

Physics 2

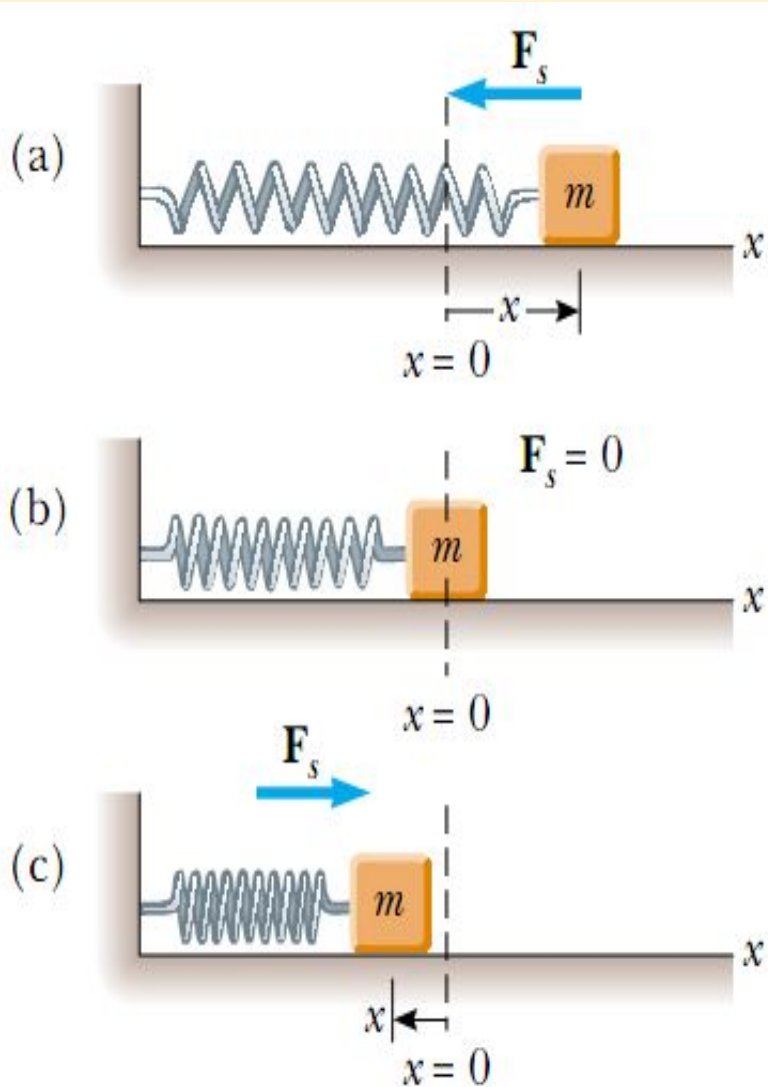
Voronkov Vladimir Vasilyevich

Lecture 1

- Oscillatory motion.
- Simple harmonic motion.
- The simple pendulum.
- Damped harmonic oscillations.
- Driven harmonic oscillations.

Harmonic Motion of Object with Spring

A block attached to a spring moving on a frictionless surface.



(a) When the block is displaced to the right of equilibrium ($x > 0$), the force exerted by the spring acts to the left.

(b) When the block is at its equilibrium position ($x = 0$), the force exerted by the spring is zero.

(c) When the block is displaced to the left of equilibrium ($x < 0$), the force exerted by the spring acts to the right.

So the force acts opposite to displacement.

- x is displacement from equilibrium position.
- Restoring force is given by Hook's law:

$$F_s = -kx$$

- Then we can obtain the acceleration:

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

- That is, the acceleration is proportional to the position of the block, and its direction is opposite the direction of the displacement from equilibrium.

Simple Harmonic Motion

$$a_x = -\frac{k}{m}x$$

- An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

Mathematical Representation of Simple Harmonic Motion

- So the equation for harmonic motion is:

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

- We can denote angular frequency as:

$$\omega^2 = \frac{k}{m}$$

- Then:
- $$\frac{d^2 x}{dt^2} = -\omega^2 x$$

- Solution for this equation is:

$$x(t) = A \cos(\omega t + \phi)$$

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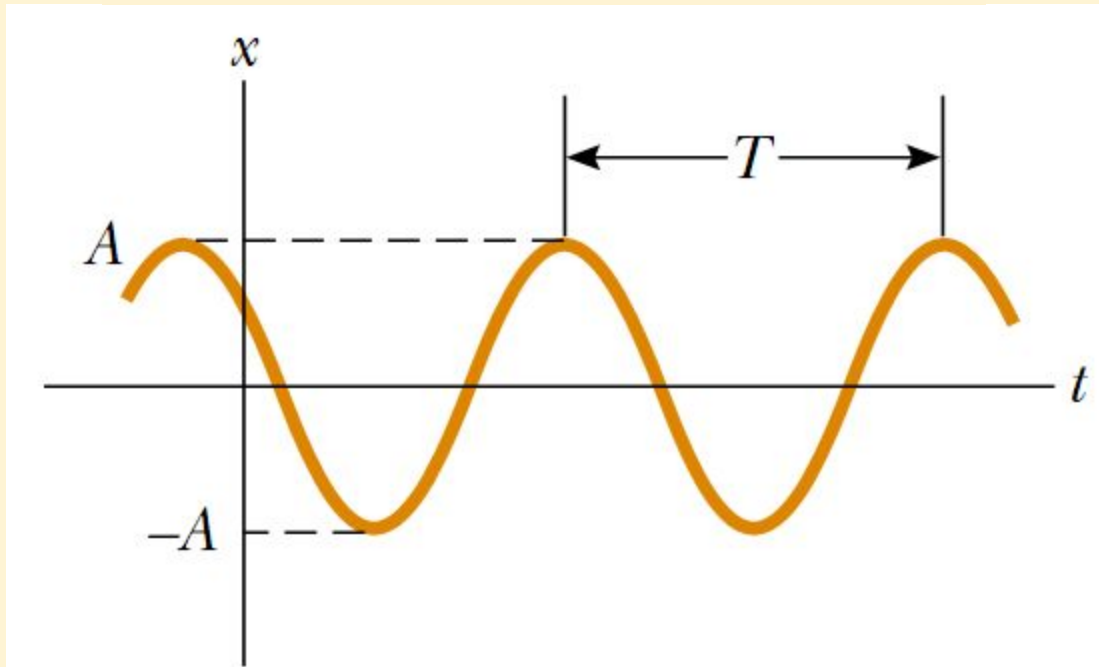
- $A = \text{const}$ is the **amplitude** of the motion
- $\omega = \text{const}$ is the **angular frequency** of the motion

$$\omega = \sqrt{\frac{k}{m}}$$

- $\phi = \text{const}$ is the **phase constant**
- $\omega t + \phi$ is the **phase of the motion**
- $T = \text{const}$ is the **period** of oscillations:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

$$x(t) = A \cos(\omega t + \phi)$$



- The inverse of the period is the **frequency** f of the oscillations:

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

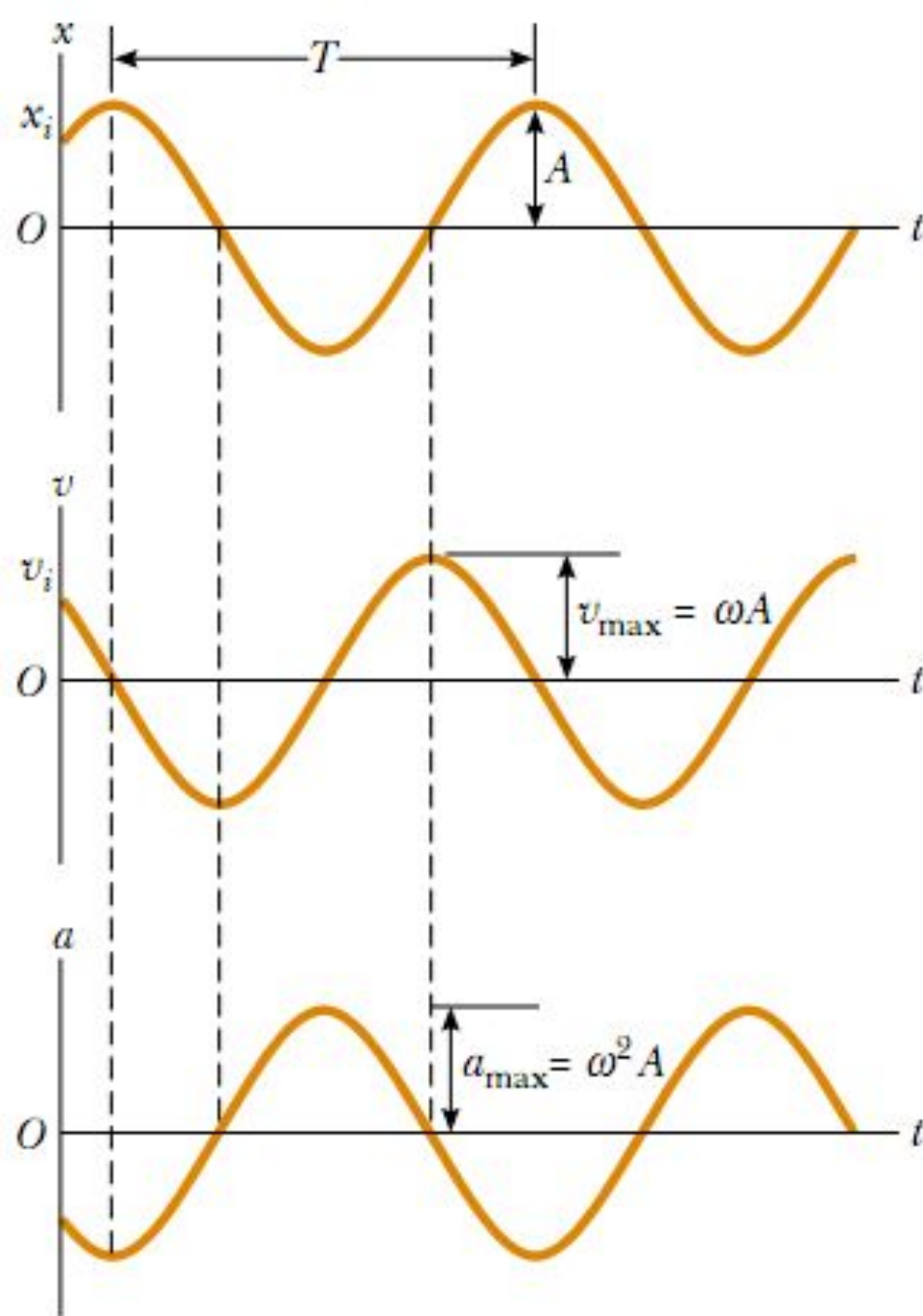
- Then the velocity and the acceleration of a body in simple harmonic motion are:

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A$$

$$a_{\max} = \omega^2 A = \frac{k}{m} A$$



- Position vs time

- Velocity vs time

At any specified time the velocity is 90° out of phase with the position.

- Acceleration vs time

At any specified time the acceleration is 180° out of phase with the position.

Energy of the Simple Harmonic Oscillator

- Assuming that:
 - no friction
 - the spring is massless
- Then the kinetic energy of system spring-body corresponds only to that of the body:

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

- The potential energy in the spring is:

$$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

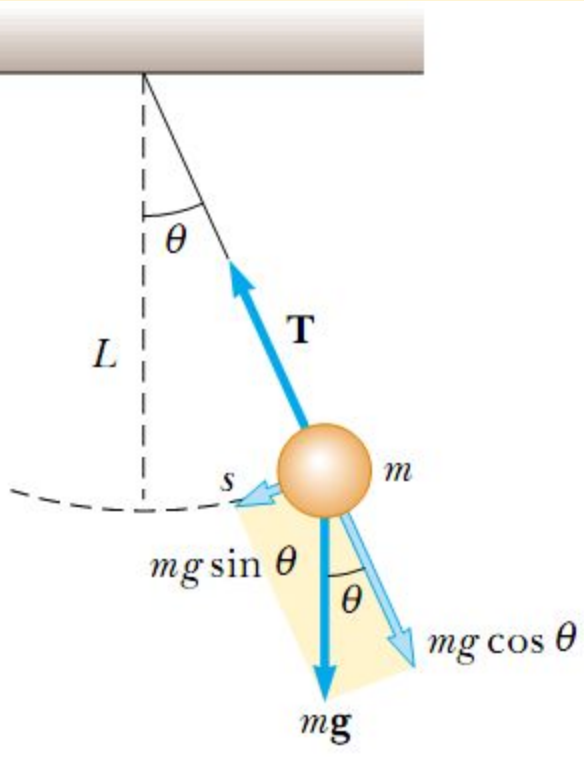
- The total mechanical energy of simple harmonic oscillator is:

$$E = K + U = \frac{1}{2} kA^2 [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$E = \frac{1}{2} kA^2$$

- That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude.

Simple Pendulum



- Simple pendulum consists of a particle-like bob of mass m suspended by a light string of length L that is fixed at the upper end.
- The motion occurs in the vertical plane and is driven by the gravitational force.
- When θ is small, a simple pendulum oscillates in simple harmonic motion about the equilibrium position $\theta = 0$. The restoring force is $-mg \sin \theta$, the component of the gravitational force tangent to the arc.

$$F_t = -mg \sin \theta = m \frac{d^2 s}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = -\frac{g}{L} \theta$$

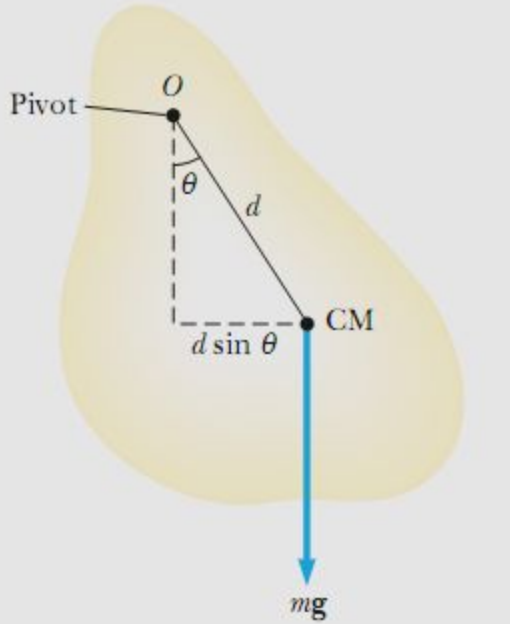
$$\theta = \theta_{\max} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

- The period and frequency of a simple pendulum depend only on the length of the string and the acceleration due to gravity.
- The simple pendulum can be used as a timekeeper because its period depends only on its length and the local value of g .

Physical Pendulum



If a hanging object oscillates about a fixed axis that does not pass through its center of mass and the object cannot be approximated as a point mass, we cannot treat the system as a simple pendulum. In this case the system is called a **physical pendulum**.

- Applying the rotational form of the second Newton's law:

$$-mgd \sin \theta = I \frac{d^2 \theta}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = - \left(\frac{mgd}{I} \right) \theta = - \omega^2 \theta$$

- The solution is: $\theta = \theta_{\max} \cos(\omega t + \phi)$

$$\omega = \sqrt{\frac{mgd}{I}}$$

- The period is $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mgd}}$

Damped Harmonic Oscillations

- In many real systems, nonconservative forces, such as friction, retard the motion. Consequently, the mechanical energy of the system diminishes in time, and the motion is **damped**. The retarding force can be expressed as $\mathbf{R} = -b\mathbf{v}$ ($b = \text{const}$ is the damping coefficient) and the restoring force of the system is $-k\mathbf{x}$ then:

$$-kx - b \frac{dx}{dt} = m \frac{d^2 x}{dt^2}$$

- The solution for small b is

$$x = Ae^{-\frac{b}{2m}t} \cos(\omega t + \phi)$$

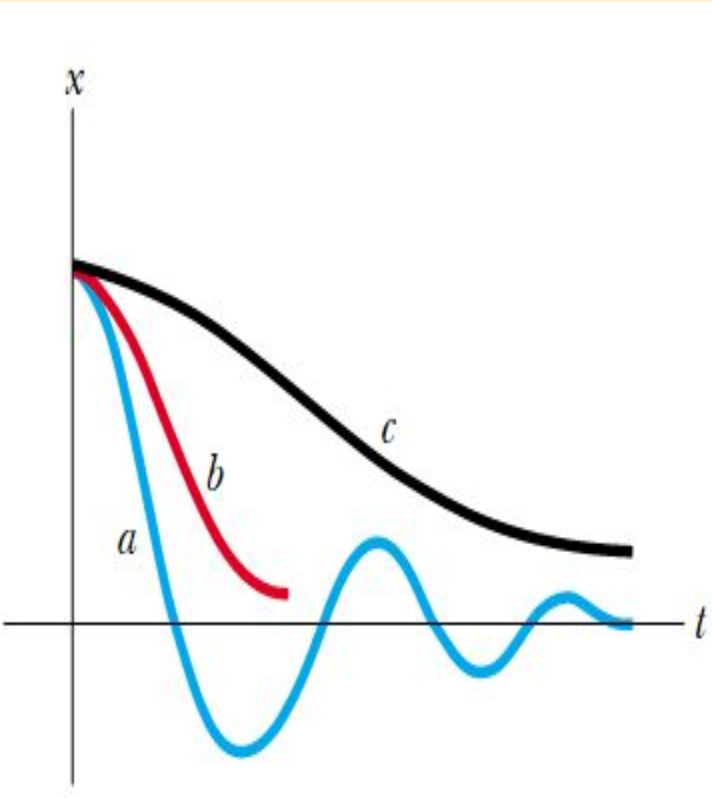
$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

- When the retarding force is small, the oscillatory character of the motion is preserved but the amplitude decreases in time, with the result that the motion ultimately ceases.

- The angular frequency can be expressed through $\omega_0 = (k/m)^{1/2}$ – the natural frequency of the system (the undamped oscillator):

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

(a) underdamped oscillator:
 $R_{max} = bV_{max} < kA$. System oscillates with damping amplitude



b) critically damped oscillator:
when b has critical value $b_c = 2m\omega_0$. System does not oscillate, just returns to the equilibrium position.

c) overdamped oscillator:
 $R_{max} = bV_{max} > kA$ and
 $b/(2m) > \omega_0$. System does not oscillate, just returns to the equilibrium position.

Driven Harmonic Oscillations

- A driven (or forced) oscillator is a damped oscillator under the influence of an external periodical force $F(t) = F_0 \sin(\omega t)$. The second Newton's law for forced oscillator is:

$$F_0 \sin \omega t - b \frac{dx}{dt} - kx = m \frac{d^2 x}{dt^2}$$

- The solution of this equation is:

$$x = A \cos(\omega t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

- The forced oscillator vibrates at the frequency of the driving force
- The amplitude of the oscillator is constant for a given driving force.
- For small damping, the amplitude is large when the frequency of the driving force is near the natural frequency of oscillation, or when $\omega \approx \omega_0$.
- The dramatic increase in amplitude near the natural frequency is called **resonance**, and the **natural frequency** ω_0 is also called the **resonance frequency** of the system.

Resonance

- So resonance happens when the driving force frequency is close to the natural frequency of the system: $\omega \approx \omega_0$. At resonance the amplitude of the driven oscillations is the largest.
- In fact, if there were no damping ($b = 0$), the amplitude would become infinite when $\omega = \omega_0$: This is not a realistic physical situation, because it corresponds to the spring being stretched to infinite length. A real spring will snap rather than accept an infinite stretch; in other words, some form of damping will ultimately occur. But it does illustrate that, at resonance, the response of a harmonic system to a driving force can be catastrophically large.

Units in Si

- spring constant k N/m=kg/s²
- damping coefficient b kg/s
- phase φ rad (or degrees)
- angular frequency ω rad/s
- frequency f 1/s
- period T s