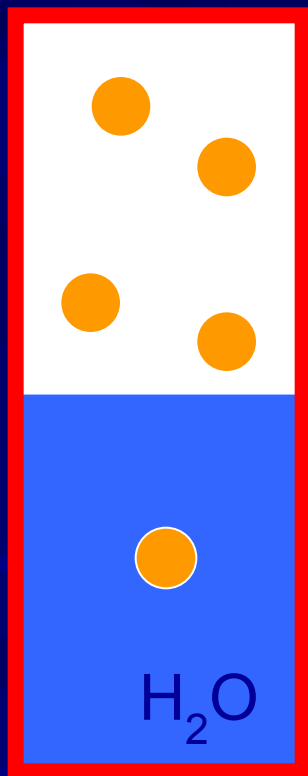


# PROTEIN PHYSICS

## LECTURES 5-6


**Elementary interactions:  
hydrophobic  
&  
electrostatic;  
SS and coordinate bonds**

# Hydrophobic effect




Henry's constant

$$(k_{H,cc})^{-1} = \frac{[\text{in gas}]}{[\text{in liquid}]}$$

for  : = 50/1

for ethanol: = 1/47000

Concentration of C<sub>6</sub>H<sub>14</sub>   
in H<sub>2</sub>O:  
50 times less  
than in gas!

WHY?

## ENTROPY:

$$S_E = k_B \cdot \ln[M_E]; \quad M_E = \text{number\_of\_states}(E)$$

Why  $k_B$ ? What is  $k_B$ ?

Because **entropy**  $S_E$  comes to the **free energy**

$$F_E = E - TS_E \quad (\text{measured in energy units}) \text{ as } TS_E,$$

and  $T$  is measured in degrees, while

$\ln[\text{number of states}]$  is dimensionless;

Thus,  $k_B$  is energy\_unit/degree

## FREE ENERGY:

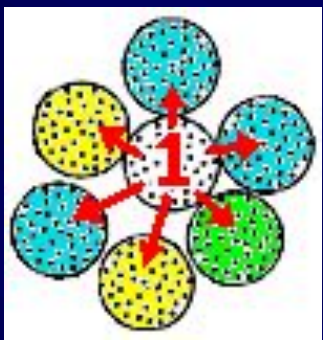
$$\text{Probability}(E) \sim M_E \cdot \exp(-E/k_B T) = \exp(-F_E/k_B T)$$

*Boltzmann*

$$F = E - TS \text{ at } V = \text{const};$$

$$G = H - TS = (E + PV) - TS \text{ at } P = \text{const} \text{ (better for experiment)}$$

-----



$G^{\text{int}}$ : “Free energy of interactions”  
 (“mean force potential”)

Chemical potential:

$$\mu \equiv G^{(1)} = G^{\text{int}} - T \cdot k_B \ln(V^{(1)}) \equiv G^{\text{int}} + T \cdot k_B \ln[C]$$

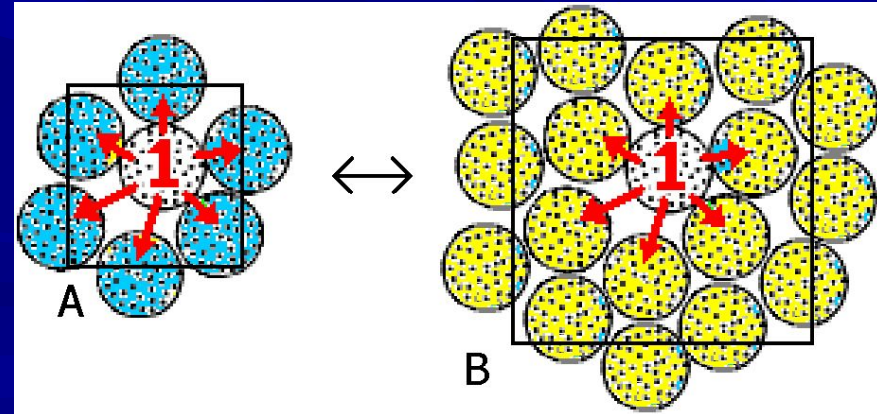
**EQUILIBRIUM** for transition

of molecule **1** from **A** to **B**:  $G_A^{(1)} = G_B^{(1)}$

chemical potentials in **A** and **B** are equal

$$\Delta G_{A \rightarrow B}^{\text{int}} \equiv G_B^{\text{int}} - G_A^{\text{int}}$$

$$\Delta G_{A \rightarrow B}^{\text{int}} = k_B T \cdot \ln([C_{\text{inA}}]/[C_{\text{inB}}])$$



**Experiment:**  $\Delta G_{A \rightarrow B}^{\text{int}} = k_B T \cdot \ln([C_{1 \text{ in } A}]/[C_{1 \text{ in } B}])$

$\Delta S_{A \rightarrow B}^{\text{int}} = -d(\Delta G_{A \rightarrow B}^{\text{int}})/dT$

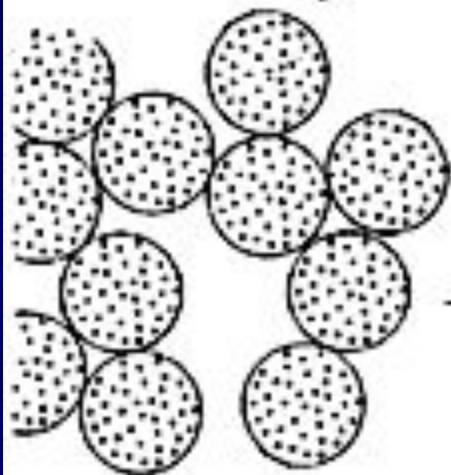
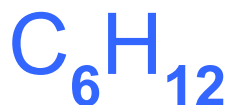
$\Delta U_{A \rightarrow B}^{\text{int}} = \Delta Q_{A \rightarrow B}^{\text{int}} + T \Delta S_{A \rightarrow B}^{\text{int}}$

$\Delta H = -7.3$   
 $T\Delta S = -2.9$   
 $\Delta G = -4.4$   
 $\Delta C_p = +8$



Gas

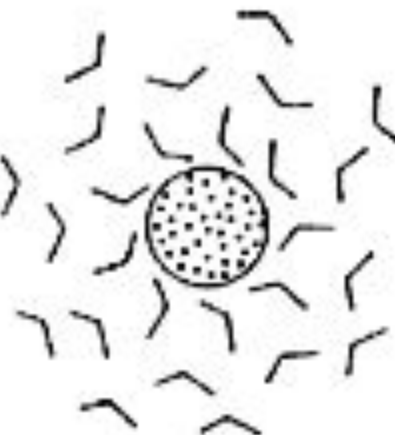
$\Delta H = -7.3$   
 $T\Delta S = -9.6$   
 $\Delta G = +2.3$   
 $\Delta C_p = +94$



Liquid

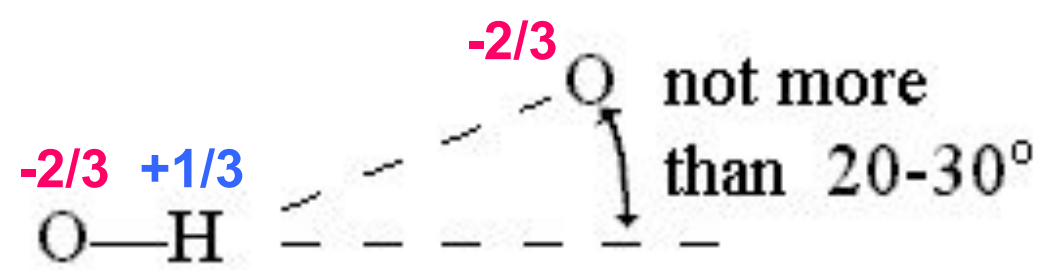
$T = 298^{\circ}K = 25^{\circ}C$

$\Delta H = 0$   
 $T\Delta S = -6.7$   
 $\Delta G = +6.7$   
 $\Delta C_p = +86$



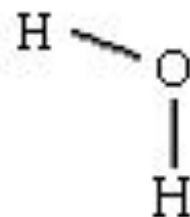
Aqueous solution

[C] of  $C_6H_{12}$   
in  $H_2O$ :  
50 times less  
than in gas;  
100000 times  
less than in  
liquid  $C_6H_{12}$



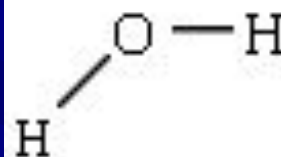
**H-bond: directed**

**Loss: S**



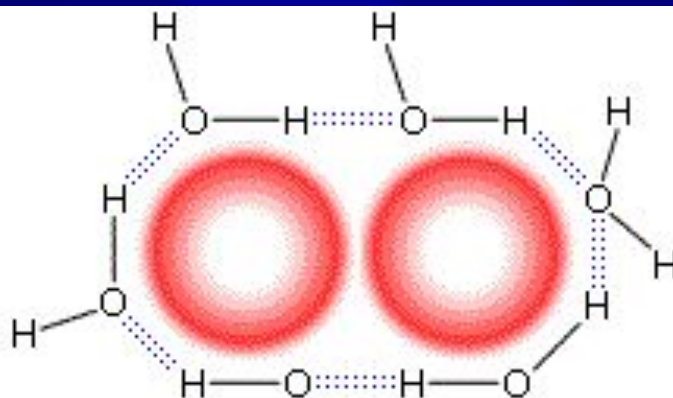
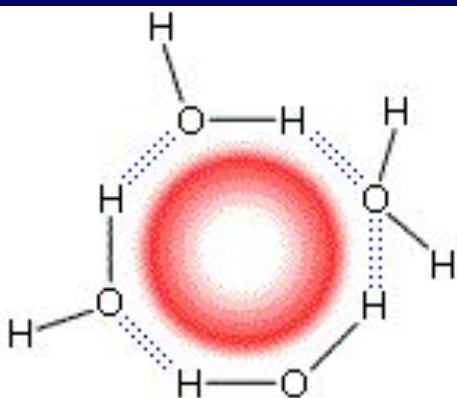
usual case

**Loss: LARGE E**

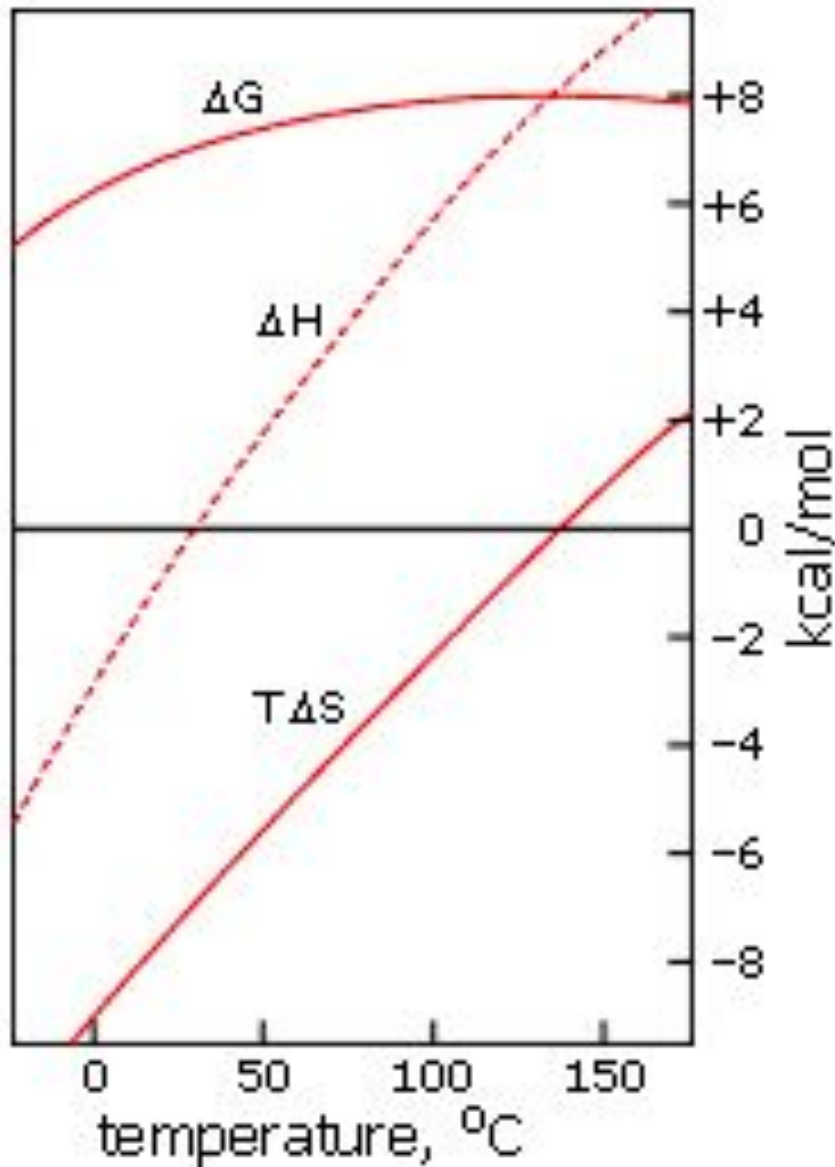


rare case

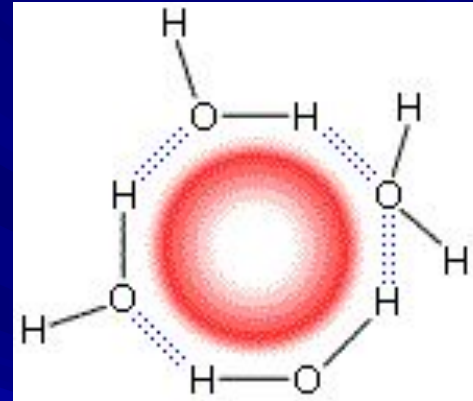
Non-polar body

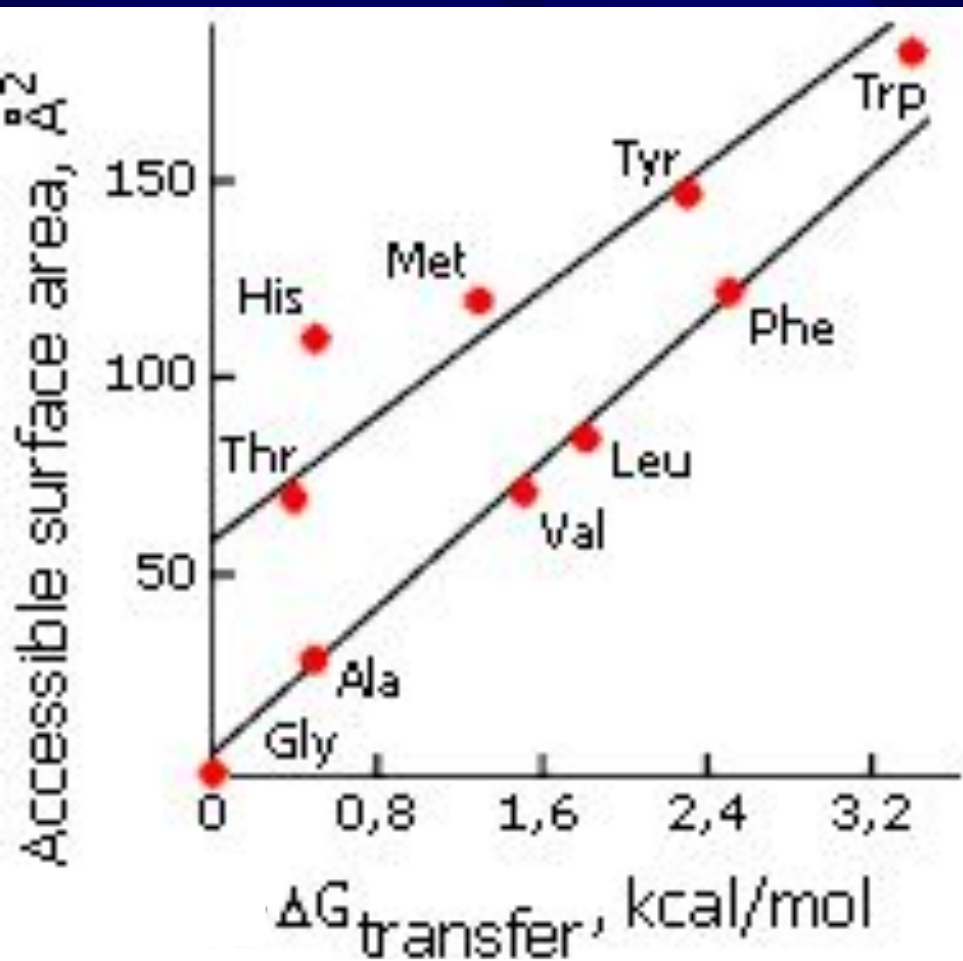


**“hydrophobic bond”**

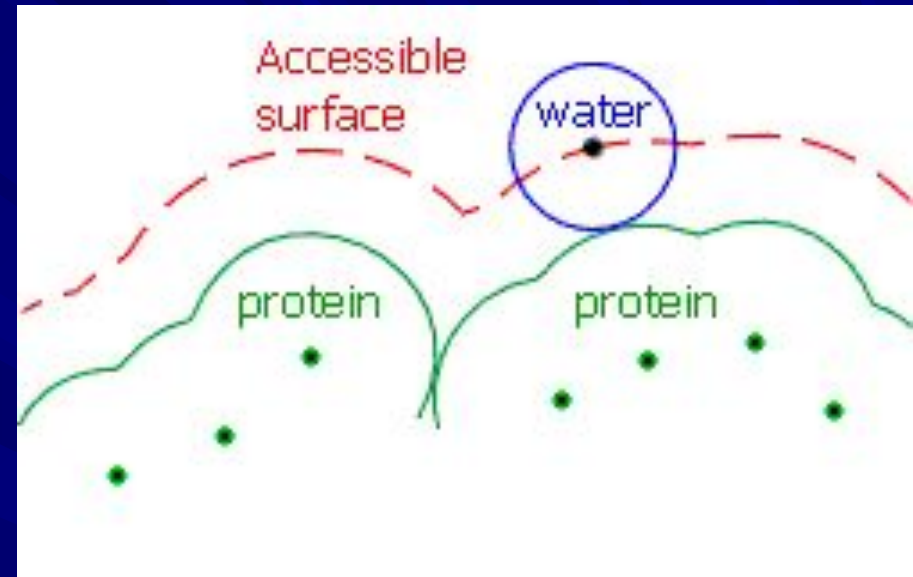


**High  
heat capacity  
 $d(\Delta H)/dT$ :  
Melting of  
“iceberg”**



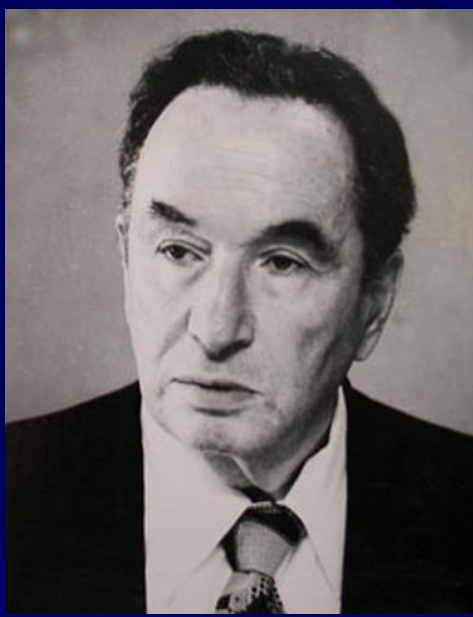


Octanol  $\rightarrow$  Water



20-25 cal/mol per  $\text{\AA}^2$  of molecular accessible non-polar surface





Семён Ефимович **Бреслер**

(1911 – 1983)

Hypothesis on a role of hydrophobic effect in protein folding



Давид Львович **Талмуд**

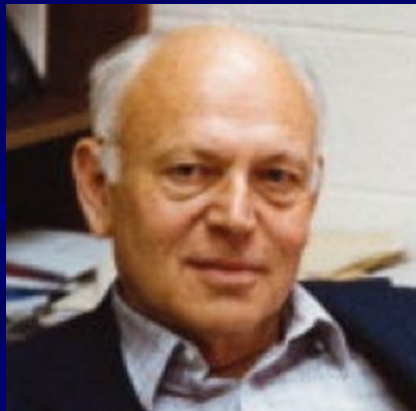
(1900 - 1973)

Hydrophobic  
effect  
&  
denaturation  
of proteins



WALTER  
KAUZMANN

1916-2009



Charles **Tanford**

(1921 - 2009)

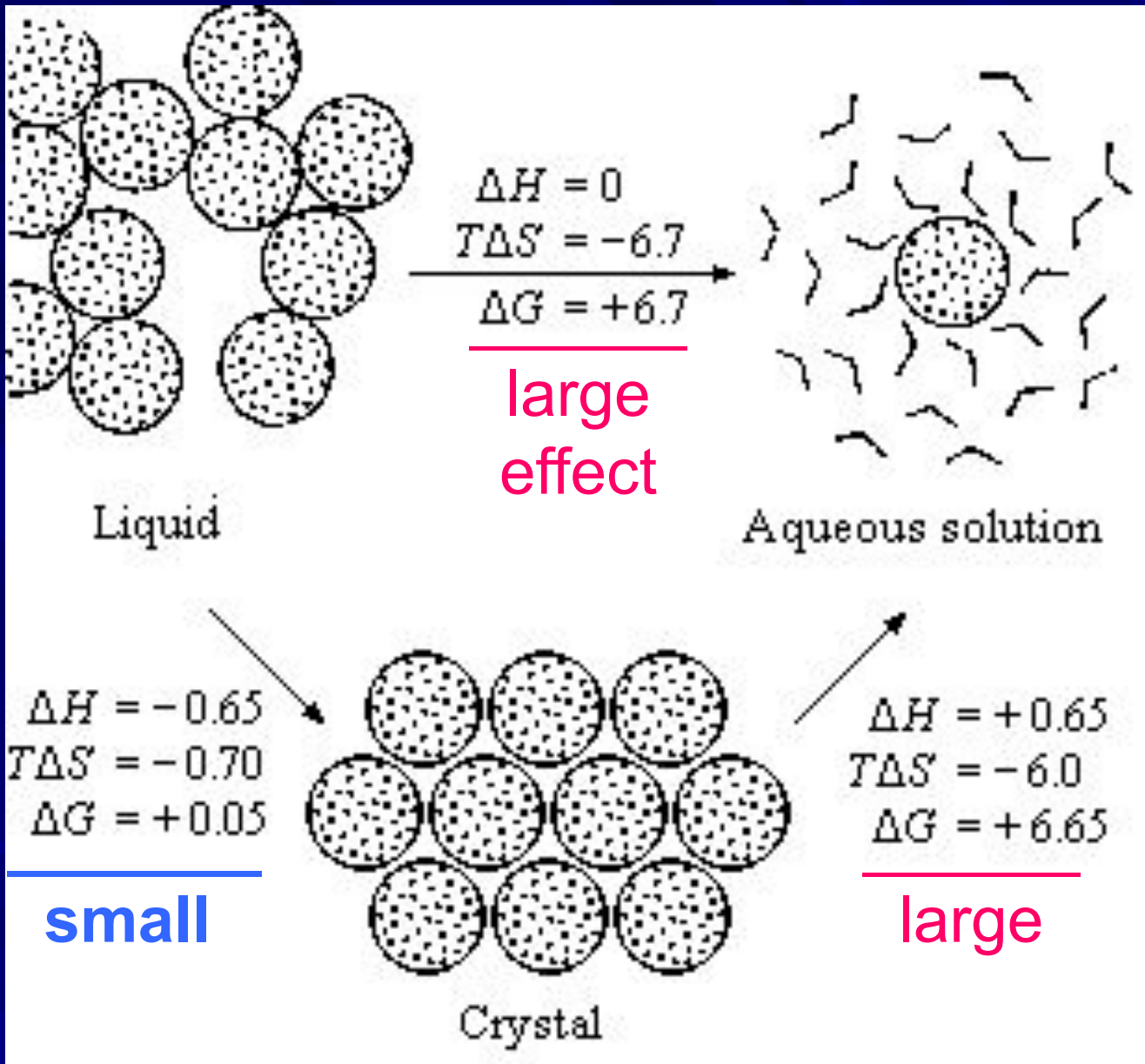
General physical  
features of  
Hydrophobic  
effect



Cyrus Homi **Chothia**,

1942

Hydrophobic  
effect  
&  
amino acid  
water-accessible  
surface



# Electrostatics in uniform media:

potential  $\phi_1 = q_1/\epsilon r$

Interaction of two charges:

$$U = \phi_1 q_2 = \phi_2 q_1 = q_1 q_2 / \epsilon r$$

$\epsilon = 1$  vacuum

$\epsilon \approx 3$  protein

$\epsilon \approx 80$  water

## Protein/water interface

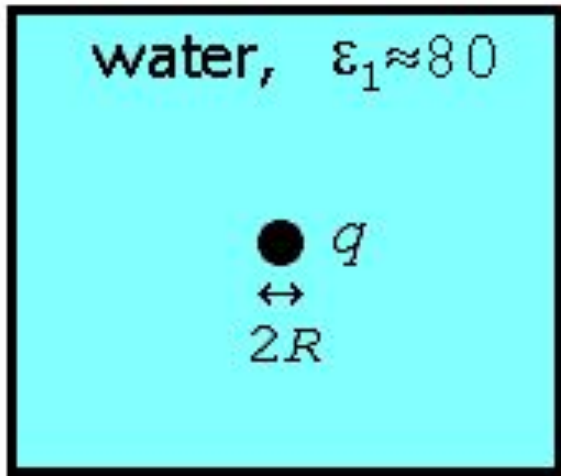
In non-uniform media:  $\epsilon_{\text{eff}} = ?$

At atomic distances:  $\epsilon_{\text{eff}} = ?$

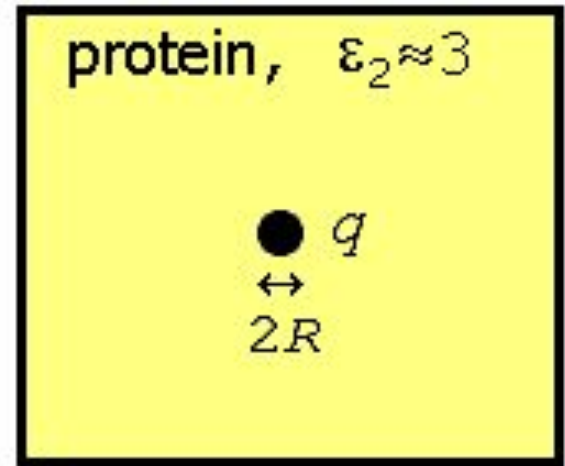
## Charles-Augustin de Coulomb



(1736-1806)



# CHARGE inside PROTEIN



$$\Delta U_{1 \Rightarrow 2} = q^2/2\epsilon_2 R - q^2/2\epsilon_1 R$$

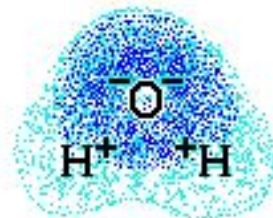
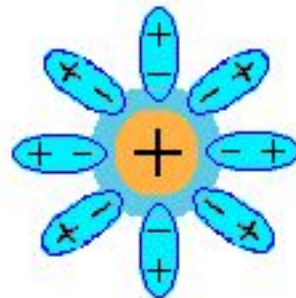
Water => vacuum:  
 $\Delta U \approx +100$

kcal/mol

—  $R=(1.5-2 \text{ \AA}) +0.1\text{\AA}$

+  $R=(1-1.5 \text{ \AA}) +0.7\text{\AA}$

++  $R=(0.5-1 \text{ \AA}) +0.7\text{\AA}$



Water => PROTEIN  
( $\epsilon \approx 3$ )

$R \approx 1.5 - 2 \text{ \AA}$

$\Delta U \approx +30 - 40 \text{ kcal/mol}$

CHARGE inside  
PROTEIN:  
**VERY BAD**

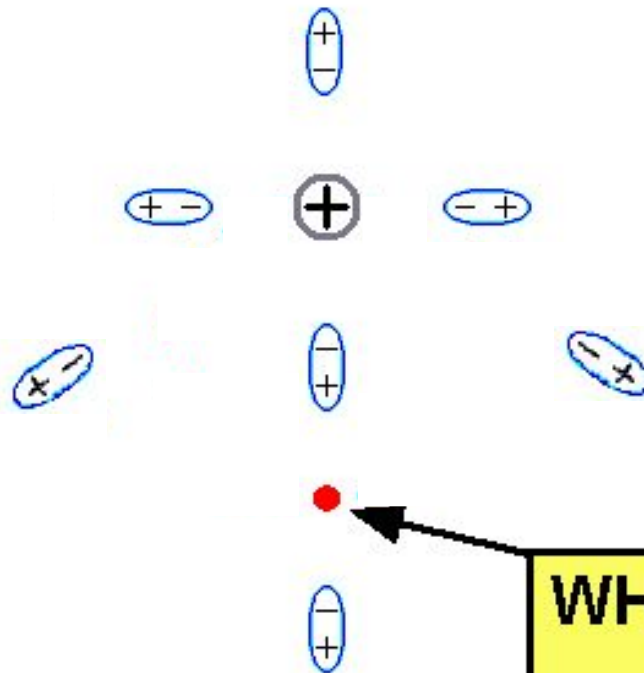
# Non-uniform media:

$$\epsilon_{\text{eff}} = ?$$

**KNOWN:**

molecules (dipoles) decrease potential:

$$U \sim 1/\epsilon r$$



**WHICH dipoles decrease potential here?**

# Non-uniform media:

$$\epsilon_{\text{eff}} = ?$$

**KNOWN:**

**molecules (dipoles) decrease potential:**

$$U \sim 1/\epsilon r$$



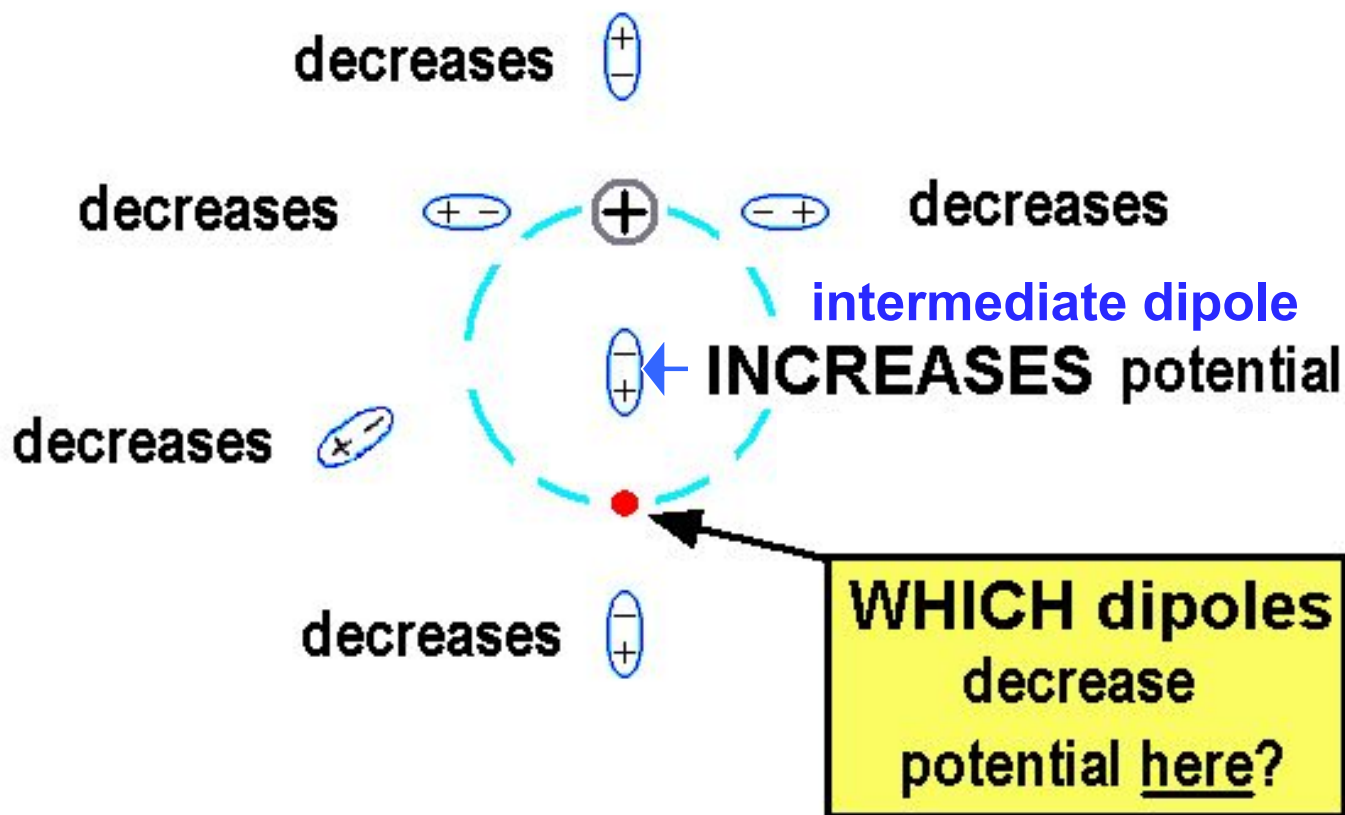
# Non-uniform media:

$$\epsilon_{\text{eff}} = ?$$

**KNOWN:**

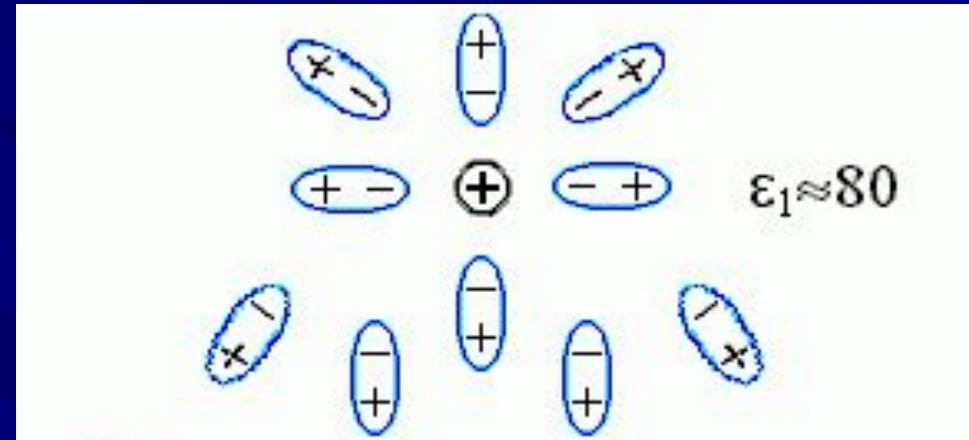
molecules (dipoles) decrease potential:

$$U \sim 1/\epsilon r$$

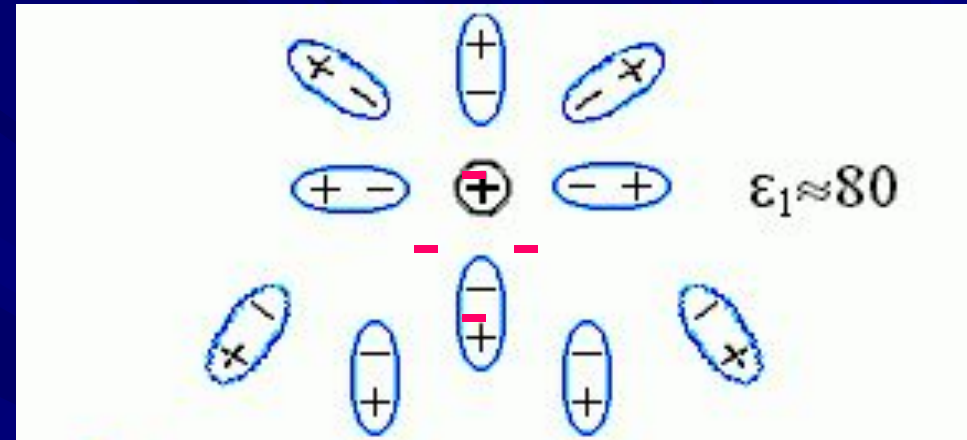




$$\phi = q/\epsilon_1 r$$

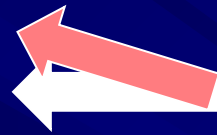


$$\phi = (q/\epsilon_1)/r$$



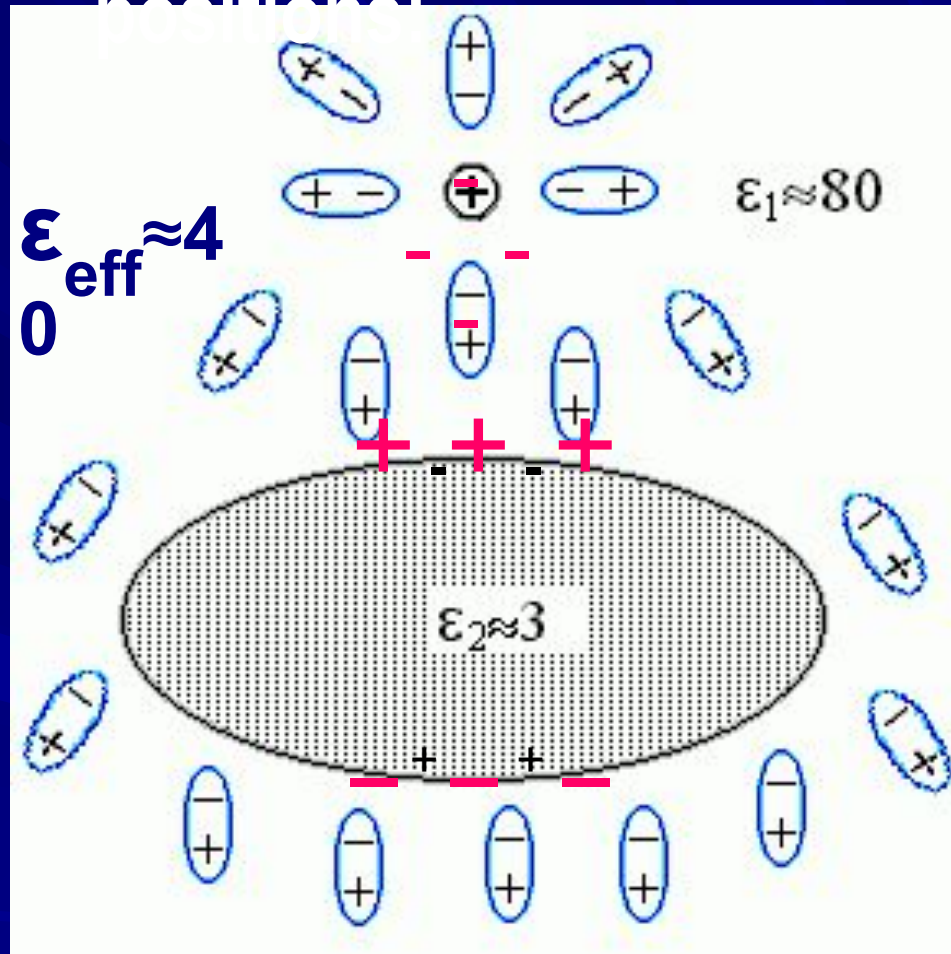
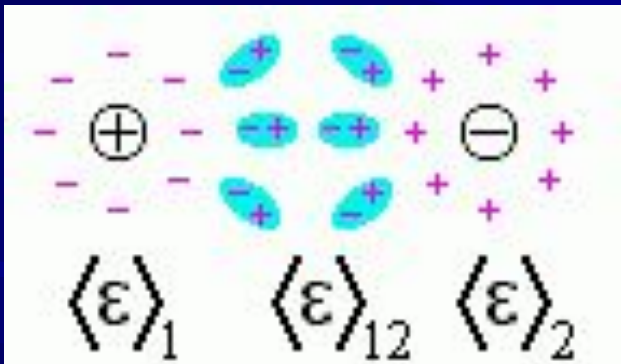
$$\phi = q/r\epsilon_{\text{eff}} \text{ in}$$

$$\vec{E} = -d\phi/d\vec{r}$$



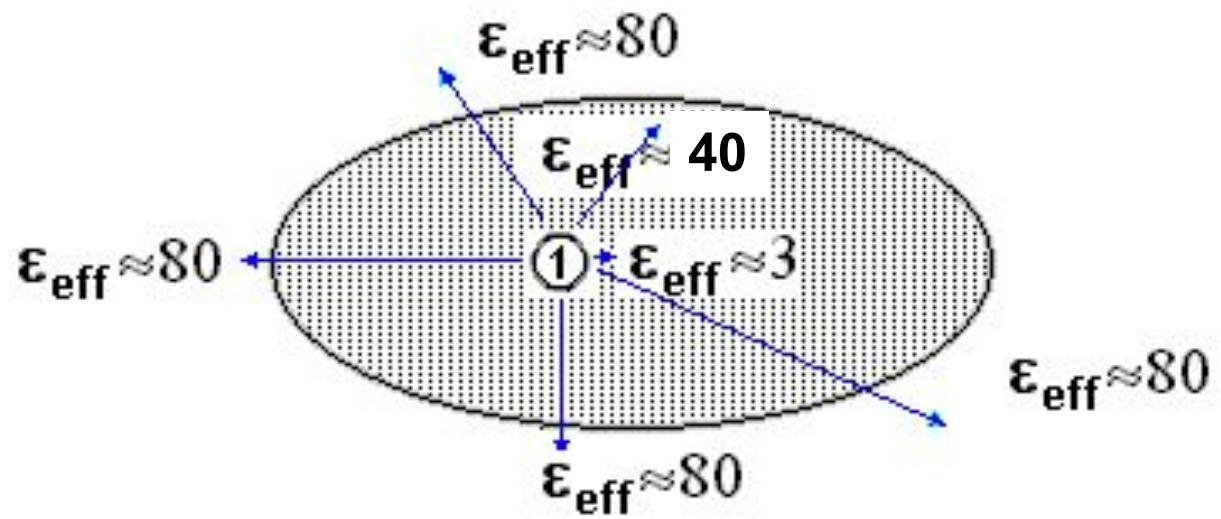
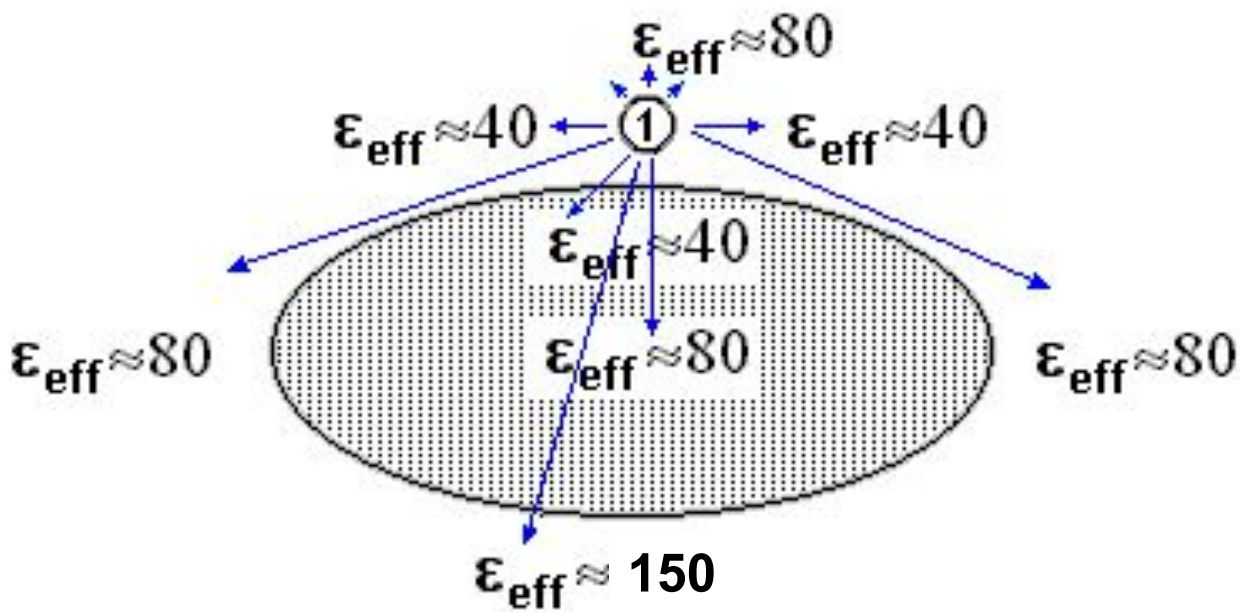
$$U = q_1 q_2 / \epsilon_{\text{eff}} r =$$

$$(q_1 / \langle \epsilon \rangle_1) \cdot (\langle \epsilon \rangle_{12} / r) \cdot (q_2 / \langle \epsilon \rangle_2)$$



$$\epsilon_{\text{eff}} \approx 150 !!$$

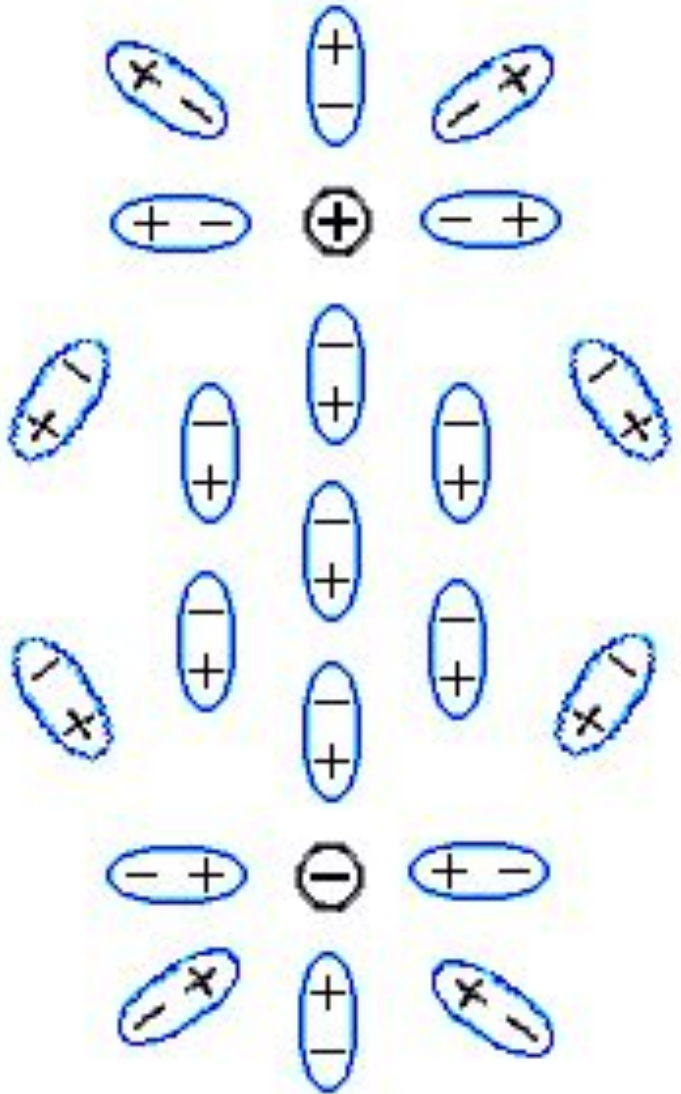
Good estimate for non-uniform media



$\epsilon_{\text{effective}}$   
in non-uniform media

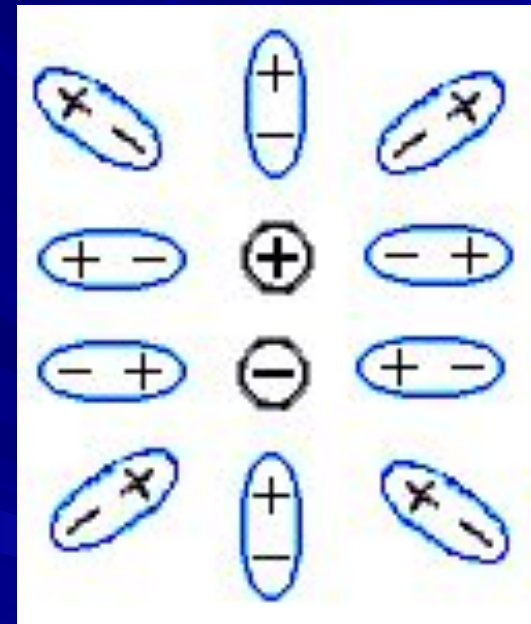
Large distance:

$$\epsilon_{\text{eff}} = \epsilon = 80$$



Atomic distance:

$$\epsilon_{\text{eff}} = ?$$

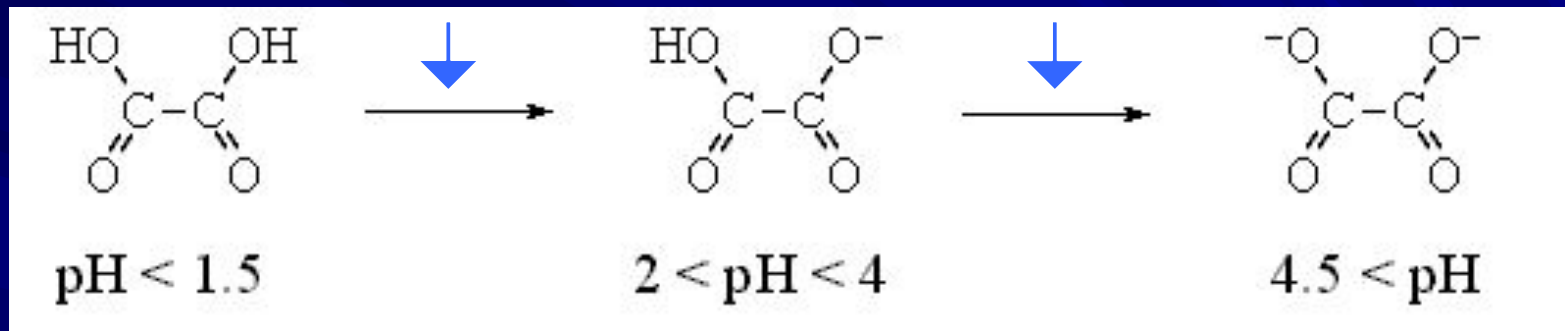


intermediate  
“vacuum”,  $\epsilon \sim 1$ ?  
but the absence  
of intermediate  
dipoles can  
only increase  
interaction...

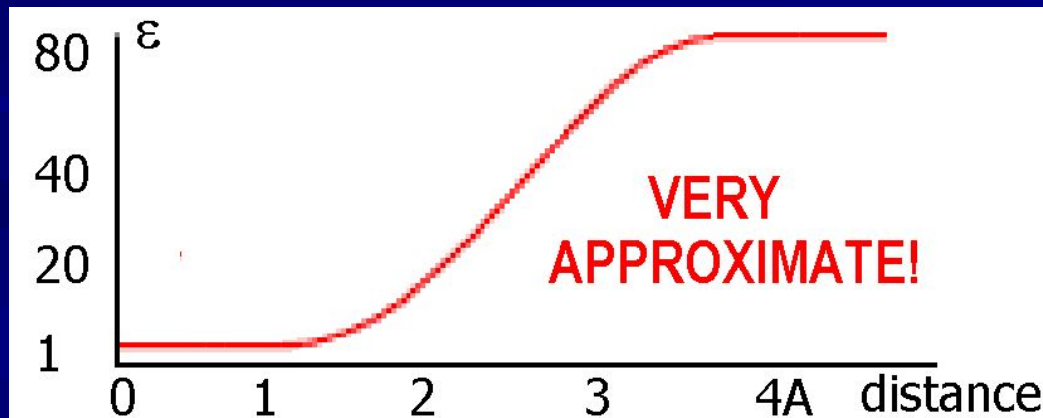
# At atomic distances in water:

1)  $\epsilon=80$  is not a bad approximation (much better than  $\epsilon = 1$  or 3 !!)  
(salt does not dissolve, if  $\epsilon < 50$  at  $3\text{\AA}$ !)

2)  $[\text{H}]_{1/2} = 10^{-1.75}$   $[\text{H}]_{1/2} = 10^{-4.25} = 10^{-1.75} \times e^{-\Delta G_{el}/RT}$

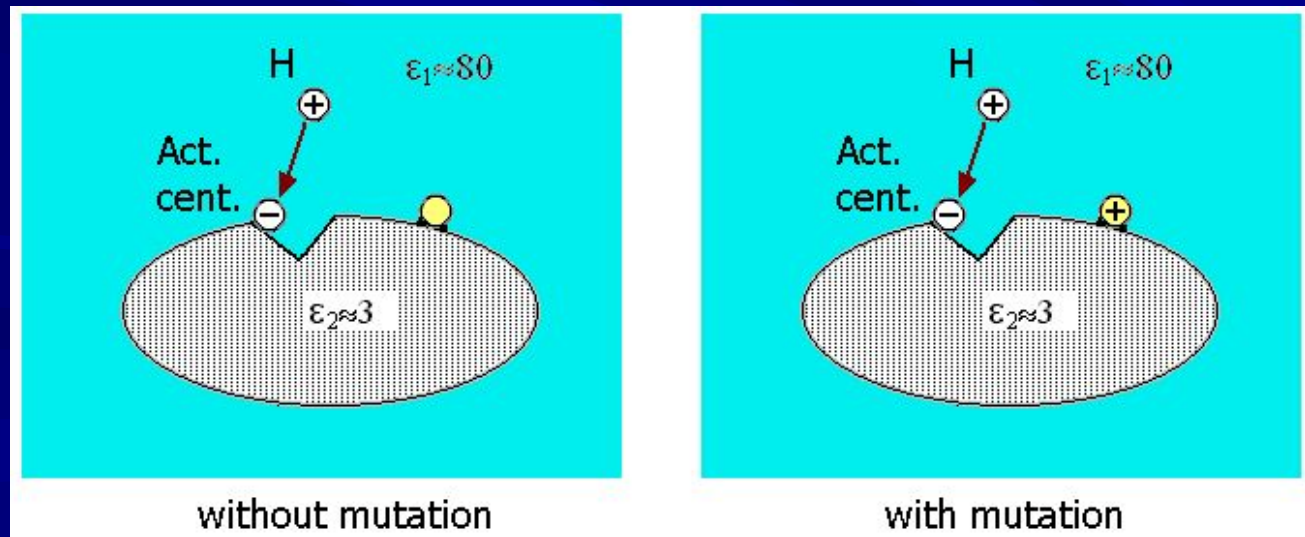
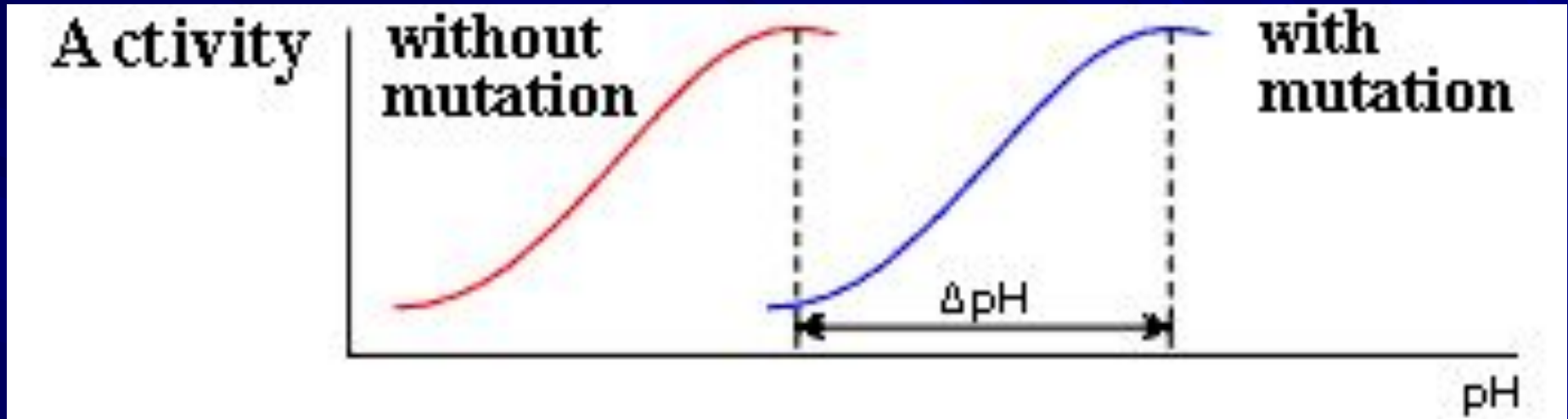


$\Delta G_{el} = 2.5 \times \ln(10) \times RT \approx 6RT \approx 3.5 \text{ kcal/mol}$  at  $\approx 2.5\text{\AA}$   
 $\epsilon \approx 30-40$  at  $\approx 2.5\text{\AA}$ !



# Protein engineering experiments:

$$\phi(r) = \Delta\text{pH} \times 2.3RT \Rightarrow \Rightarrow \epsilon_{\text{eff}}(r)$$



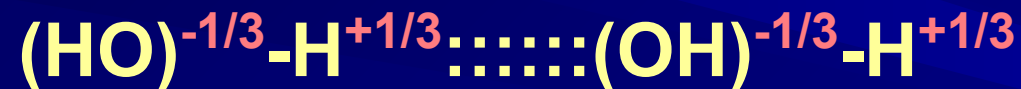
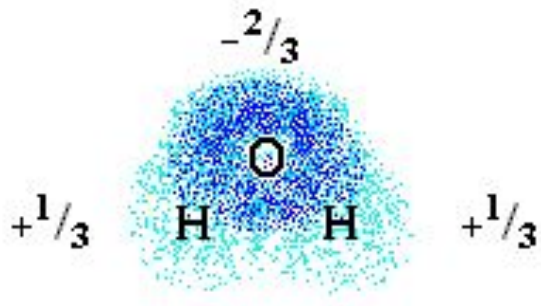
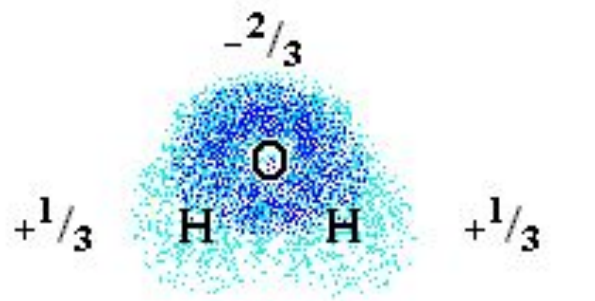


*Sir Alan Roy Fersht, 1943*

Protein engineering



# Dipole interactions (e.g., H-bonds):



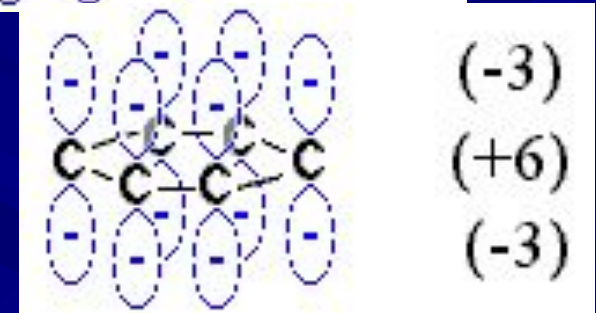
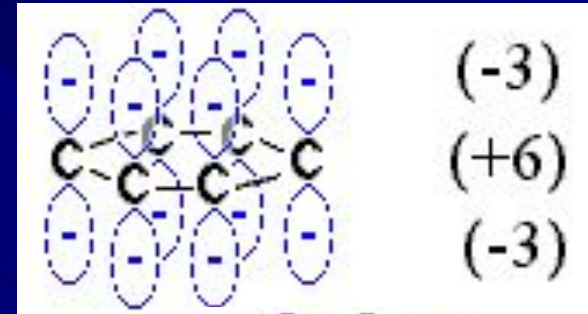
Also: charge-dipole, dipole-quadrupole, etc.

Potentials:

$$\phi_{\text{dipole}} \sim 1/\epsilon r^2$$

$$\phi_{\text{quadrupole}} \sim 1/\epsilon r^3$$

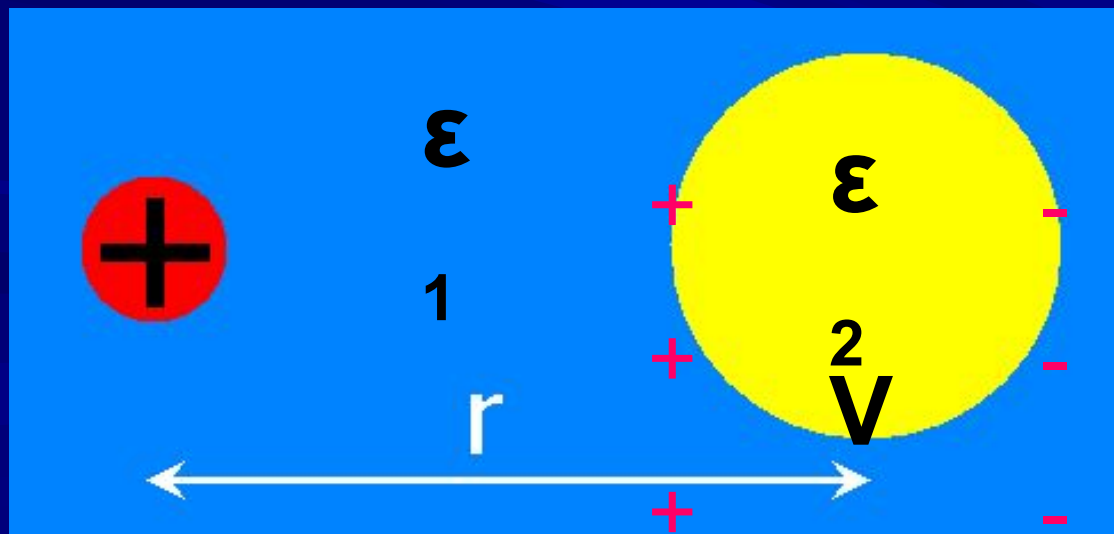
# Quadruple interactions



Electrostatic interactions also occur between charge ( $q$ ) and non-charged **body**, if its  $\epsilon_2$  differs from the media's  $\epsilon_1$ :

$$U \sim q \cdot [1/\epsilon_2 - 1/\epsilon_1] \cdot [\epsilon_2 / (\epsilon_1 + \epsilon_2 / 2)] \cdot V \cdot (1/r^4) \quad \text{at large } r$$

In water: repulsion of charges from non-polar molecules (since here  $\epsilon_1 \gg \epsilon_2$ );  
 in vacuum (where  $\epsilon_1 < \epsilon_2$ ): just the opposite!



# Debye-Hückel screening of electrostatic by ions:

$$U = [q_1 q_2 / \epsilon r] \cdot \exp(-r/D) ;$$

$$\text{in water: } D = 3\text{\AA} \cdot I^{-1/2} ;$$

$$\text{Ionic strength } I = \frac{1}{2} \sum_i C_i (Z_i^{\text{ion}})^2 .$$

$$\text{Usually: } I \approx 0.1 \text{ [mol/liter]}; \quad D \approx 8\text{\AA}.$$

**Electrostatics** is an example of a multi-body  
(charge1, charge2, media, ions) interaction

## Electrostatics is T-dependent;

$$U = (1/\varepsilon) \cdot (q_1 q_2 / r)$$

is free energy ( $U = H - TS$ );

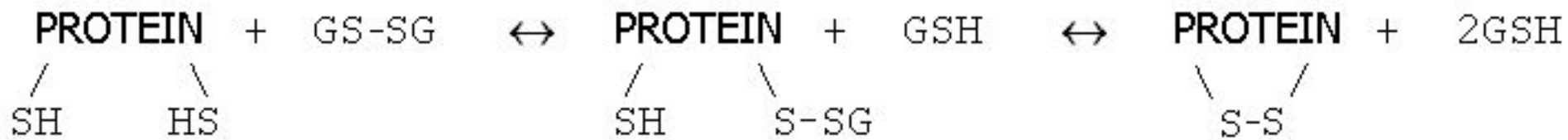
$$\begin{aligned} TS &= T \cdot (-dU/dT) = -T \cdot [d(1/\varepsilon)/dT] \cdot (q_1 q_2 / r) = \\ &= [d \ln(\varepsilon) / d \ln T] \cdot U \end{aligned}$$

in water: when T grows from 273° to 293°K (by 7%),  
ε decreases from 88 to 80 (by 10%):

$$-TS \approx 1.3 U; \quad H \approx -0.3 U$$

In water the entropic term ( $-TS$ ) is the main  
for electrostatics!

# S-S bonds (Cys-Cys)

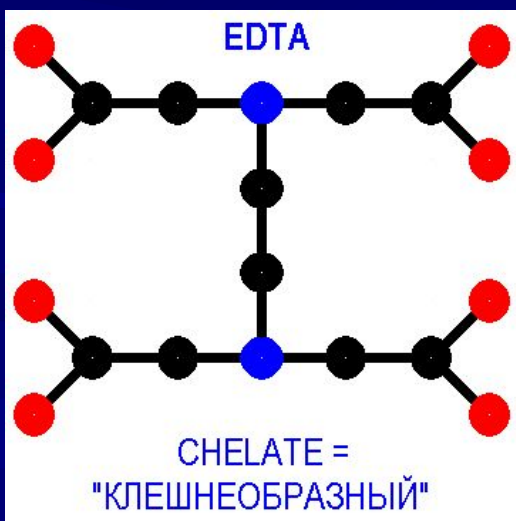
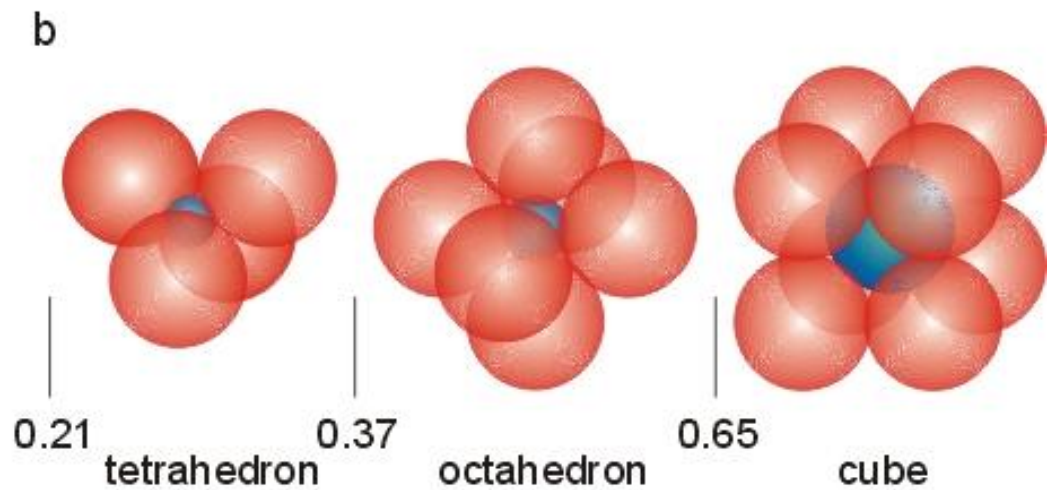
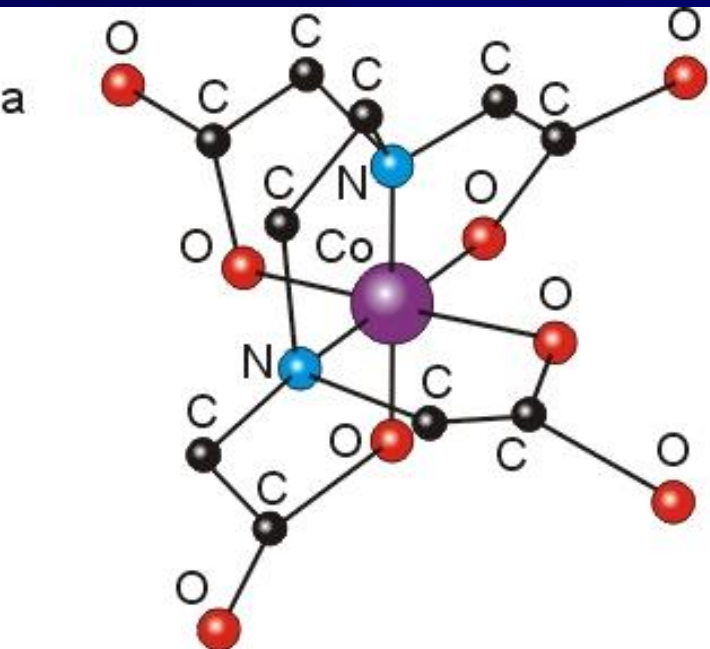


exchange:

entropic force

S-S bond is not stable  
within a cell

# Coordinate bonds (with $Zn^{++}$ , $Fe^{+++}$ , ...)



exchange:

entropic

force