta \simeq 5/9$ pour $r_d = 10\,\Omega$

Fig. 10. Noise power at resonance $P(f_0)$ measured at SQUID output, resonant frequency f_0, and width of resonance Δf, as functions of Φ/Φ_0 for 20-turn SQUID, at two different temperatures: (a–c) $T = 4.2$ K, (d–f) $T = 1.5$ K. The bias current I was constant for each set of data: (a) $I = 4.0\ \mu A$, (b) $I = 5.0\ \mu A$, (c) $I = 6.0\ \mu A$, (d) $I = 5.0\ \mu A$, (e) $I = 6.0\ \mu A$, (f) $I = 7.0\ \mu A$. The resonant frequency f_0^0 and bandwidth Δf^0 of the unloaded circuit are shown as dashed lines.



T = 4.2 K

← f₀ unloaded

← Δf₀ unloaded

T = 1.5 K

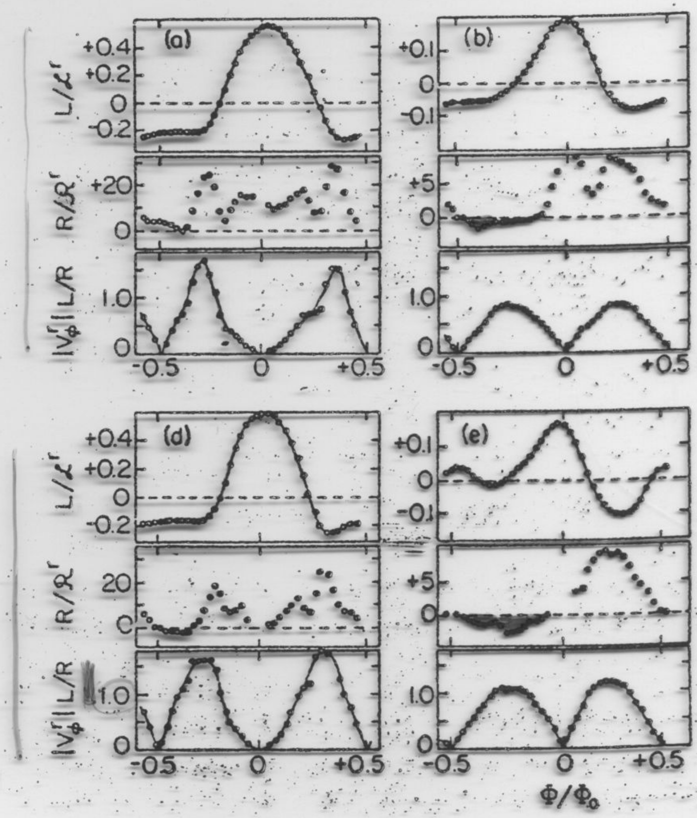


Fig. 11. Dynamic inductance $L/\mathcal{L}^r$, dynamic resistance $R/\mathcal{R}^r$, and flux-to-voltage transfer function $|V_\Phi^r|$ versus Φ/Φ_0 obtained from Fig. 10. Note that the scales for $L/\mathcal{L}^r$ and $R/\mathcal{R}^r$ in (a) and (d) differ from those for the remaining figures.