KAZAKH BRITISH TECHNICAL UNIVERSITY

Physics 1 for KMA

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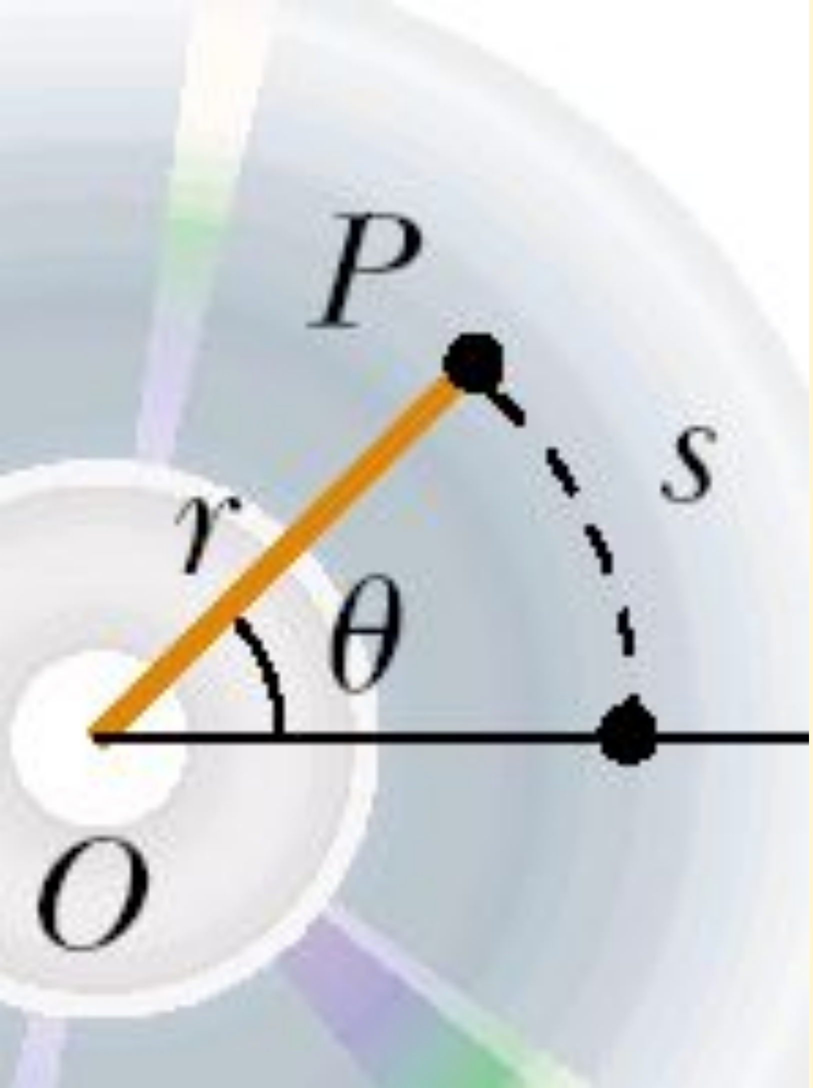
Lecture 4

- Rotation of rigid bodies.
- Angular momentum and torque.
- Properties of fluids.
- Flotation.
- Bernulli equation.

Rotation of Rigid Bodies

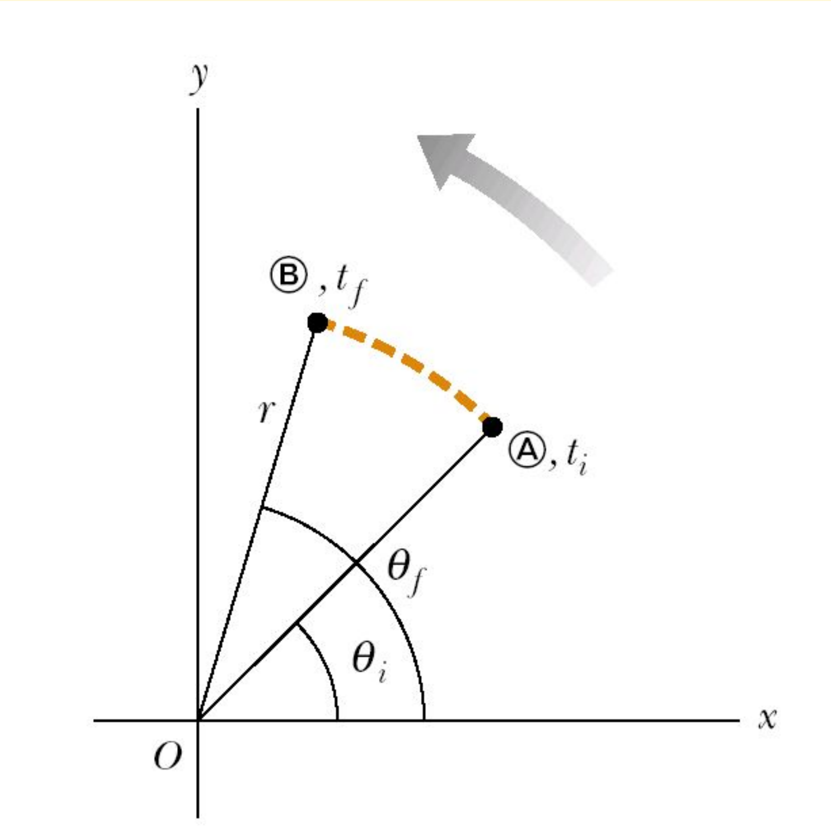
- When a rigid object is rotating about a *fixed* axis, every particle of the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration. So the rotational motion of the entire rigid object as well as individual particles in the object can be described by three angles. Using these three angles we can greatly simplify the analysis of rigid-object rotation.

Radians



$$\theta = \frac{s}{r}$$

Angular kinematics



- Angular displacement:

$$\Delta\theta \equiv \theta_f - \theta_i$$

- Instantaneous angular speed:

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Instantaneous angular acceleration:

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Angular and linear quantities

- Every particle of the object moves in a circle whose center is the axis of rotation.
- Linear velocity: $v = r\omega$
- Tangential acceleration: $a_t = r\alpha$
- Centripetal acceleration: $a_c = \dfrac{v^2}{r} = r\omega^2$

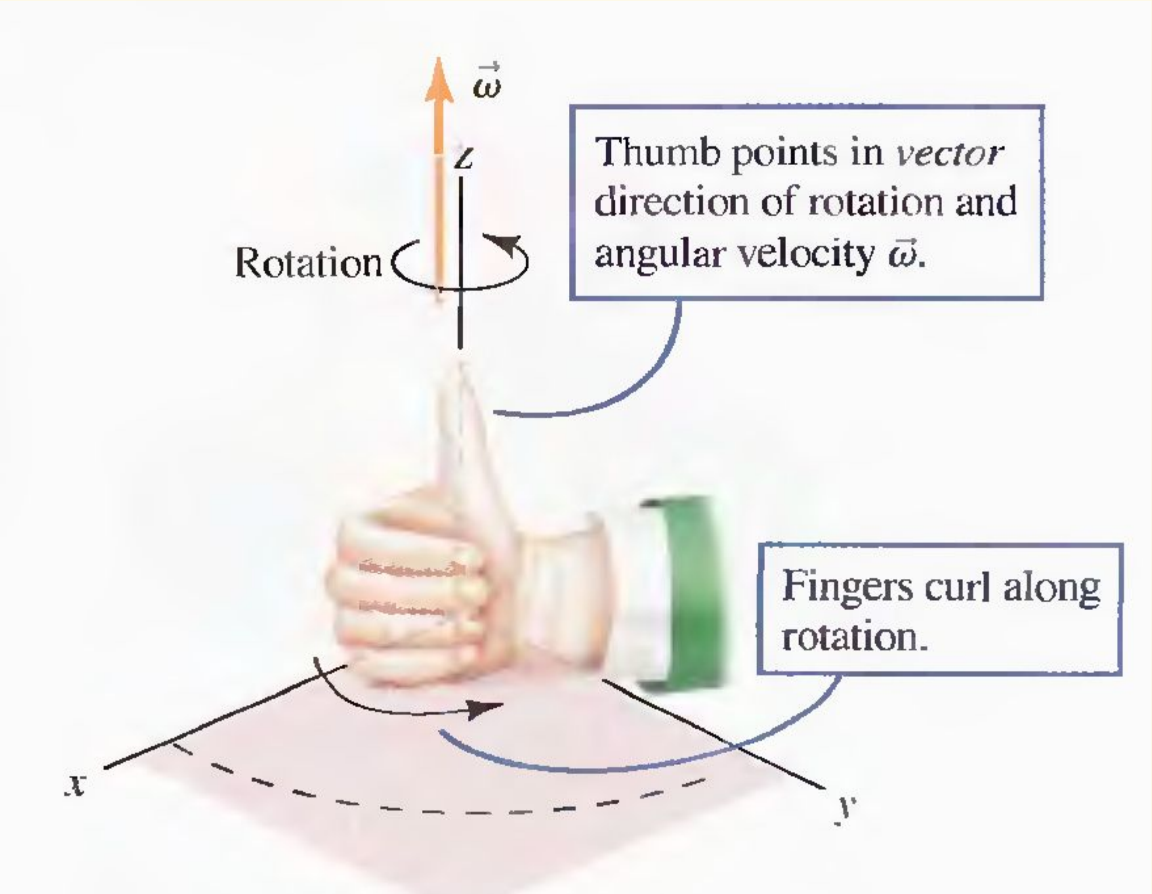
Total linear acceleration

- Tangential acceleration is perpendicular to the centripetal one, so the magnitude of total linear acceleration is

$$a = \sqrt{a_t{}^2 + a_r{}^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

Angular velocity

- Angular velocity is a vector.

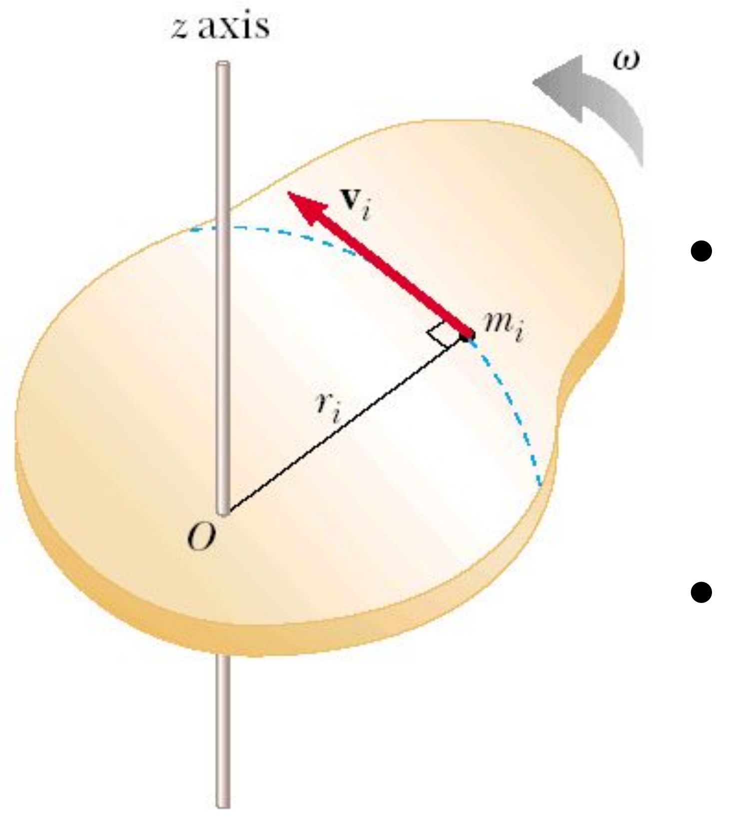


The right hand rule is applied: If the fingers of your right hand curl along with the rotation your thumb will give the direction of the angular velocity.

Rotational Kinetic Energy

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right) \omega^2$$



- Moment of rotational inertia

$$I \equiv \sum_i m_i r_i^2$$

- Rotational kinetic energy

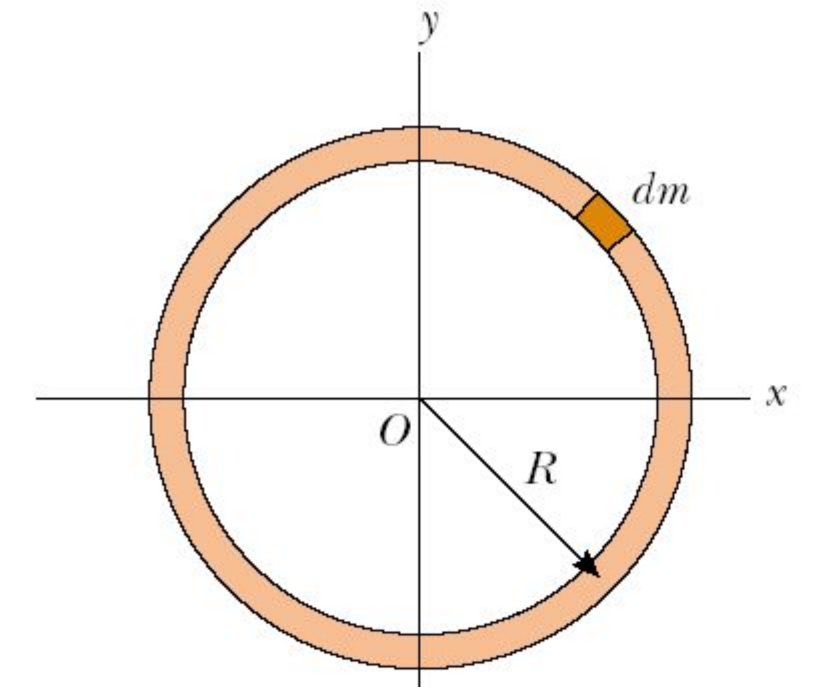
$$K_R = \frac{1}{2} I \omega^2$$

Calculations of Moments of Inertia

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i{}^2 \Delta m_i = \int r^2 \, dm$$

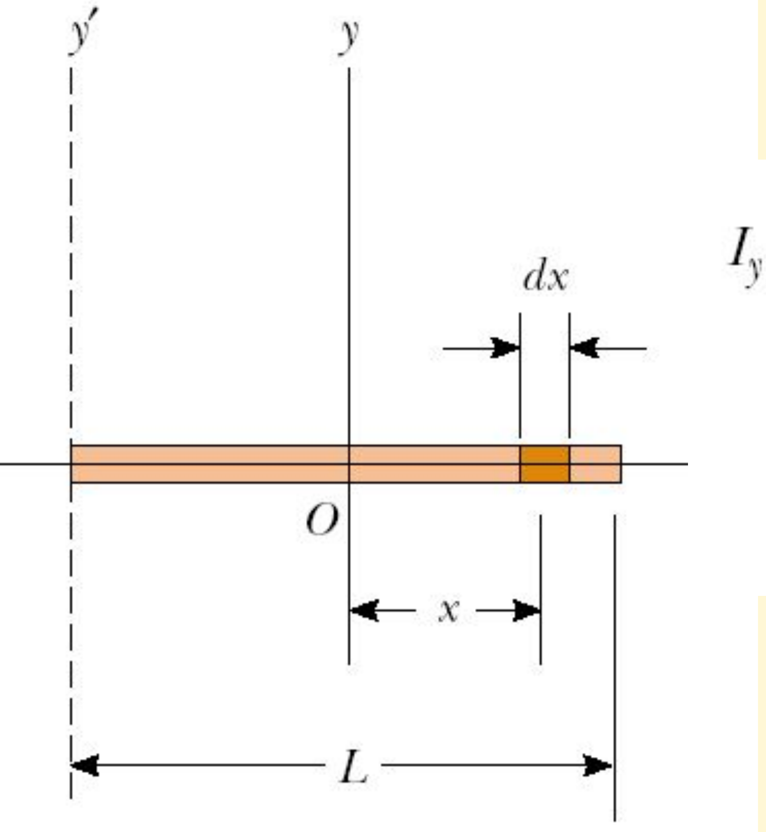
$$I = \int \rho r^2 \, dV$$

Uniform Thin Hoop



$$I_z = \int r^2\, dm = R^2 \int dm = \boxed{MR^2}$$

Uniform Rigid Rod



$$dm = \lambda\, dx = \frac{M}{L}\, dx$$

$$I_y = \int r^2\, dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\, dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2\, d$$

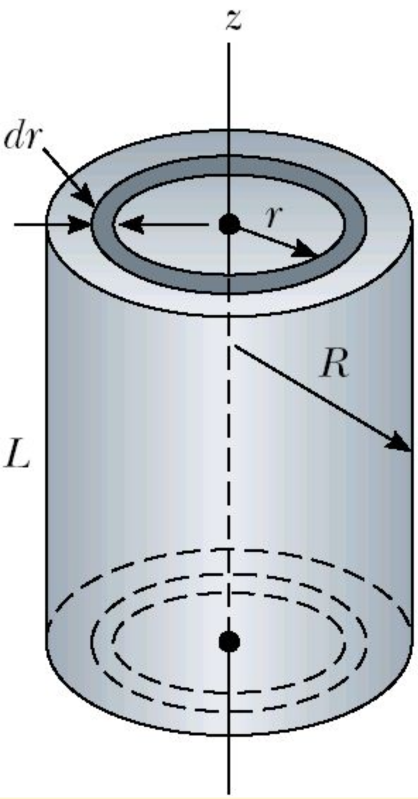
$$= \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{1}{12} ML^2$$

Uniform Solid Cylinder

$$dV = L\,dA = L(2\pi r)\,dr.$$

$$dm = \rho\, dV = 2\pi\rho L r\, dr.$$

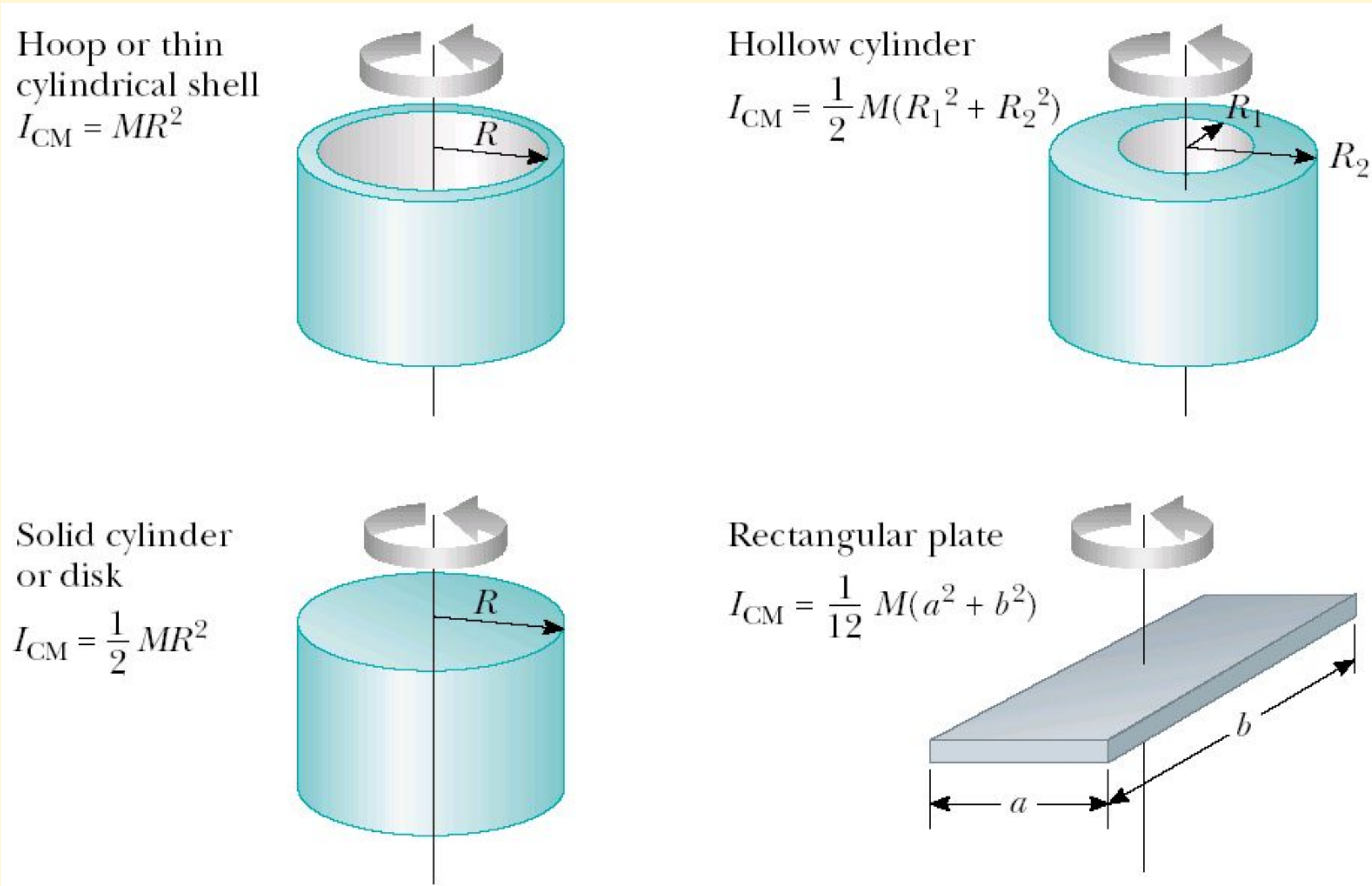
$$I_z = \int r^2\, dm = \int r^2(2\pi\rho L r\, dr) = 2\pi\rho L \int_0^R r^3 dr = \tfrac{1}{2}\pi\rho L R^4$$



$$\rho = M/V = M/\pi R^2 L.$$

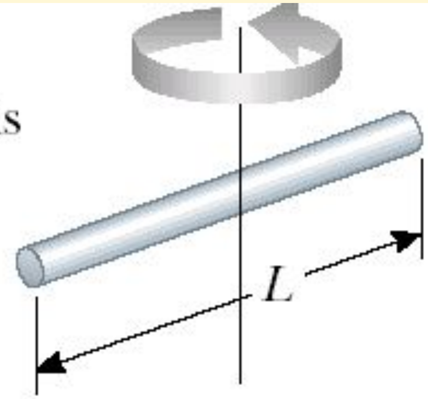
$$I_z = \tfrac{1}{2} M R^2$$

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries



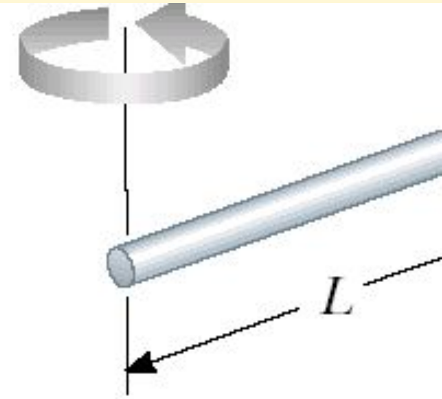
Long thin rod with rotation axis through center

$$I_{CM} = \frac{1}{12} ML^2$$



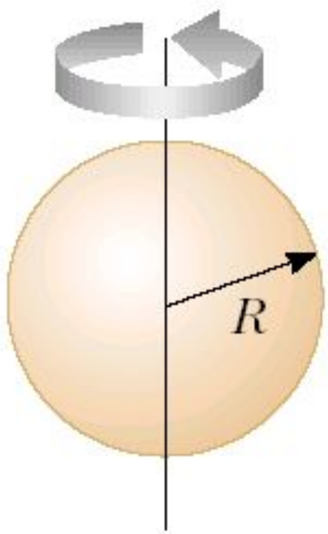
Long thin rod with rotation axis through end

$$I = \frac{1}{3} ML^2$$



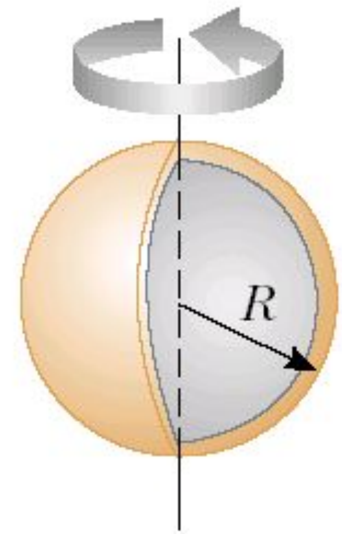
Solid sphere

$$I_{CM} = \frac{2}{5} MR^2$$



Thin spherical shell

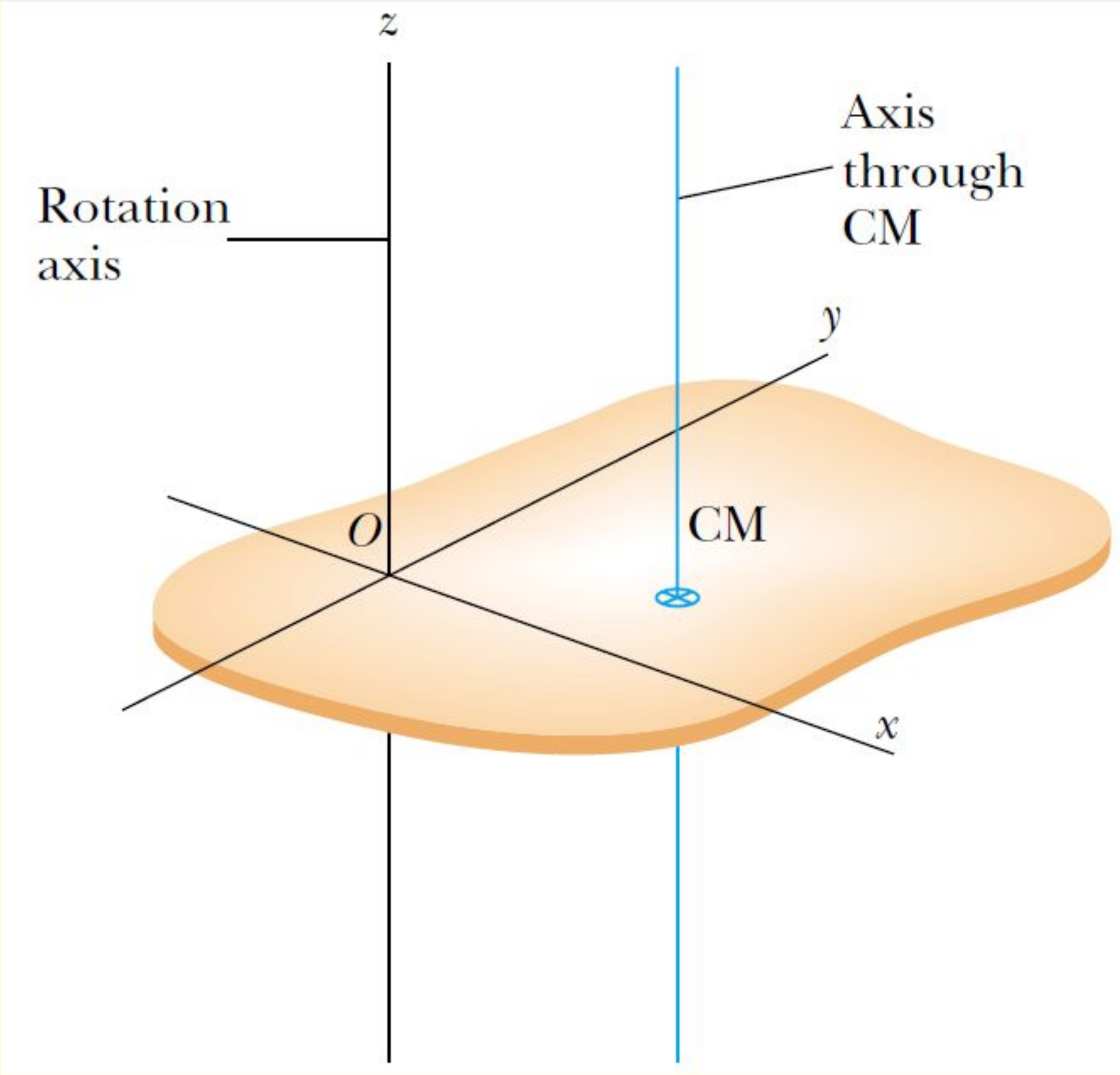
$$I_{CM} = \frac{2}{3} MR^2$$



Parallel-axis theorem

- Suppose the moment of inertia about an axis through the center of mass of an object is I_{CM}. Then the moment of inertia about any axis parallel to and a distance D away from this axis is

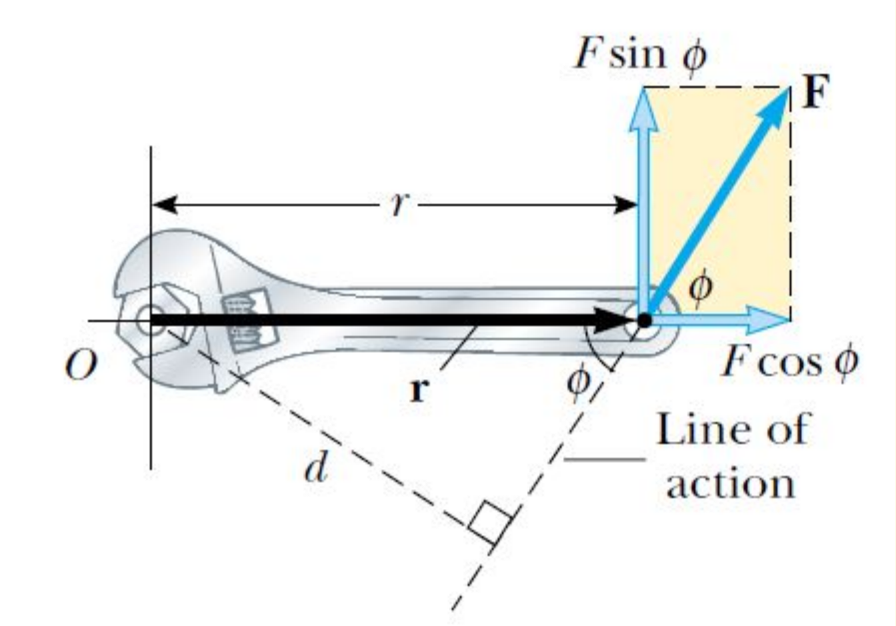
$$\boxed{I = I_{\mathrm{CM}} + MD^2}$$


z
Rotation axis
Axis through CM
y
O
CM
x

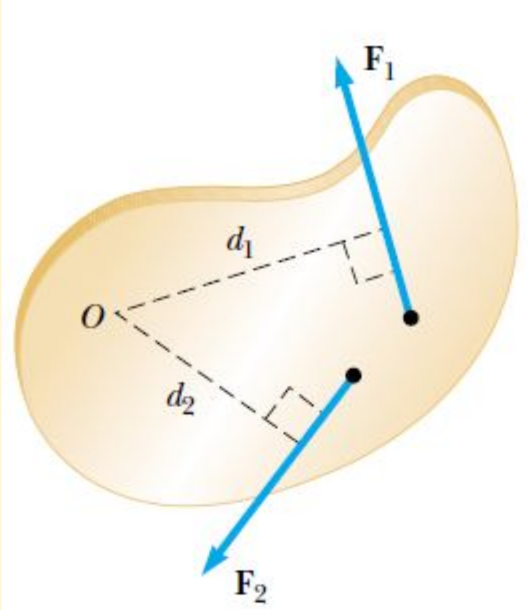
Torque

When a force is exerted on a rigid object pivoted about an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about some axis is measured by a vector quantity called torque τ (Greek tau).

$$\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F}$$



The force F has a greater rotating tendency about axis *O* as *F* increases and as the moment arm *d* increases. The component *F* sinφ tends to rotate the wrench about axis *O*.



tends to rotate the object
wise about *O*, and F_2 tends to
wise.

The force F_1 tends to rotate the object counterclockwise about *O*, and F_2 tends to rotate it clockwise.

We use the convention that the sign of the torque resulting from a force is positive if the turning tendency of the force is counterclockwise and is negative if the turning tendency is clockwise. Then

$$\sum \tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$$

Torque is not Force
Torque is not Work

Torque should not be confused with force. Forces can cause a change in linear motion, as described by Newton's second law. Forces can also cause a change in rotational motion, but the effectiveness of the forces in causing this change depends on both the forces and the moment arms of the forces, in the combination that we call *torque*. Torque has units of force times length—newton · meters in SI units—and should be reported in these units.

Do not confuse torque and work, which have the same units but are very different concepts.

Rotational Dynamics

$$\Sigma \mathbf{F} = d\mathbf{p}/dt$$

$$\mathbf{r} \times \sum \mathbf{F} = \sum \boldsymbol{\tau} = \mathbf{r} \times \frac{d\mathbf{p}}{dt}$$

Let's add $\frac{d\mathbf{r}}{dt} \times \mathbf{p}$ which equals zero, as $d\mathbf{r}/dt = \mathbf{v}$ and $\mathbf{v}$ and $\mathbf{p}$ are parallel.

Then: $$\sum \boldsymbol{\tau} = \mathbf{r} \times \frac{d\mathbf{p}}{dt} + \frac{d\mathbf{r}}{dt} \times \mathbf{p}$$

$$\sum \boldsymbol{\tau} = \frac{d(\mathbf{r} \times \mathbf{p})}{dt}$$

$$\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$$

Rotational analogue of Newton's second law

- Quantity ***L*** is an instantaneous angular momentum.

$$\sum \boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

- The torque acting on a particle is equal to the time rate of change of the particle's angular momentum.

Net External Torque

The net external torque acting on a system about some axis passing through an origin in an inertial frame equals the time rate of change of the total angular momentum of the system about that origin:

$$\sum \boldsymbol{\tau}_{\text{ext}} = \frac{d\mathbf{L}_{\text{tot}}}{dt}$$

Angular Momentum of a Rotating Rigid Object

- Angular momentum for each particle of an object:

$$L_i = m_i r_i^2 \omega$$

- Angular momentum for the whole object:

$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega = \left(\sum_i m_i r_i^2\right)\omega$$

- Thus:

$$L_z = I\omega$$

Angular acceleration

$$\frac{dL_z}{dt} = I\frac{d\omega}{dt} = I\alpha$$

$$\sum \tau_{\text{ext}} = I\alpha$$

The Law of Angular Momentum Conservation

- The total angular momentum of a system is constant if the resultant external torque acting on the system is zero, that is, if the system is isolated.

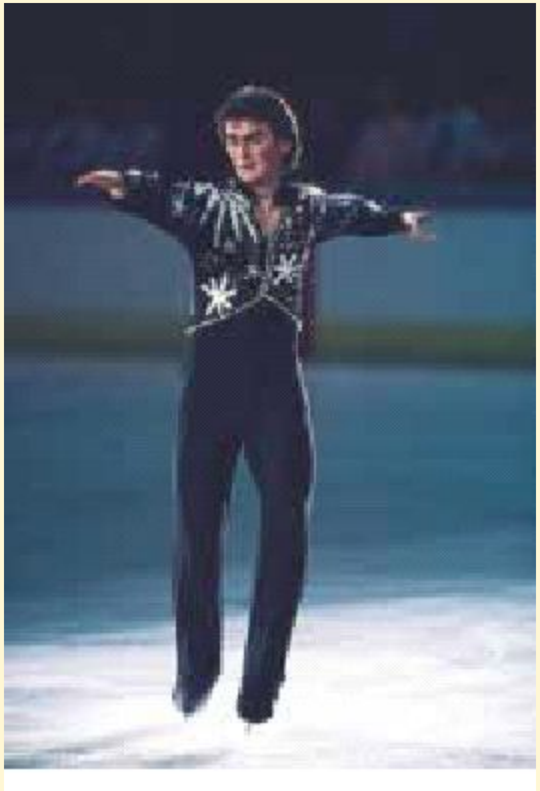
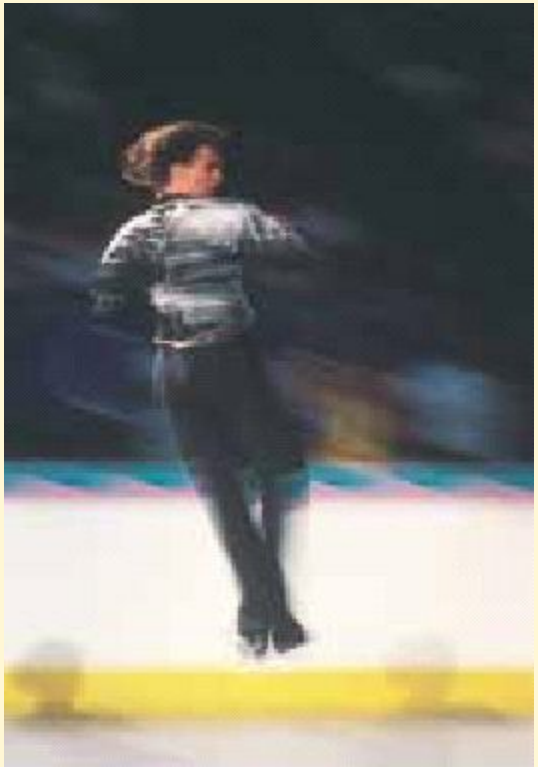
$$\sum \boldsymbol{\tau}_{\text{ext}} = \frac{d\mathbf{L}_{\text{tot}}}{dt} = 0$$

$$\mathbf{L}_{\text{tot}} = \text{constant}$$

$$\mathbf{L}_{\text{tot}} = \text{constant}$$

$$I_i \omega_i = I_f \omega_f = \text{constant}$$

- Change in internal structure of a rotating body can result in change of its angular velocity.

- When a rotating skater pulls his hands towards his body he spins faster.

Three Laws of Conservation for an Isolated System

$$\left.\begin{aligned} E_i &= E_f \\ \mathbf{p}_i &= \mathbf{p}_f \\ \mathbf{L}_i &= \mathbf{L}_f \end{aligned}\right\}$$

Full mechanical energy, linear momentum and angular momentum of an isolated system remain constant.

Work-Kinetic Theory for Rotations

- Similarly to linear motion:

$$dW \equiv \vec{\tau} \cdot d\vec{\theta}.$$

$$W = \int_{\theta_0}^{\theta} \tau \, d\theta = \int_0^t I \frac{d\omega}{dt} \omega dt = \int_0^t I \frac{1}{2} \frac{d\omega^2}{dt} dt$$

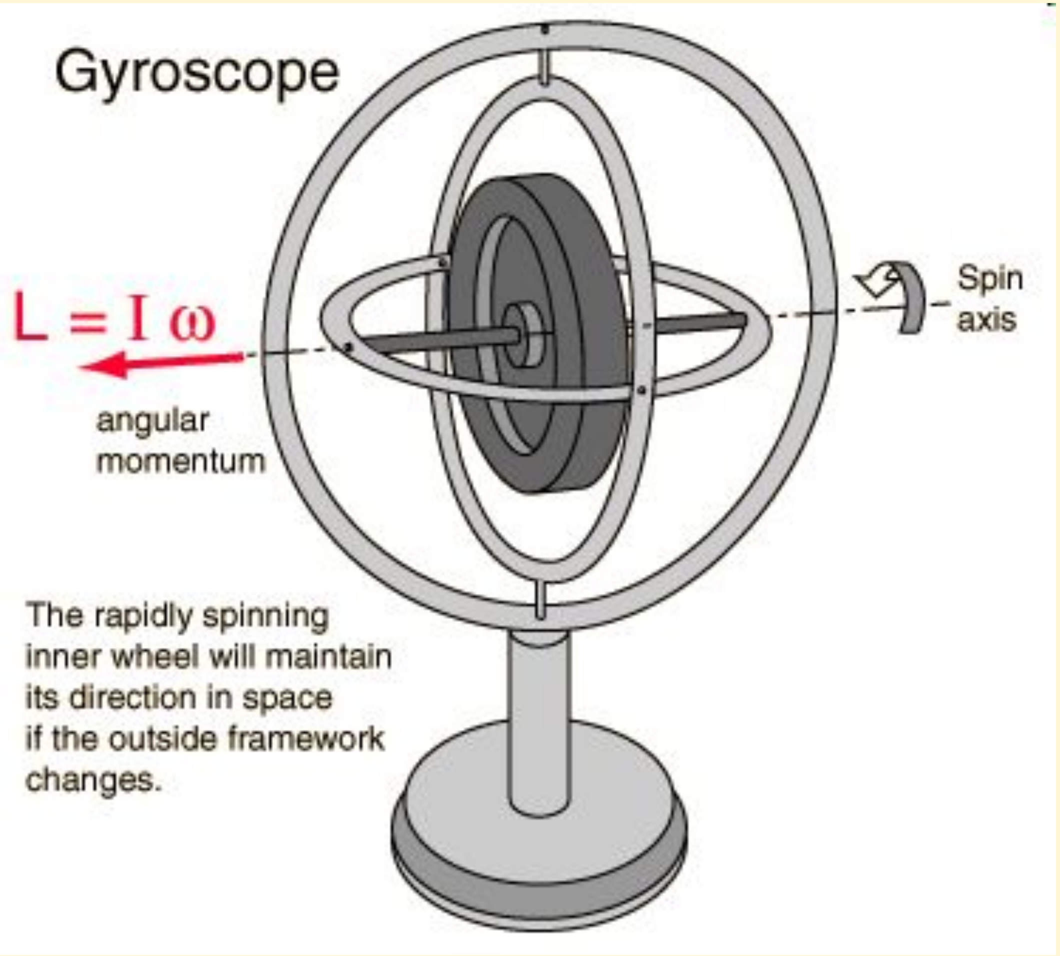
$$= \frac{1}{2} I \int_{\omega_0^2}^{\omega^2} d\omega^2 = \frac{1}{2} I(\omega^2 - \omega_0^2) = K - K_0.$$

- The net work done by external forces in rotating a symmetric rigid object about a fixed axis equals the change in the object’s rotational energy.

Equations for Rotational and Linear Motions

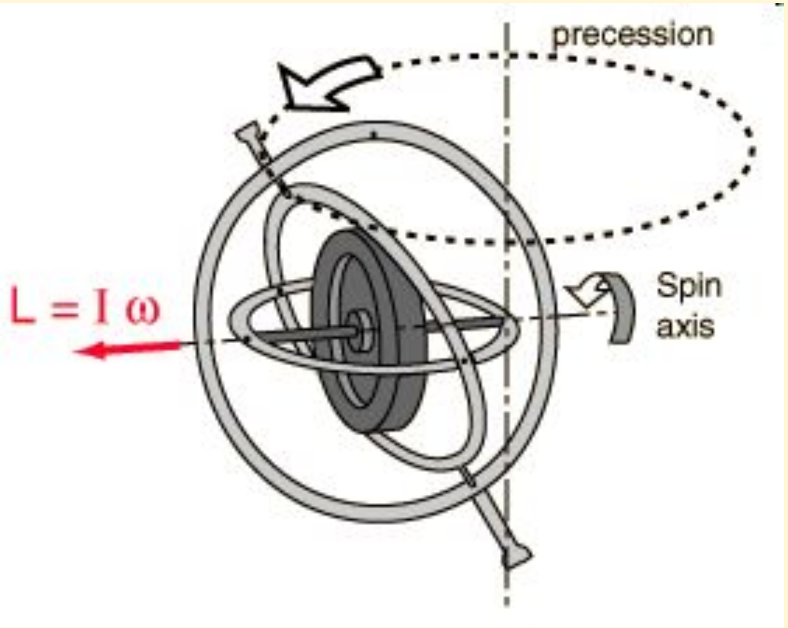
Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha_t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

Gyroscope



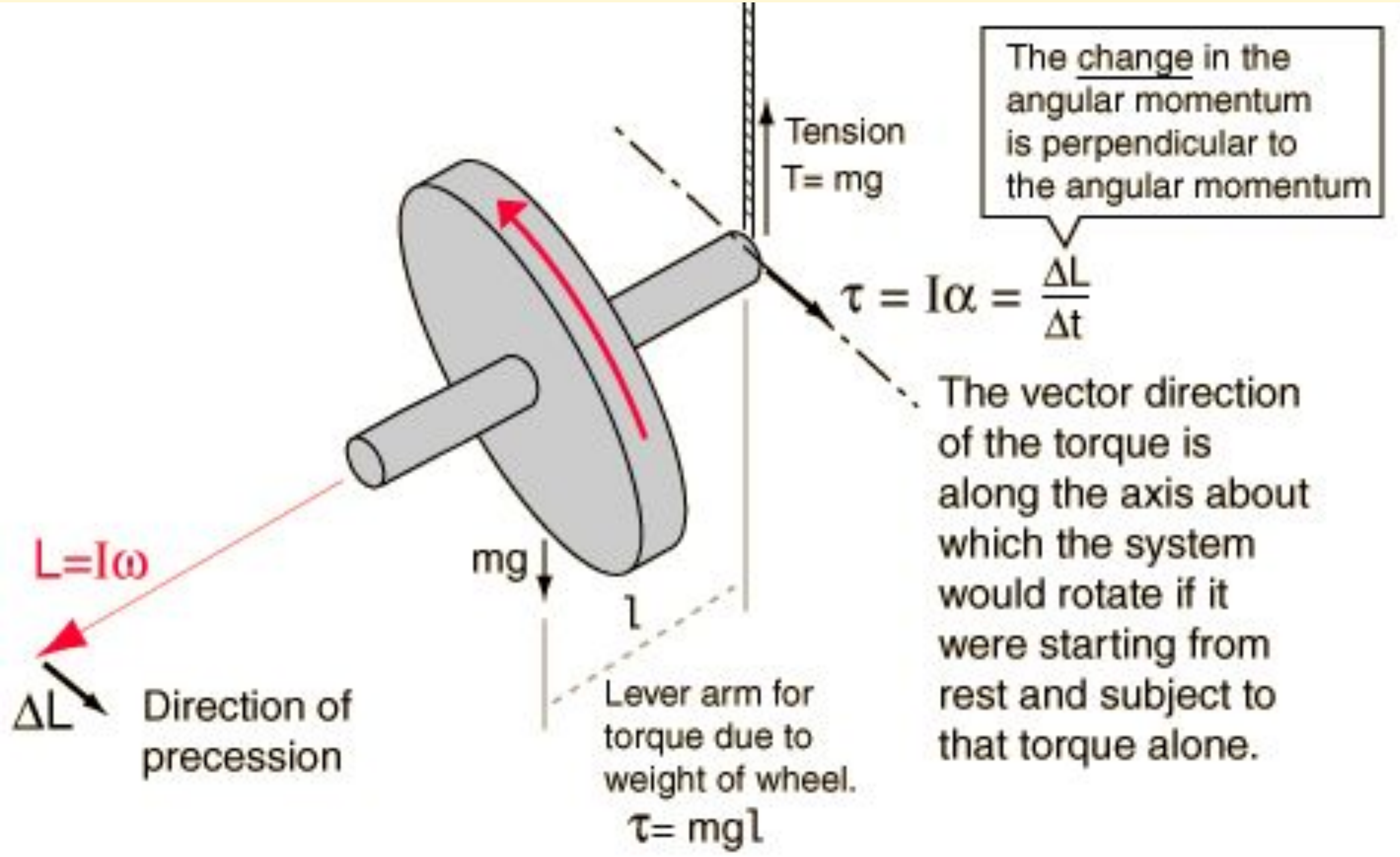
One typical type of gyroscope is made by suspending a relatively massive rotor inside three rings called gimbals. Mounting each of these rotors on high quality bearing surfaces insures that very little torque can be exerted on the inside rotor.

At high speeds, the gyroscope exhibits extraordinary stability of balance and maintains the direction of the high speed rotation axis of its central rotor. The implication of the conservation of angular momentum is that the angular momentum of the rotor maintains not only its magnitude, but also its direction in space in the absence of external torque. The classic type gyroscope finds application in gyro-compasses.



If a gyroscope is tipped, the gimbals will try to reorient to keep the spin axis of the rotor in the same direction. If released in this orientation, the gyroscope will precess in the direction shown because of the torque exerted by gravity on the gyroscope.

Precession of Spinning Wheel



Fluids and liquids

Relative density

- **Relative density** or **specific gravity** is the ratio of the density of a substance to the density of a given reference material. Specific gravity usually means relative density with respect to water.

$$RD = \frac{\rho_{\mathrm{substance}}}{\rho_{\mathrm{reference}}}$$

- If the **reference material** is **water** then a substance with a relative density (or specific gravity) less than 1 will float in water. For example, an ice cube, with a relative density of about 0.91, will float. A substance with a relative density greater than 1 will sink.

Substance	Relative density
Alcohol	0.82
Mercury	13.95
Paraffin	0.80
Petrol	0.72
Water (4°C)	1.00
Sea water	1.02
Aluminum	2.72
Brass	8.48
Cadmium	8.57
Chromium	7.03
Copper	8.79
Cast iron	7.20
Lead	11.35
Nickel	8.73

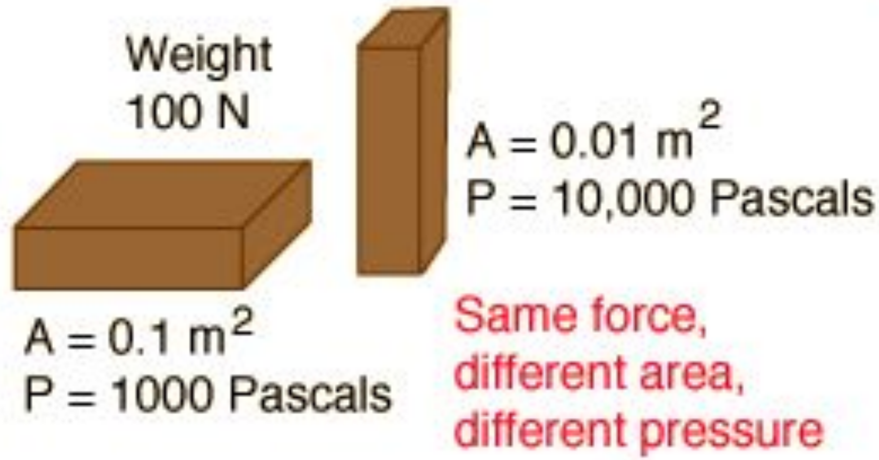
Substance	Relative density
Nylon	1.12
PVC	1.36
Rubber	0.96
Steel	7.82
Tin	7.28
Zinc	7.12
Acetylene	0.0017
Dry air	0.0013
Carbon dioxide	0.00198
Carbon monoxide	0.00126
Hydrogen	0.00009
Nitrogen	0.00125
Oxygen	0.00143

- Specific volume of a substance is the ratio of the substance's volume to its mass. It is the reciprocal of density and is an intrinsic property of matter:

$$v = \frac{V}{m} = \rho^{-1}$$

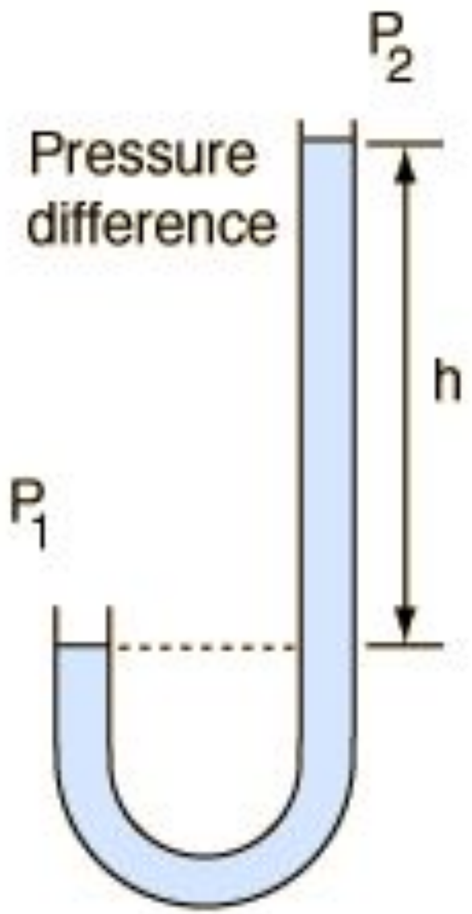
Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$



$$P = \frac{Force}{Area} = \frac{F}{A} = \frac{F \cdot d}{A \cdot d} = \frac{W}{V} = \frac{Energy}{Volume}$$

Manometer



- The difference in fluid height in a liquid column manometer is proportional to the pressure difference.
- $P_1 - P_2 = \rho g h$

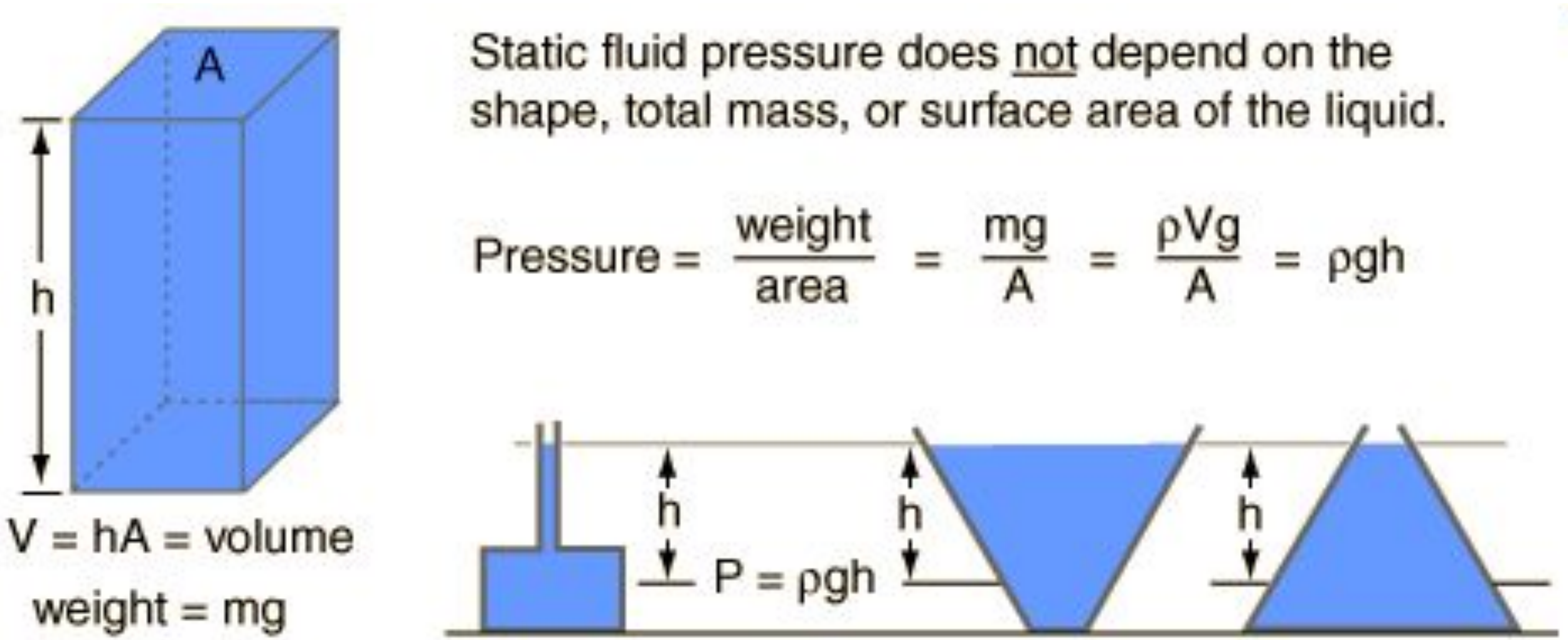
Static Fluid Pressure

$P_{\text{static fluid}} = \rho g h$ where $\rho = m/V$ = fluid density

g = gravitational acceleration

h = depth of fluid

- The pressure exerted by a static fluid depends only upon the depth of the fluid, the density of the fluid, and the acceleration of gravity



Pressure Thrust

- Thrust is a total force in a particular direction. The unit of thrust, therefore is the same as that of force: Newtons (N). Pressure is the force or thrust applied per unit area.

$$F = P \cdot A$$

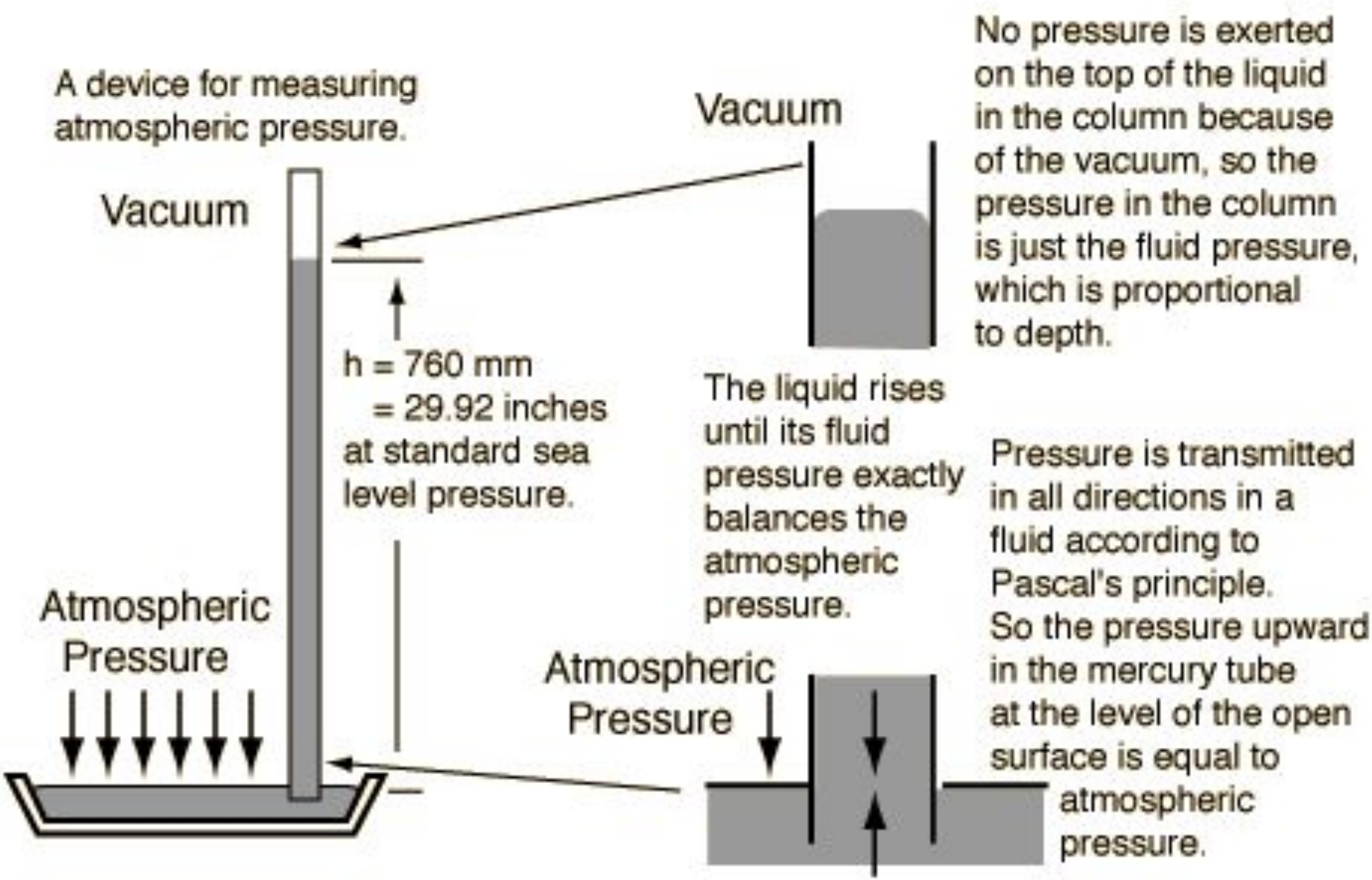
Atmospheric Pressure

- The surface of the earth is at the bottom of an atmospheric sea. The standard atmospheric pressure is measured in various units:
- 1 atmosphere = 760 mmHg = 101.3 KPa
- The **bar** is a unit of pressure defined as 100 kilopascals. It is about equal to the atmospheric pressure on Earth at sea level.
- The unit **mmHg** is often called **torr**, particularly in vacuum applications: 760 mmHg = 760 torr

Atmospheric constituents

Component	Volume Percentage	Partial Pressure (mmHg)	Molecular Mass
Nitrogen (N_2)	78.08	593.4	28.013
Oxygen (O_2)	20.95	159.2	31.998
Argon (Ar)	0.93	7.1	39.948
Carbon dioxide (CO_2)	0.03	0.2	43.999
	99.99%	759.9 mmHg	28.95 avg

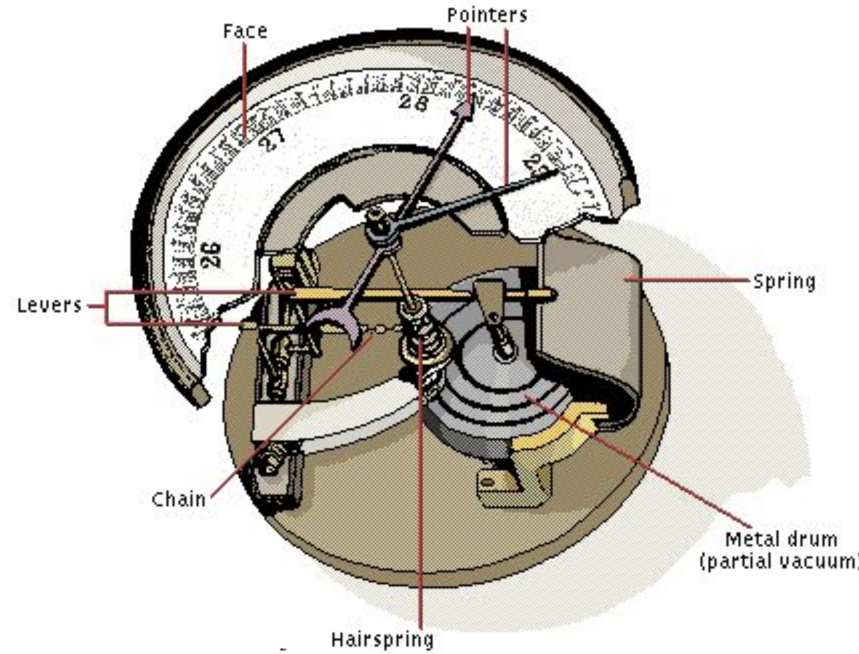
Barometer


A device for measuring atmospheric pressure.
Vacuum
Vacuum
No pressure is exerted on the top of the liquid in the column because of the vacuum, so the pressure in the column is just the fluid pressure, which is proportional to depth.
h = 760 mm
= 29.92 inches
at standard sea level pressure.
The liquid rises until its fluid pressure exactly balances the atmospheric pressure.
Pressure is transmitted in all directions in a fluid according to Pascal's principle.
So the pressure upward in the mercury tube at the level of the open surface is equal to atmospheric pressure.
Atmospheric Pressure
Atmospheric Pressure

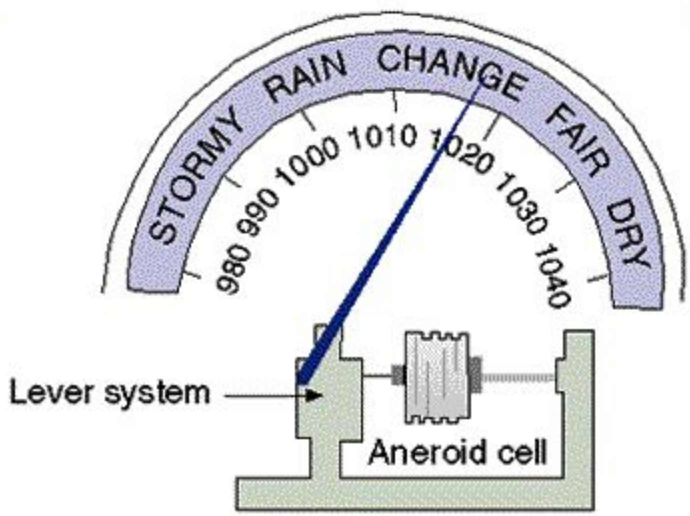
Aneroid barometer

- An aneroid barometeru ses a small, flexible metal box called an aneroid cell (capsule), which is made from an alloy of beryllium and copper. The evacuated capsule (or usually more capsules) is prevented from collapsing by a strong spring. Small changes in external air pressure cause the cell to expand or contract. This expansion and contraction drives mechanical levers such that the tiny movements of the capsule are amplified and displayed on the face of the aneroid barometer.

ANEROID BAROMETER

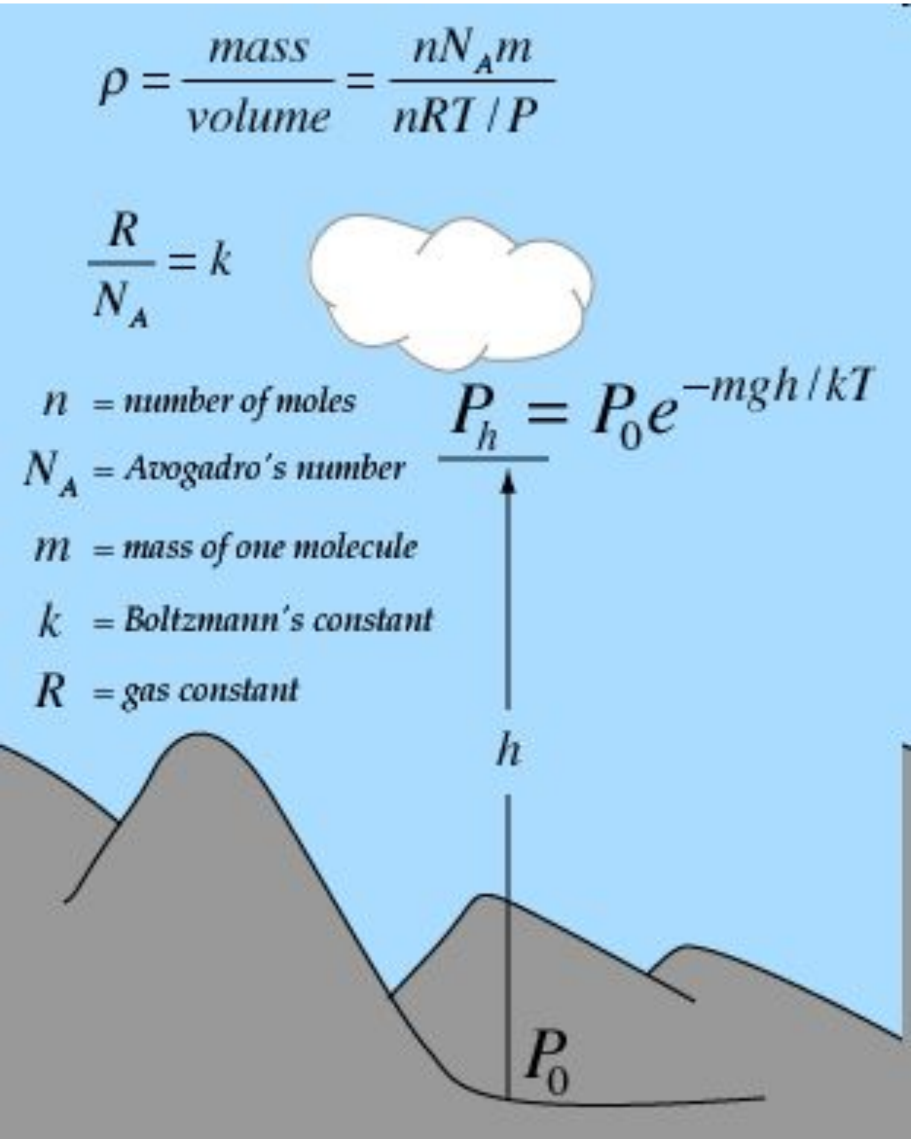


Face
Pointers
Spring
Levers
Chain
Metal drum
(partial vacuum)
Hairspring



STORMY RAIN CHANGE FAIR DRY
980 990 1000 1010 1020 1030 1040
Lever system
Aneroid cell

The Barometric Formula



μ_{air}=28.9644 g/mol

m_{air}= μ_{air}/*Na*

Pascal's Principle

- **Pressure exerted anywhere in a confined incompressible fluid is transmitted equally in all directions throughout the fluid such that the pressure ratio (initial difference) remains the same.**

$\downarrow F_1 = 10\text{ N}$ Applied force to the stopper

$A_1 = 5\text{ cm}^2$

$$P_1 = \frac{10\text{ N}}{5\text{ cm}^2} = 2\text{ N/cm}^2$$

Like a liquid lever, changing areas in an enclosed fluid permit multiplication of force

Pressure is transmitted undiminished in an enclosed static fluid.

$$F_2 = P_2 A_2 = (2\text{N/cm}^2)(500\text{ cm}^2)$$

$$= 1000\text{ N!!}$$

plus the force from the weight of the liquid.

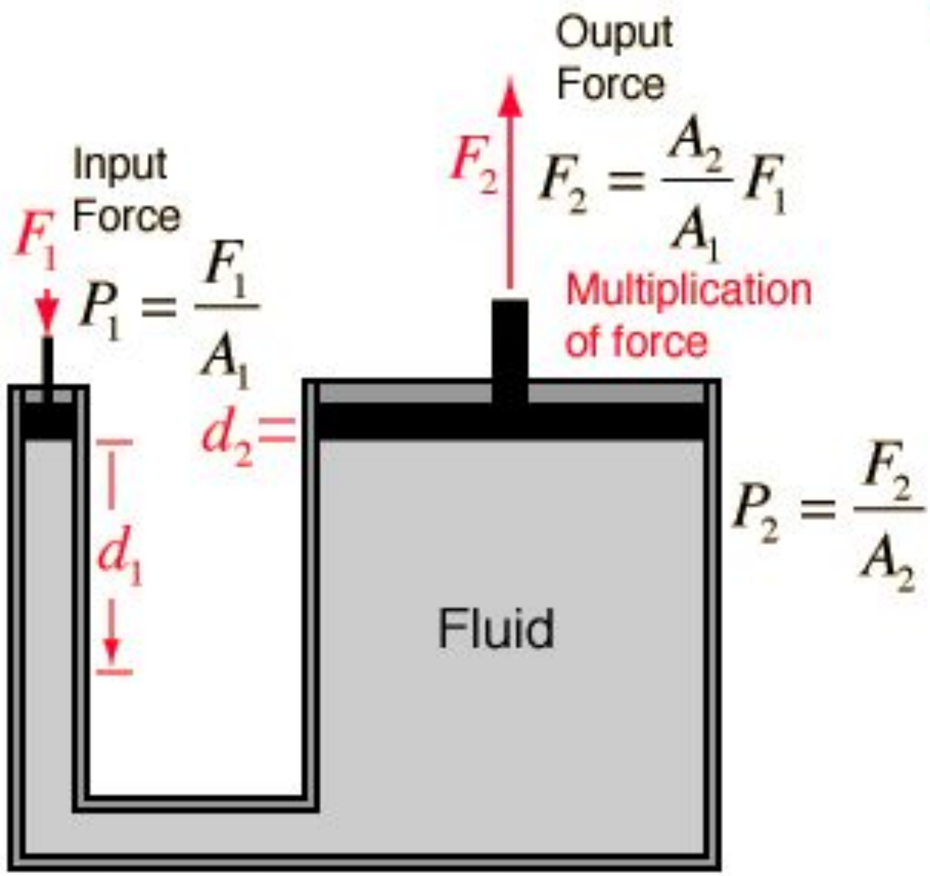
Resulting force on bottom of jug.

$A_2 = 500\text{ cm}^2$

$$P_2 = P_1 + \rho g h$$

Static fluid pressure

Hydraulic Press

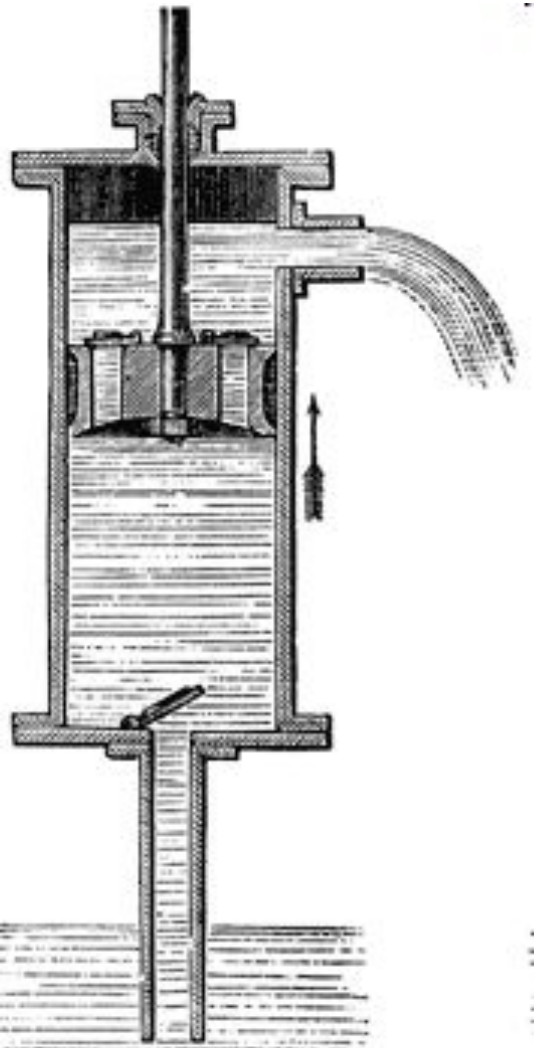


$$F_1 d_1 = F_2 d_2$$

$$d_1 = \frac{F_2}{F_1} d_2 = \frac{A_2}{A_1} d_2$$

You have to pay for the multiplied output force by exerting the smaller input force through a larger distance.

Lift pump

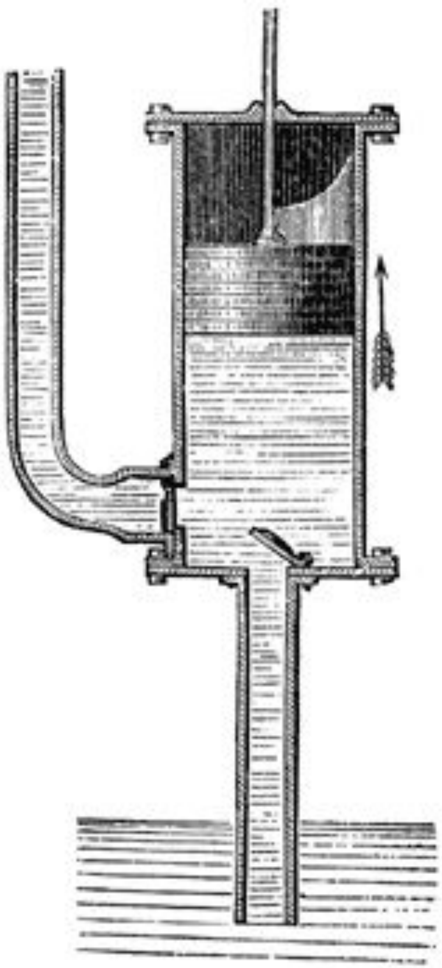


The lift pump, also known as a **suction pump**, operates as follows:

- on the **upstroke** of the plunger, the lower valve opens, the upper valve (situated on or in the plunger itself) is closed, and the low air pressure produced in the barrel allows atmospheric pressure on the surface of the water source, down below, to make the water move up the downpipe and eventually fill the barrel below the plunger.
- On the **downstroke**, the lower valve closes, the upper one opens, and water is forced into the barrel above the upper valve. On the next upstroke, the water above the plunger is forced out of the spout, located at the top of the barrel, at the same time as the volume below the barrel fills up with water again.

Force pump

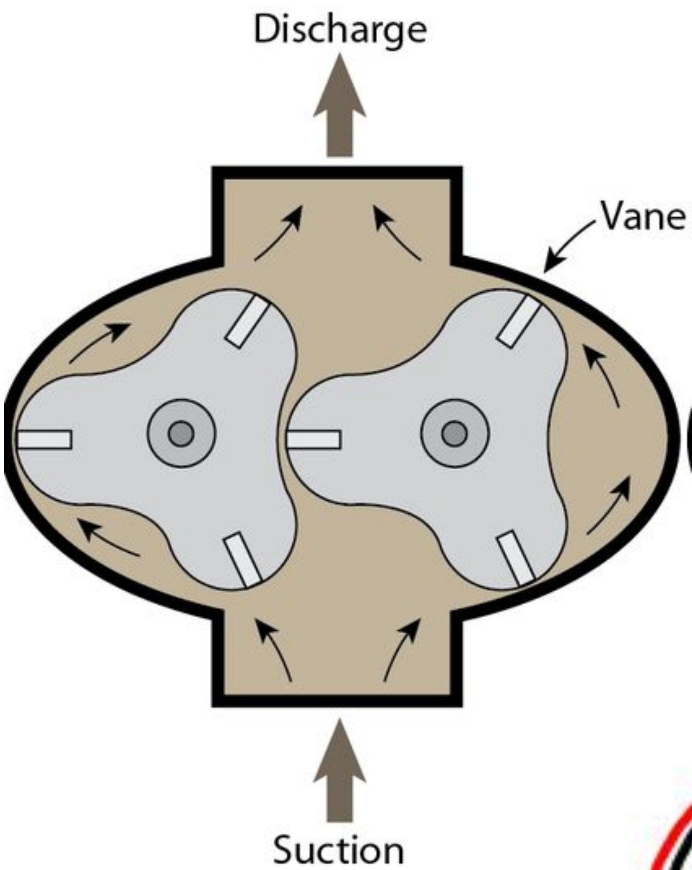
The force pump, also known as a **pressure pump**, operates as follows:



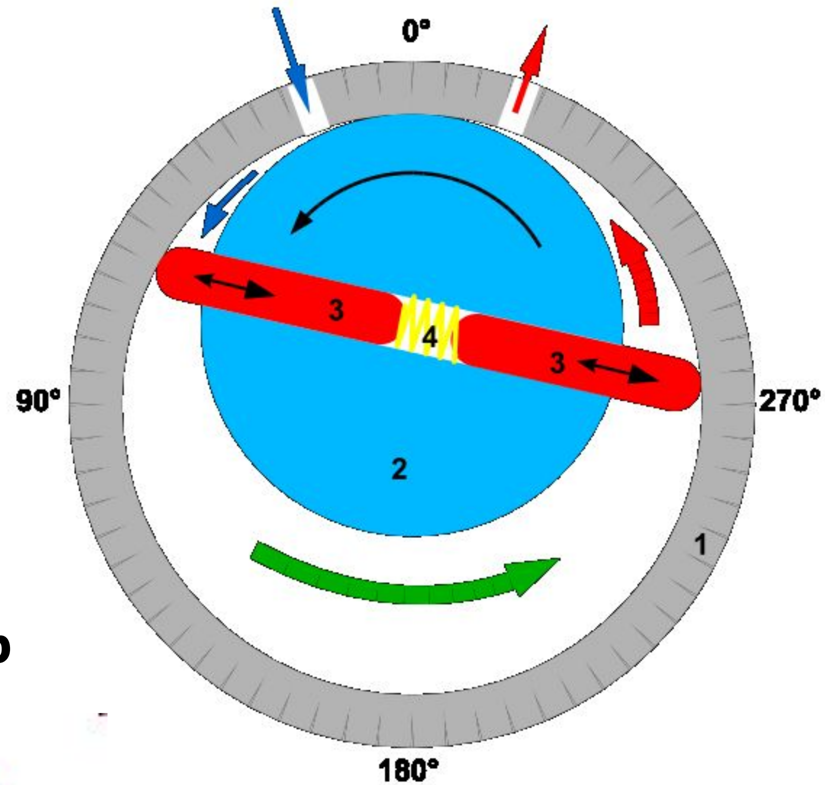
- on the **upstroke** of the plunger, the outlet or delivery valve is closed and the inlet valve opens. The low air pressure produced in the barrel causes the water below to move up the downpipe and eventually fill the barrel.
- On the **downstroke**, the inlet valve closes, the outlet valve opens, and the water is forced out via the outlet pipe, which is located at the bottom of the barrel.

Rotary Pumps

Lobe Pump



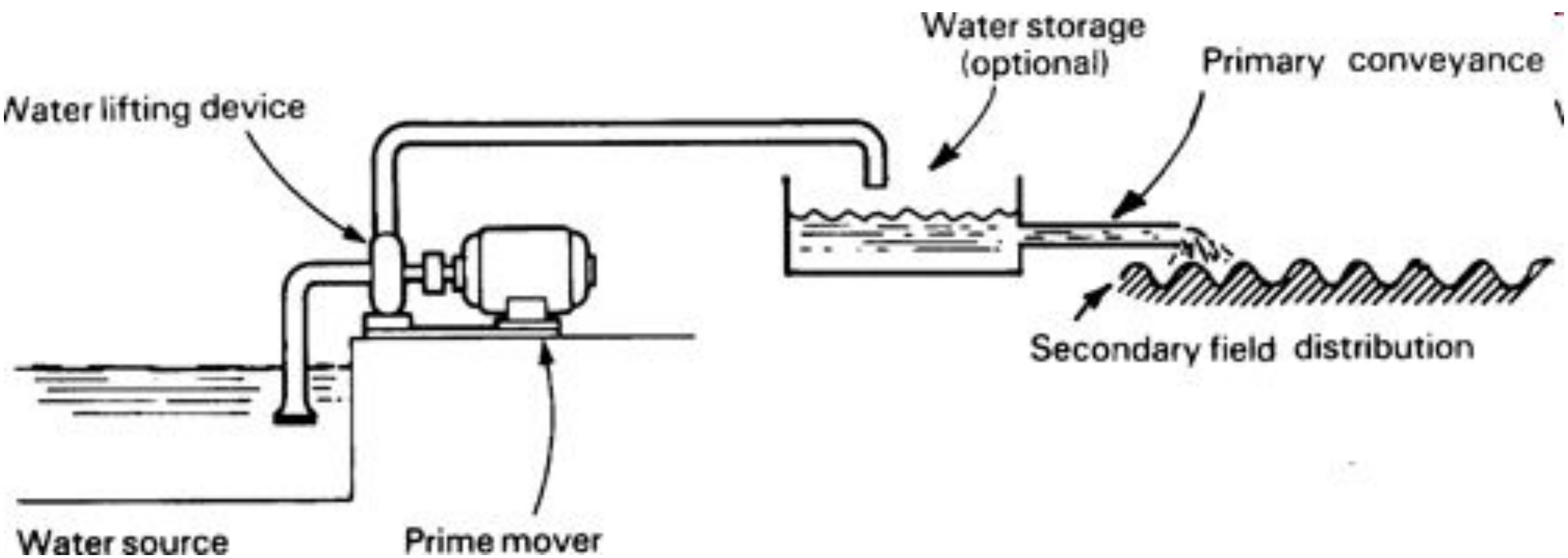
Rotary vane pump



Scroll pump

Height limitation

Total Dynamic Head (TDH) is the total equivalent height that a fluid is to be pumped, taking into account friction losses in the pipe.



Total Dynamic Height

TDH = Static Height + Static Lift + Friction Loss

Static Height is the maximum height reached by the pipe after the pump (also known as the 'discharge head').

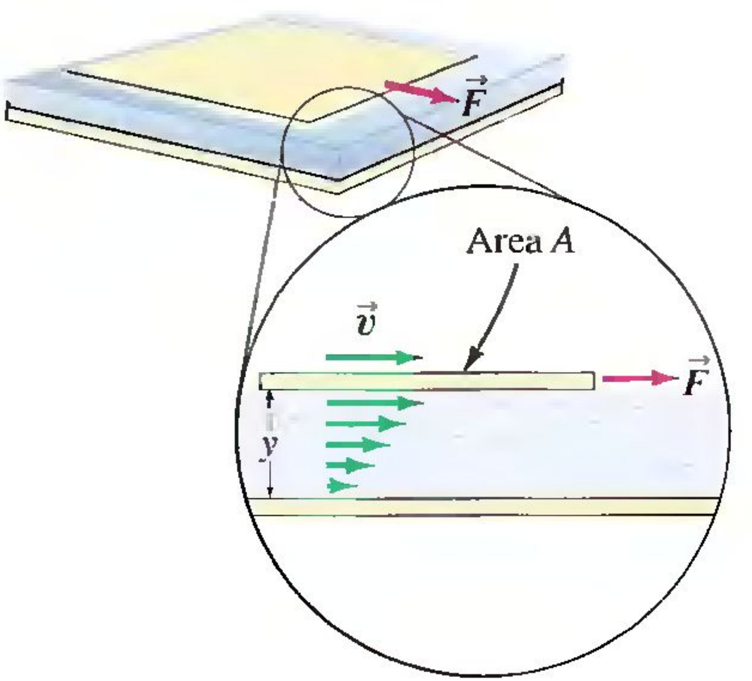
Static Lift is the height the water will rise before arriving at the pump (also known as the suction head).

Friction Loss - in any real moving fluid, energy is dissipated due to friction; turbulence dissipates even more energy for high Reynolds number flows. Friction loss is divided into two main categories, "major losses" associated with energy loss per length of pipe, and "minor losses" associated with bends, fittings, valves, etc.

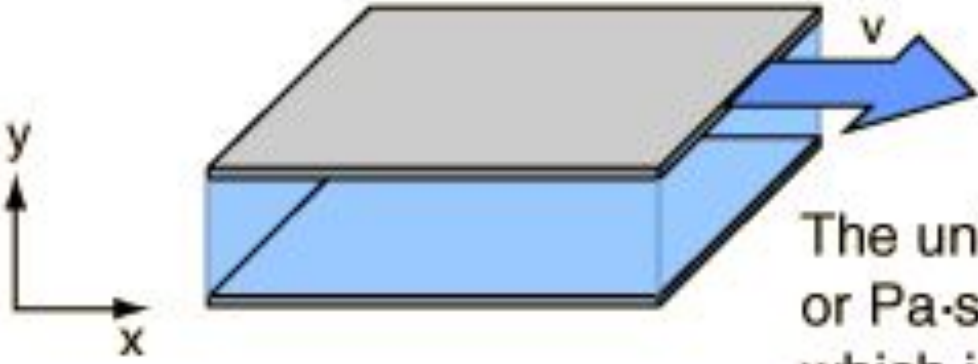
$$h_{total} = \frac{P_2 - P_1}{\rho g}$$

Viscosity

The **resistance to flow** of a fluid and the **resistance to the movement** of an object through a fluid are usually stated in terms of the **viscosity** of the fluid.



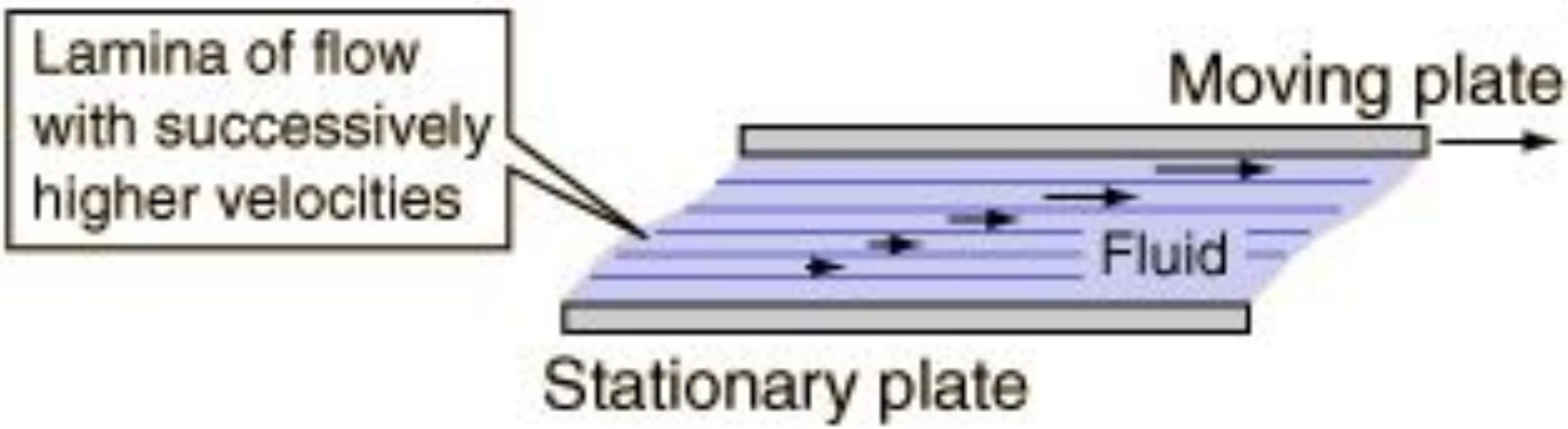
$$F = \eta A \frac{dv}{dy}$$



$$F_{viscous} = \eta A \frac{\Delta v_x}{\Delta y}$$

The units of viscosity η are then N s/m^2 or Pa·s. The CGS unit is dyne sec/cm^2 which is called Poise. The viscosity of water at 20°C is 0.01 Poise. The viscosity of blood at body temperature is about 0.03 Poise. The Pa·s is called a Poisuille and is equal to 10 Poise.

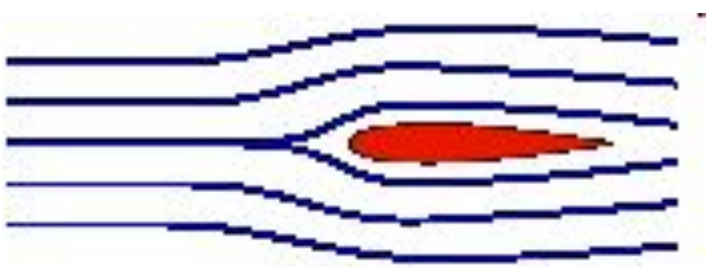
Experimentally, under conditions of laminar flow, the force required to move a plate at constant speed against the resistance of a fluid is proportional to the area of the plate and to the velocity gradient perpendicular to the plate. The constant of proportionality is called the viscosity .



Lamina of flow with successively higher velocities
Moving plate
Fluid
Stationary plate

Drag force due viscosity

In a viscous fluid, a boundary layer is formed. This causes a net drag due to skin friction. Further, because the ideal pressure now acts on the boundary layer, as opposed to the ship, and the boundary layer grows along the length of the ship, the net opposing forces are greater than the net supporting forces. This further adds to the resistance.



Effect of Temperature on Viscosity

The temperature dependence of liquid viscosity is the phenomenon by which liquid viscosity tends to decrease (or, alternatively, its fluidity tends to increase) as its temperature increases.

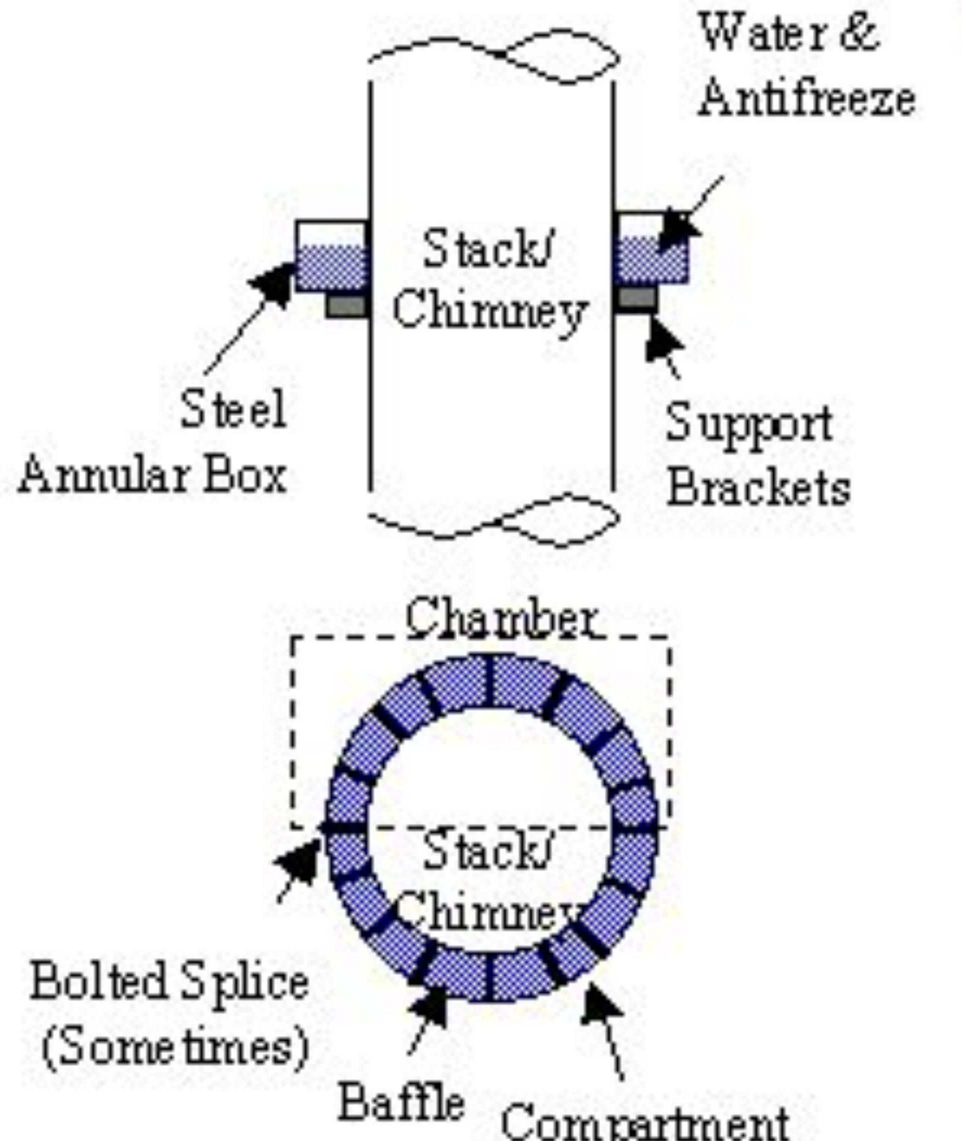
$$\eta = \eta_0 \cdot \exp\left(-b \cdot T\right)$$

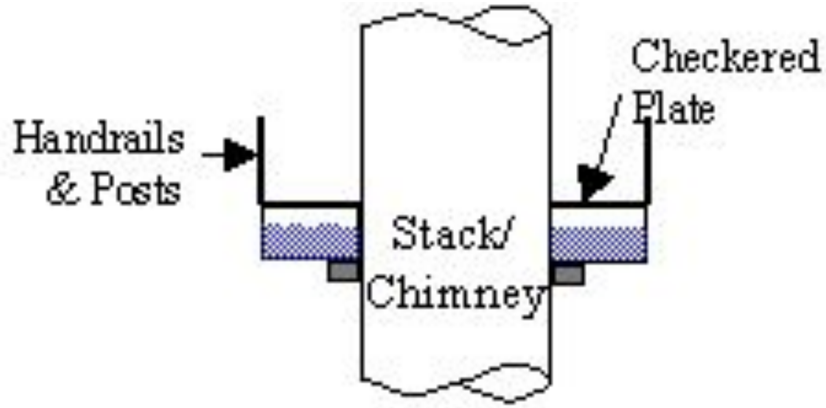
here η_0 and *b* are constants.

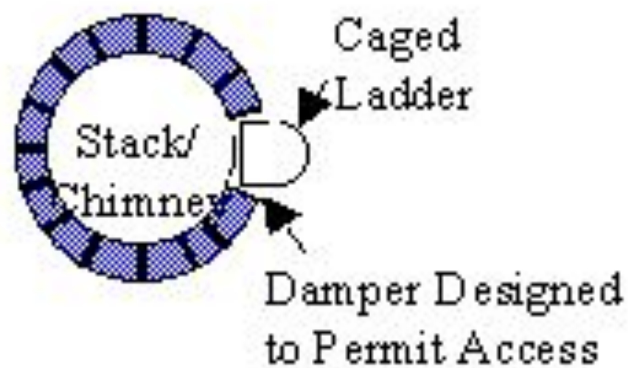
This is an empirical model that usually works for a limited range of temperatures.

Liquid Damping

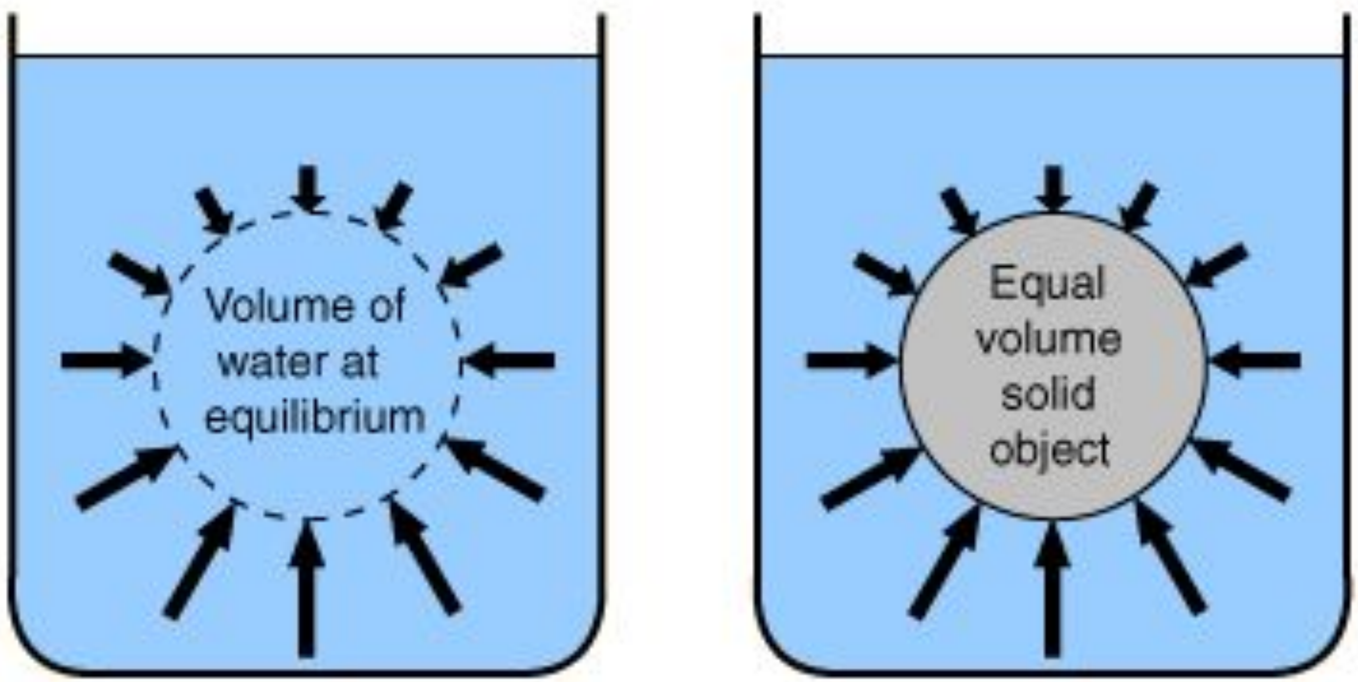
- Damping is an effect that reduces the amplitude of oscillations in an oscillatory system
- Fluid viscous damping is a way to add energy dissipation to the lateral system of a building structure. A fluid viscous damper dissipates energy by pushing fluid through an orifice, producing a damping pressure which creates a force. These damping forces are 90 degrees out of phase with the displacement driven forces in the structure. This means that the damping force does not significantly increase the seismic loads for a comparable degree of structural deformation.


Water &
Antifreeze
Stack/
Chimney
Steel
Annular Box
Support
Brackets
Chamber
Stack/
Chimney
Bolted Splice
(Sometimes)
Baffle
Compartment


Handrails
& Posts
Checkered
Plate
Stack/
Chimney


Caged
Ladder
Stack/
Chimney
Damper Designed
to Permit Access

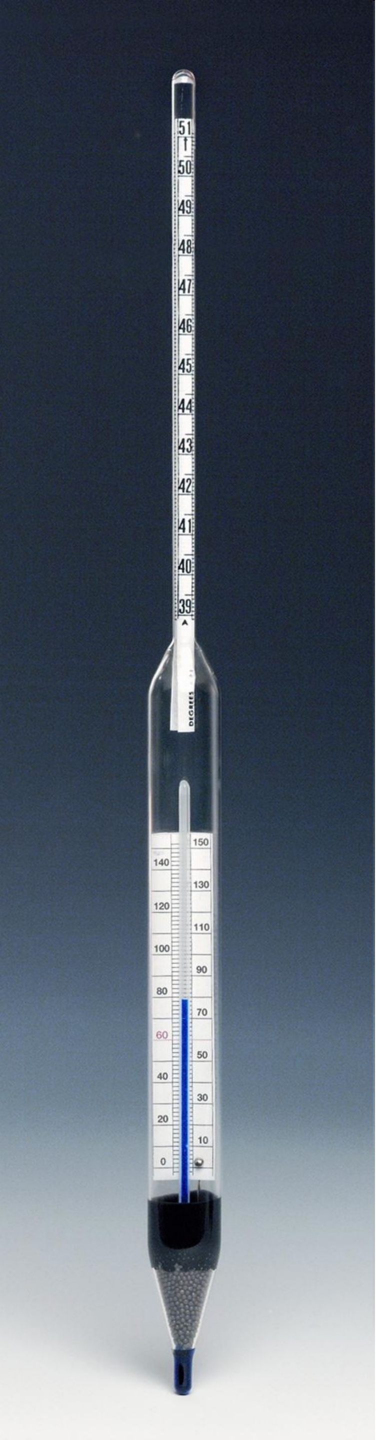
Buoyancy



Buoyancy arises from the fact that fluid pressure increases with depth and from the fact that the increased pressure is exerted in all directions (Pascal's principle) so that there is an unbalanced upward force on the bottom of a submerged object.

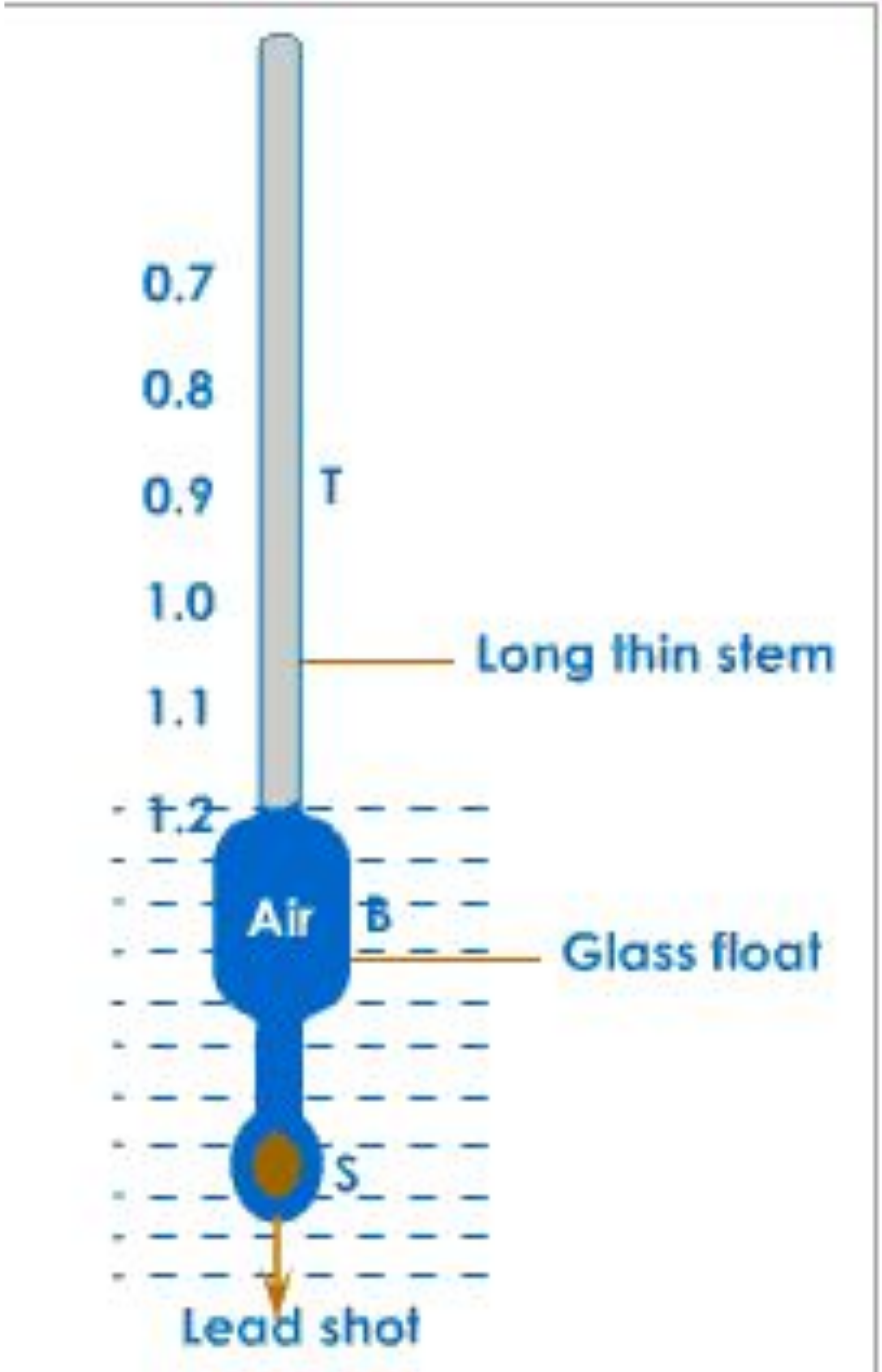
Archimedes' Principle

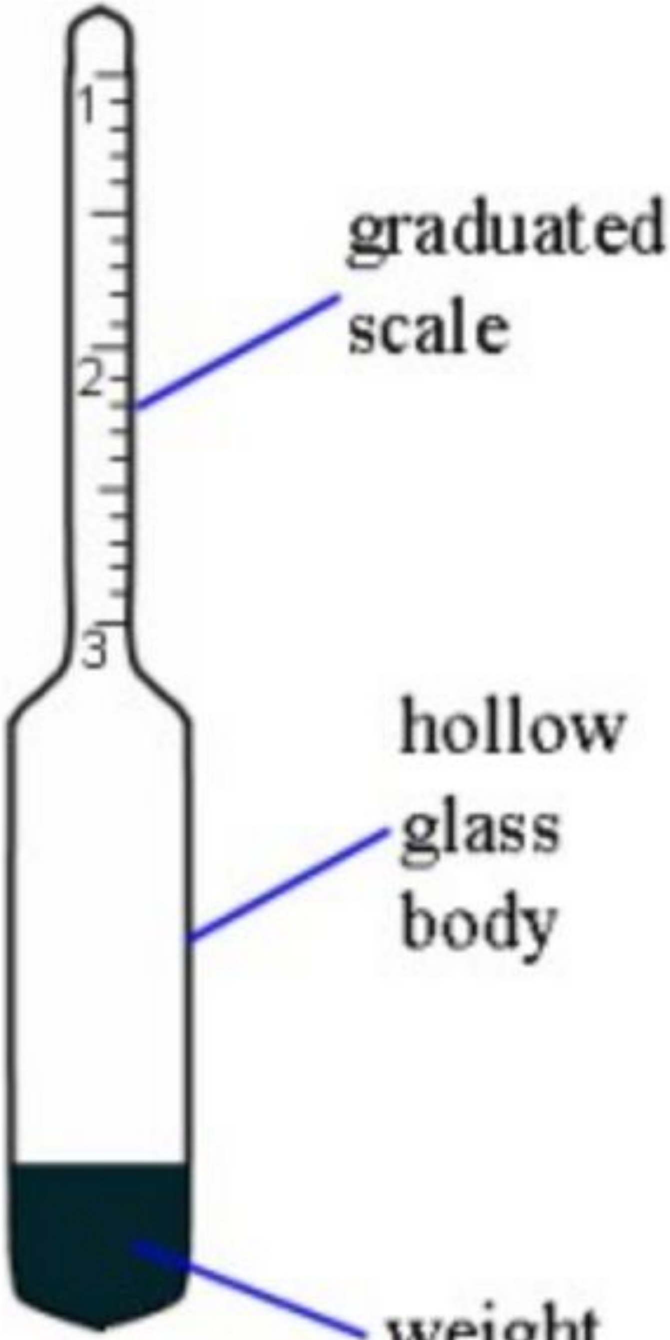
- **The buoyant force on a submerged object is equal to the weight of the fluid displaced.**
- The upward **thrust** which the surrounding fluid exerts on an object is referred to as the force of buoyancy.

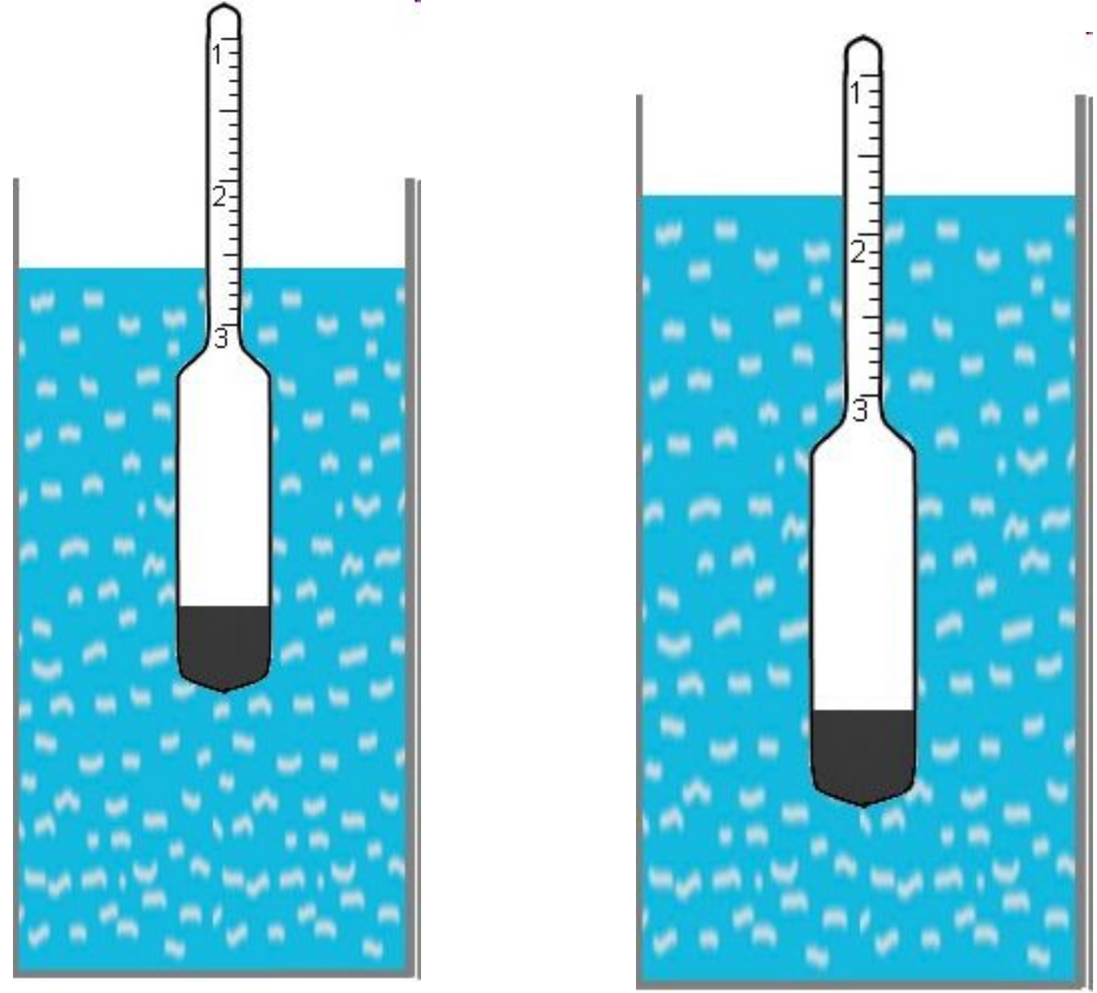


Hydrometer

- A hydrometer is an instrument used to measure the **specific gravity** (or **relative density**) of liquids; that is, the ratio of the density of the liquid to the density of water.
- A hydrometer is usually made of glass and consists of a **cylindrical stem** and a **bulb weighted with mercury** or lead shot to make it float upright. The liquid to be tested is poured into a tall container, often a **graduated cylinder**, and the hydrometer is gently lowered into the liquid until it floats freely. The point at which the surface of the liquid touches the stem of the hydrometer is noted. Hydrometers usually contain a **scale** inside the stem, so that the specific gravity can be read directly. A variety of scales exist, and are used depending on the context.
- Hydrometers may be calibrated for different uses, such as a **lactometer** for measuring the density (creaminess) of milk, a **saccharometer** for measuring the density of sugar in a liquid, or an **alcoholometer** for measuring higher levels of alcohol in spirits.

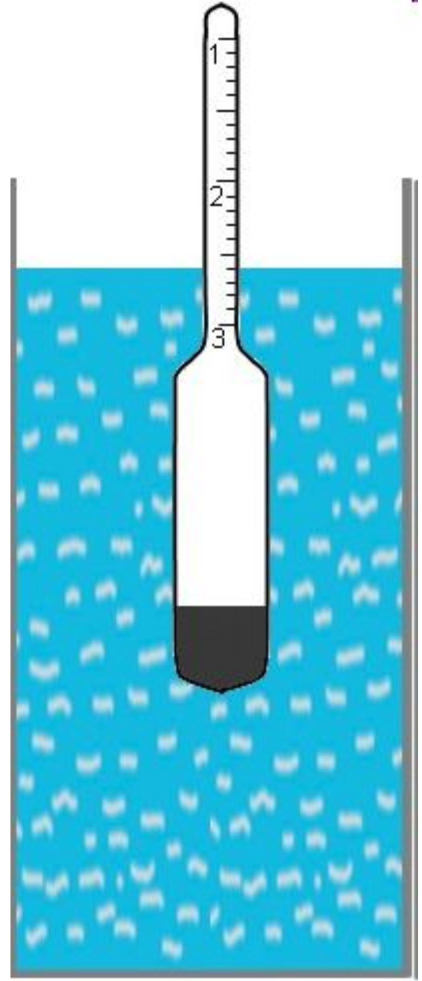

0.7
0.8
0.9
1.0
1.1
1.2
T
Long thin stem
Air
B
Glass float
S
Lead shot





Determine, what liquid is denser?

This liquid is lighter.



This liquid is denser.

This liquid is lighter.

Fluid Kinetic Energy

- The kinetic energy of a moving fluid is more useful in applications like the Bernoulli equation when it is expressed as kinetic energy per unit volume

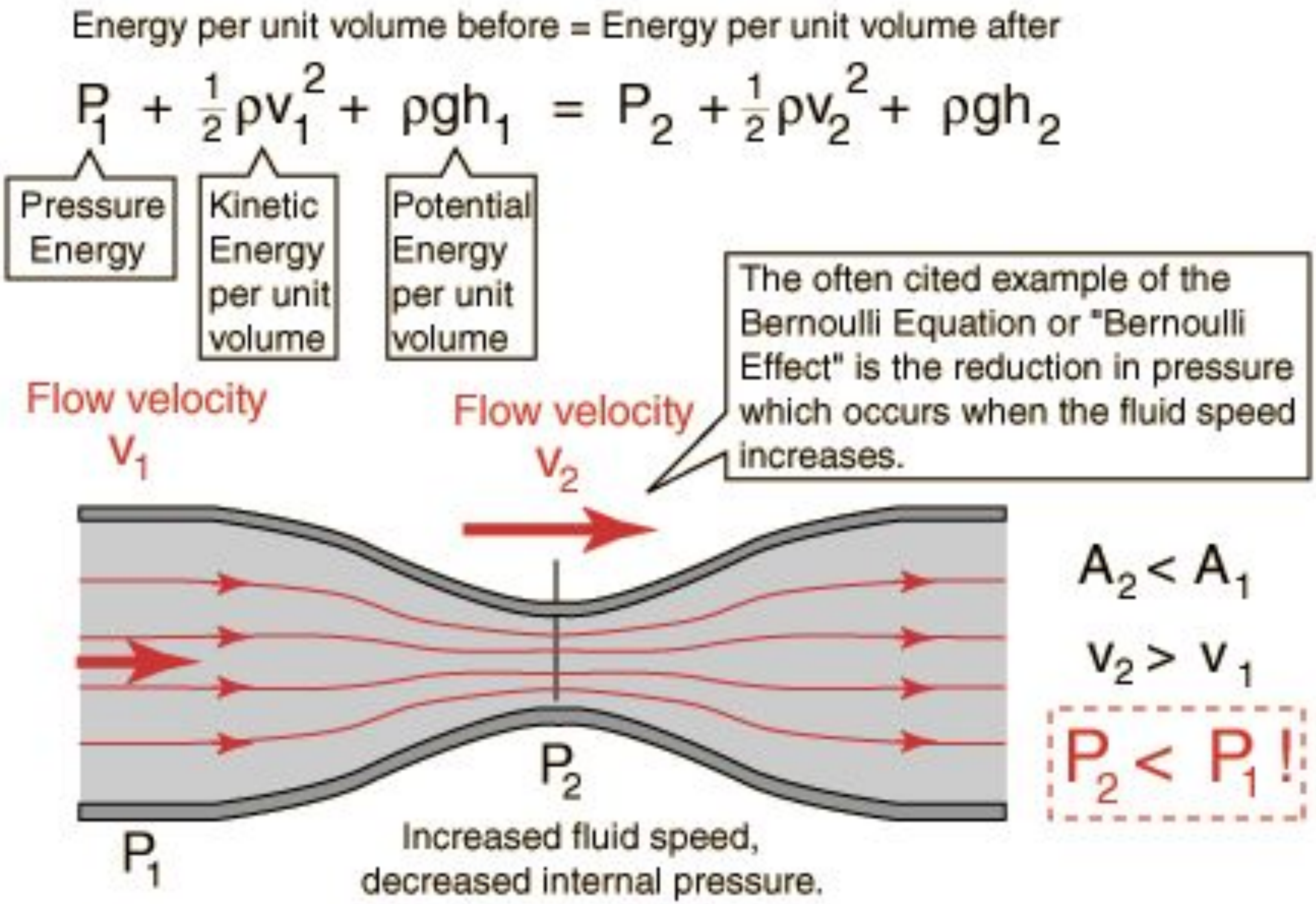
$$\frac{Kinetic\ energy}{Volume} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\rho v^2$$

Fluid Potential Energy

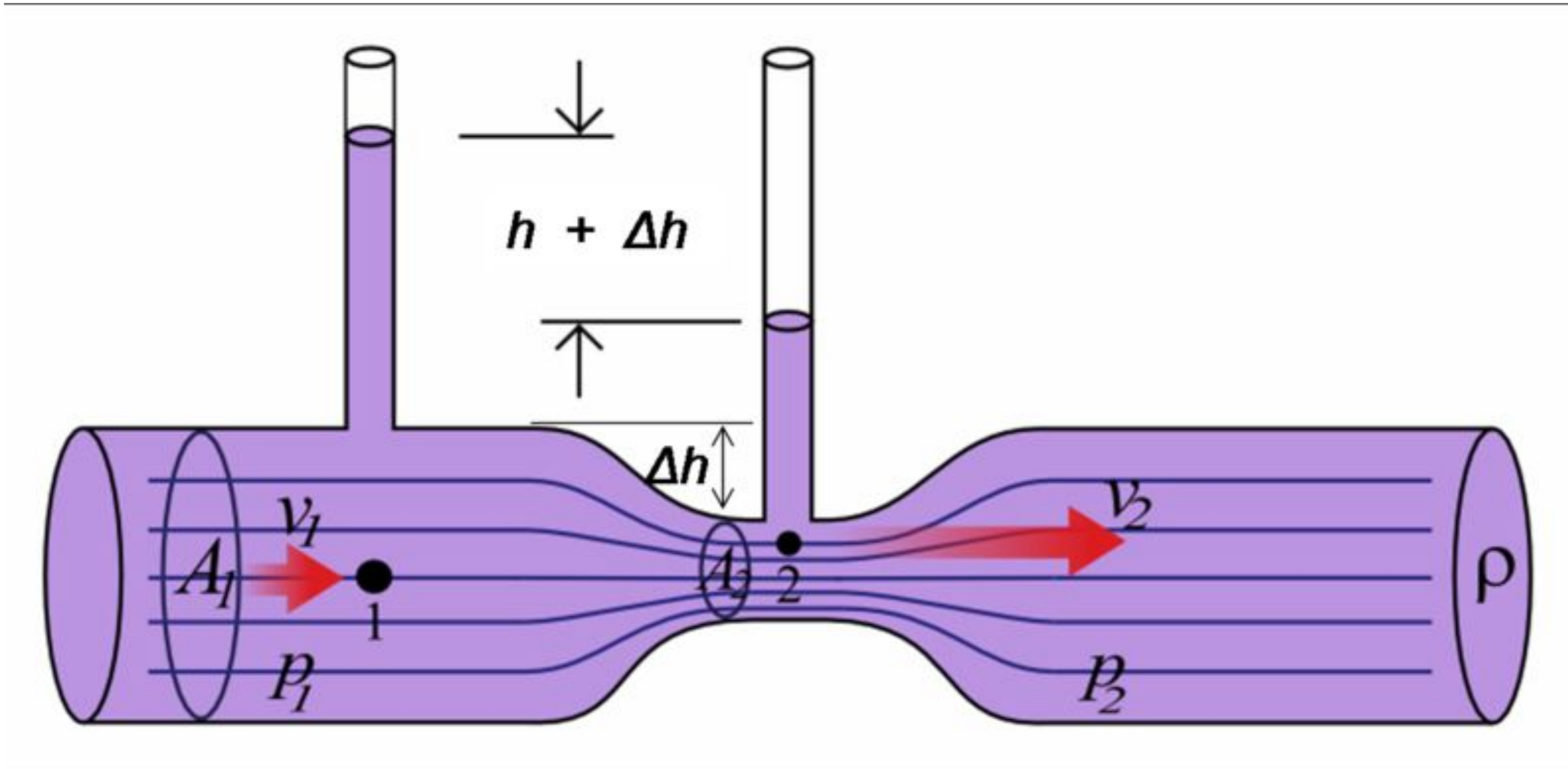
- The potential energy of a moving fluid is more useful in applications like the Bernoulli equation when is expressed as potential energy per unit volume

$$\frac{Potential\ energy}{Volume} = \frac{mgh}{V} = \rho gh$$

Bernoulli Equation



Venturi meter



The Venturi effect is the **reduction in fluid pressure that results when a fluid flows through a constricted section of pipe**.

The Venturi effect is named after Giovanni Battista Venturi (1746–1822), an Italian physicist.

Venturi effect

Since

$$\begin{cases} Q = v_1 A_1 = v_2 A_2 \\ p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2), \end{cases}$$

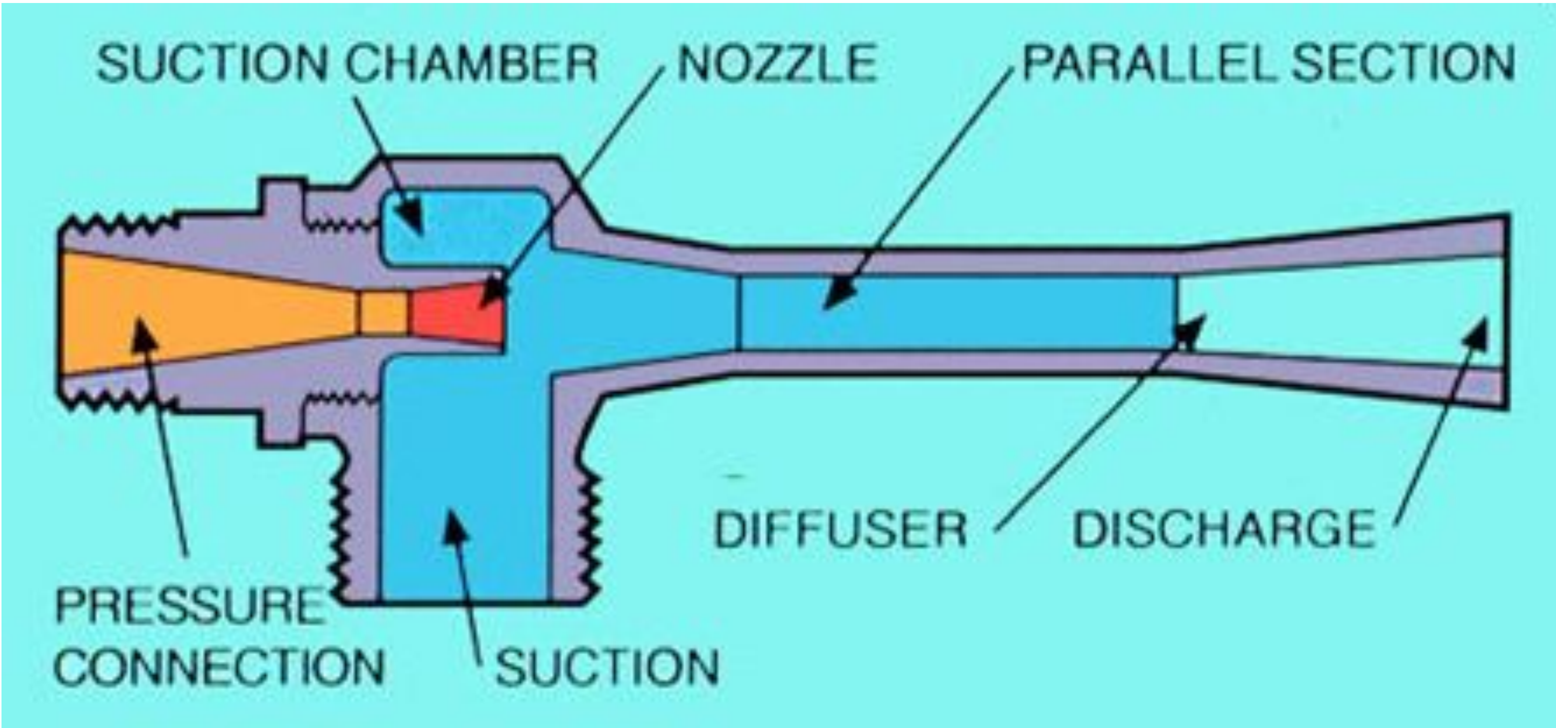
then

$$Q = A_1 \sqrt{\frac{2\,(p_1 - p_2)}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}} = A_2 \sqrt{\frac{2\,(p_1 - p_2)}{\rho\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}}.$$

Q is volumetric flow rate

So Venturi meter can be used to measure the flow rate.

Water Eductor



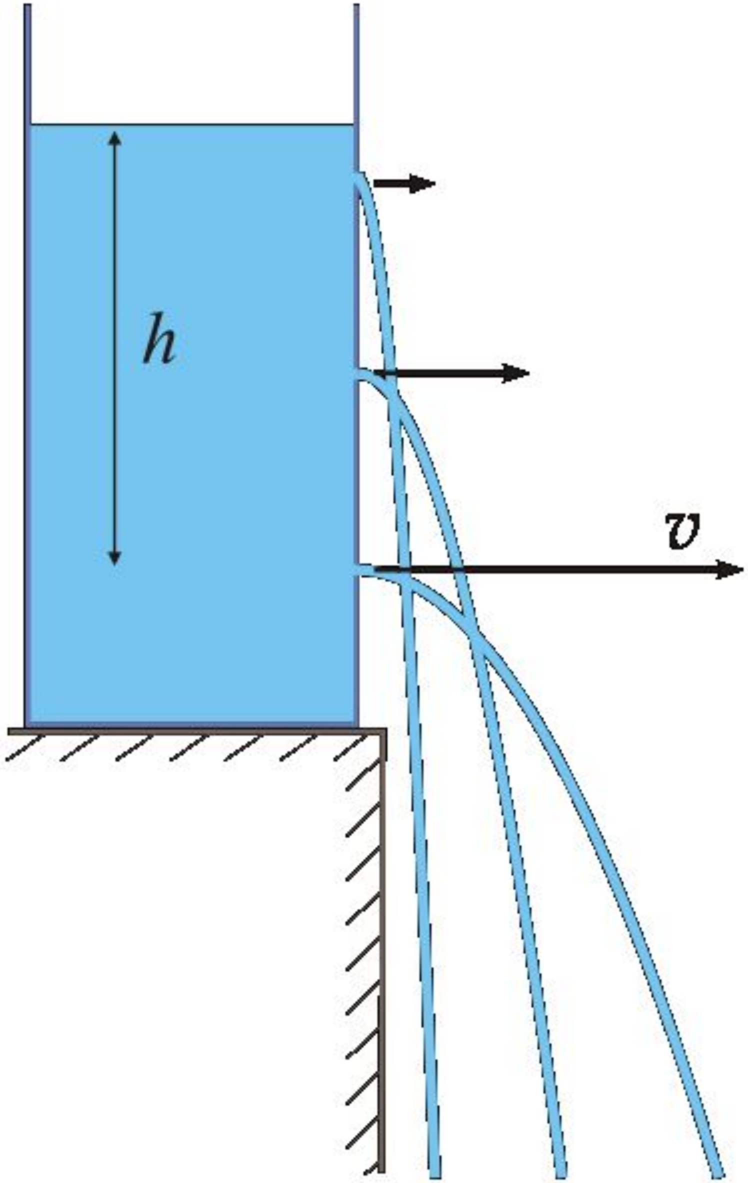
- Liquid Jet Eductors use the kinetic energy of a motive liquid to entrain another liquid, completely mix the two, and then discharge the mixture against a counter pressure and are used for pumping and mixing operations.

Converging
Nozzle

Diffuser (or Venturi)

Body - holds diffuser
and nozzle in position

Torricelli's law



$$gz + \frac{p_{atm}}{\rho} = \frac{v^2}{2} + \frac{p_{atm}}{\rho}$$
$$\Rightarrow v^2 = 2gz$$
$$\Rightarrow v = \sqrt{2gz}$$