Quick Quiz 1 If a fly collides with the windshield of a fast-moving bus, which object experiences an impact force with a larger magnitude? (a) the fly (b) the bus (c) the same force is experienced by both.

Quick Quiz 2 In a free-body diagram for a single object, you draw (a) the forces acting on the object and the forces the object exerts on other objects, or (b) only the forces acting on the object.

Quick Quiz 3 Which of the following is *impossible* for a car moving in a circular path? (a) the car has tangential acceleration but no centripetal acceleration. (b) the car has centripetal acceleration but no tangential acceleration. (c) the car has both centripetal acceleration and tangential acceleration.

Course of lectures «Contemporary Physics: Part1»

Lecture №4

Energy and Energy Transfer. Potential Energy.

Work Done by a Constant Force

(a)

(b)

(c)

Figure 6.1 An eraser being pushed along a chalkboard tray.

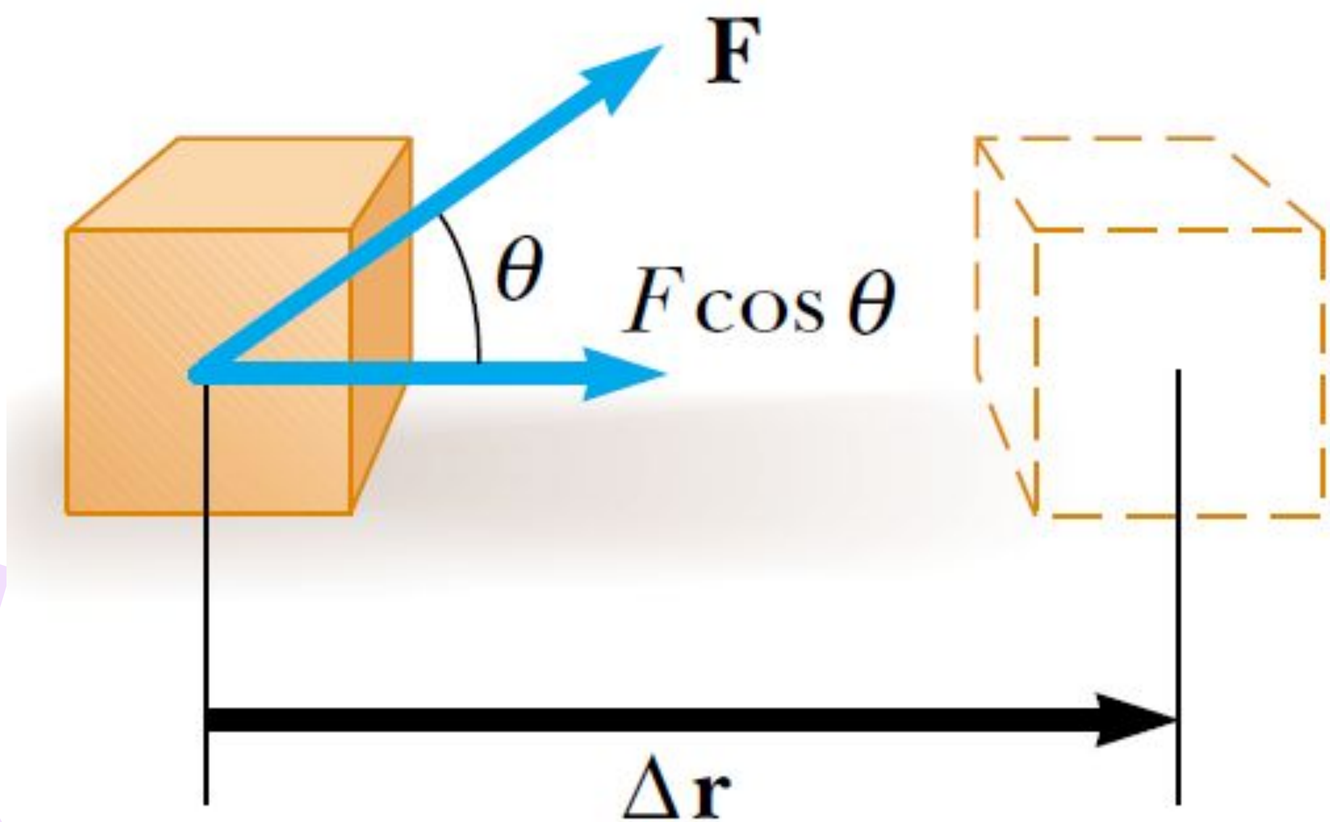
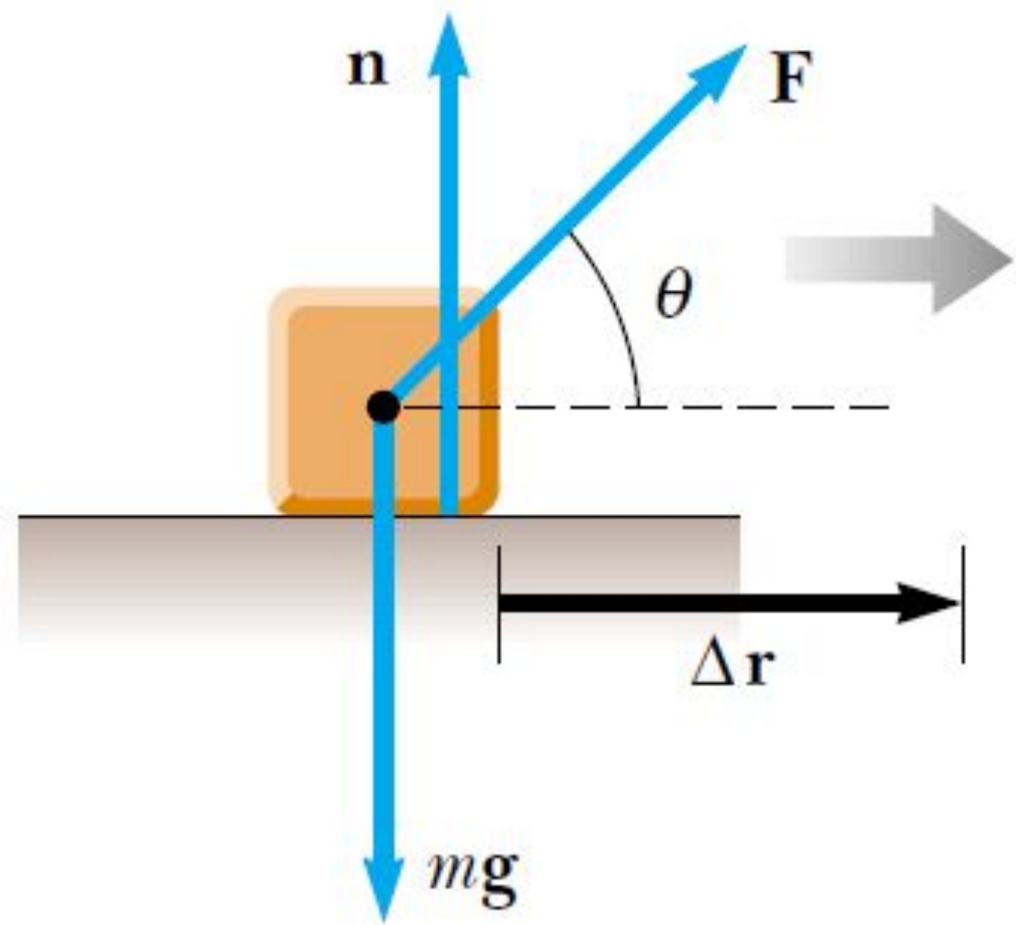


Figure 6.2 If an object undergoes a displacement Δr under the action of a constant force **F**, the work done by the force is *F*Δ*r*cos*θ*.

The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δ r of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and displacement vectors:

$$\boxed{W \equiv F\,\Delta r \cos\theta} \qquad \textbf{(6.1)}$$



$$W = F\Delta r$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the **newton·meter** (N·m). This combination of units is used so frequently that it has been given a name of its own: the **joule** (J).

Figure 6.3 When an object is displaced on a frictionless, horizontal surface, the normal force **n** and the gravitational force m**g** do no work on the object. In the situation shown here, **F** is the only force doing work on the object.

An important consideration for a system approach to problems is to note that **work is an energy transfer**. If W is the work done on a system and W is positive, energy is transferred *to* the system; if W is negative, energy is transferred *from* the system. Thus, if a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary. This will result in a change in the energy stored in the system.

Work Done by a Varying Force

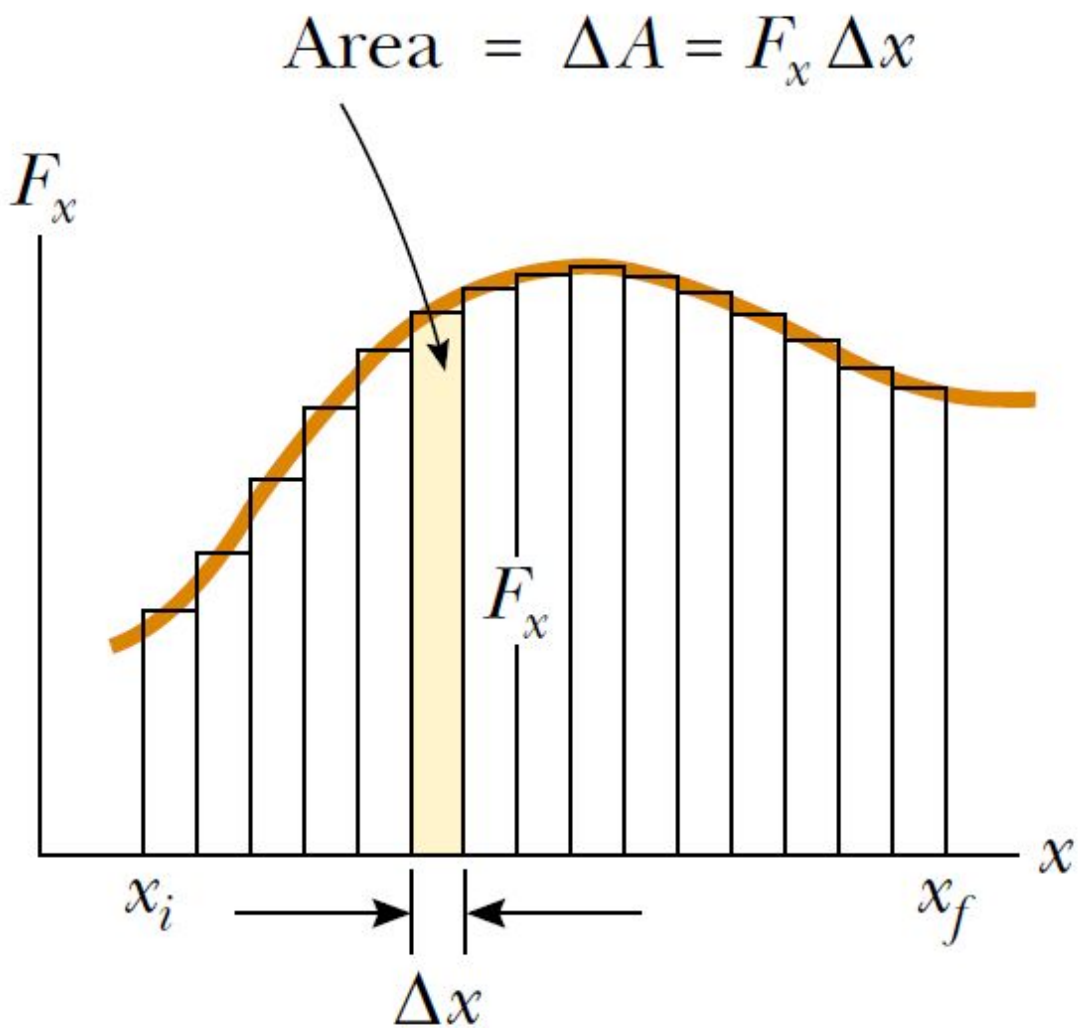


Figure 6.4 The work done by the force

$$\boxed{W \approx F_x \Delta x}$$

$$\boxed{W \approx \sum_{x_i}^{x_f} F_x \Delta x}$$

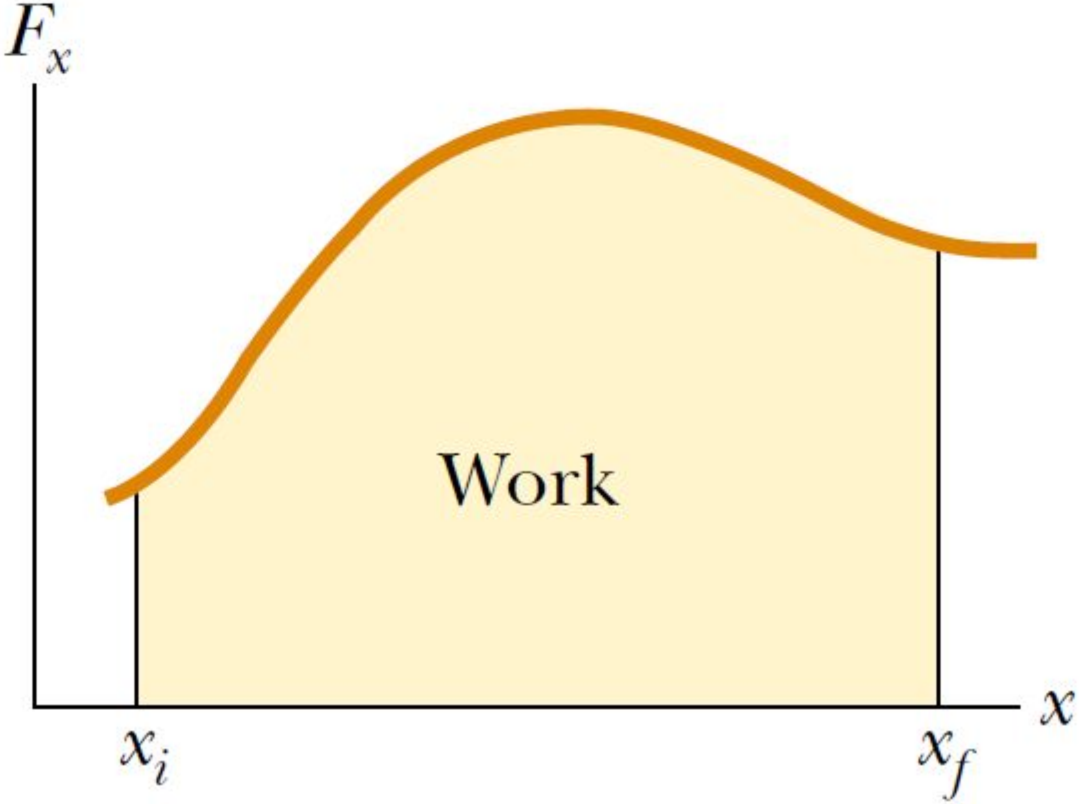


Figure 6.5 The work done by the component F_x *of* the varying force as the particle moves from x_i to x_f is exactly equal to the area under this curve.

$$\boxed{W = \int_{x_i}^{x_f} F_x \, dx}$$

$$\boxed{\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx}$$

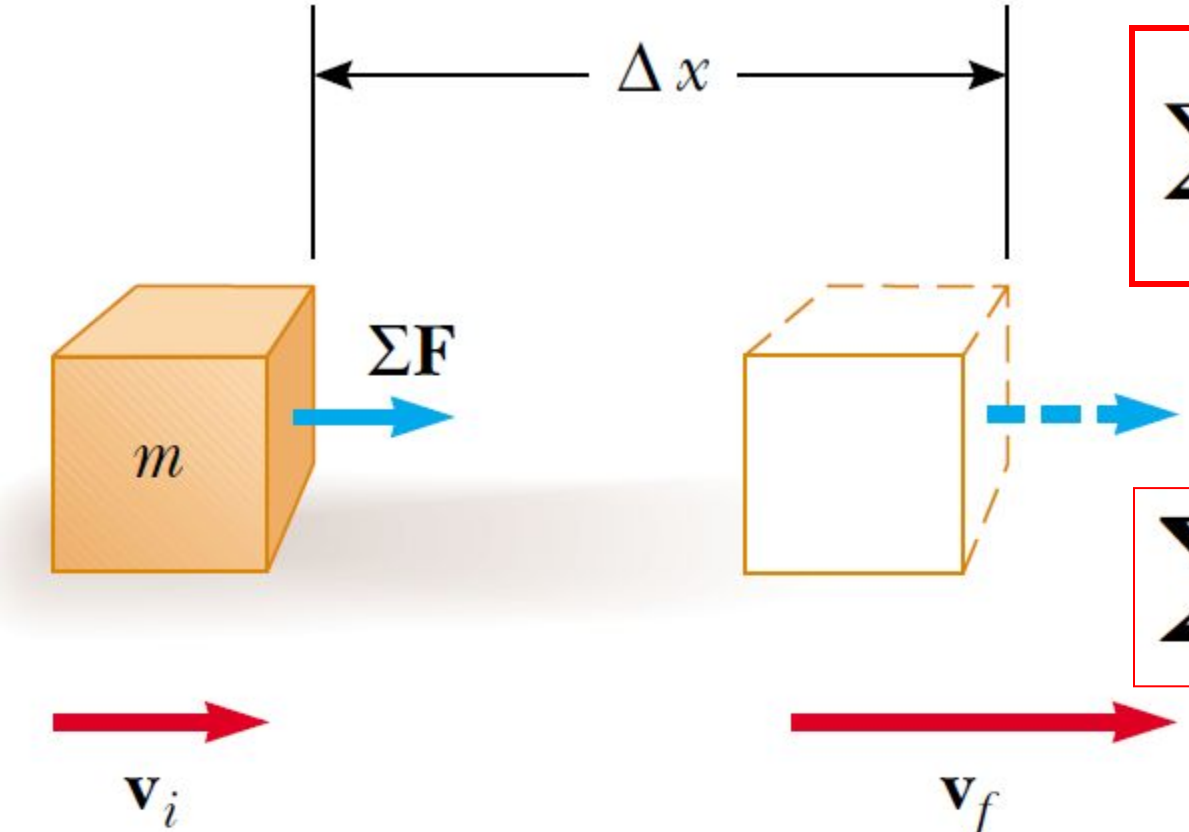
(6.2)

$$\boxed{\sum W = W_{\text{net}} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx}$$

(6.3)

Work Done by a Spring

Kinetic Energy and the Work–Kinetic Energy Theorem



$$\boxed{\sum W = \int_{x_i}^{x_f} \sum F\,dx}$$

(6.4)

$$\boxed{\sum F = ma}$$

Figure 6.6 An object undergoing a displacement $\Delta\mathbf{r}=\Delta x\hat{\mathbf{i}}$ and a change in velocity under the action of a constant net force $\Sigma\mathbf{F}$.

$$\sum W = \int_{x_i}^{x_f} madx = \int_{x_i}^{x_f} m\frac{dv}{dt}dx =$$

$$= \int_{x_i}^{x_f} m\frac{dv}{dx}\frac{dx}{dt}dx = \int_{v_i}^{v_f} mv\,dv$$

$$\boxed{\sum W = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2} \quad \textbf{(6.5)}$$

where v_i is the speed of the block when it is at $x = x_i$ and v_f is its speed at x_f.

$$K \equiv \frac{1}{2}mv^2 \qquad \textbf{(6.6)}$$

Kinetic energy is a scalar quantity and has the same units as work.

$$\sum W = K_f - K_i = \Delta K \qquad \textbf{(6.7)}$$

Equation 6.7 is an important result known as the work–kinetic energy theorem:

In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

Figure 6.7 Energy transfer mechanisms. (a) Energy is transferred to the block by work; (b) energy leaves the radio from the speaker by mechanical waves; (c) energy transfers up the handle of the spoon by heat.

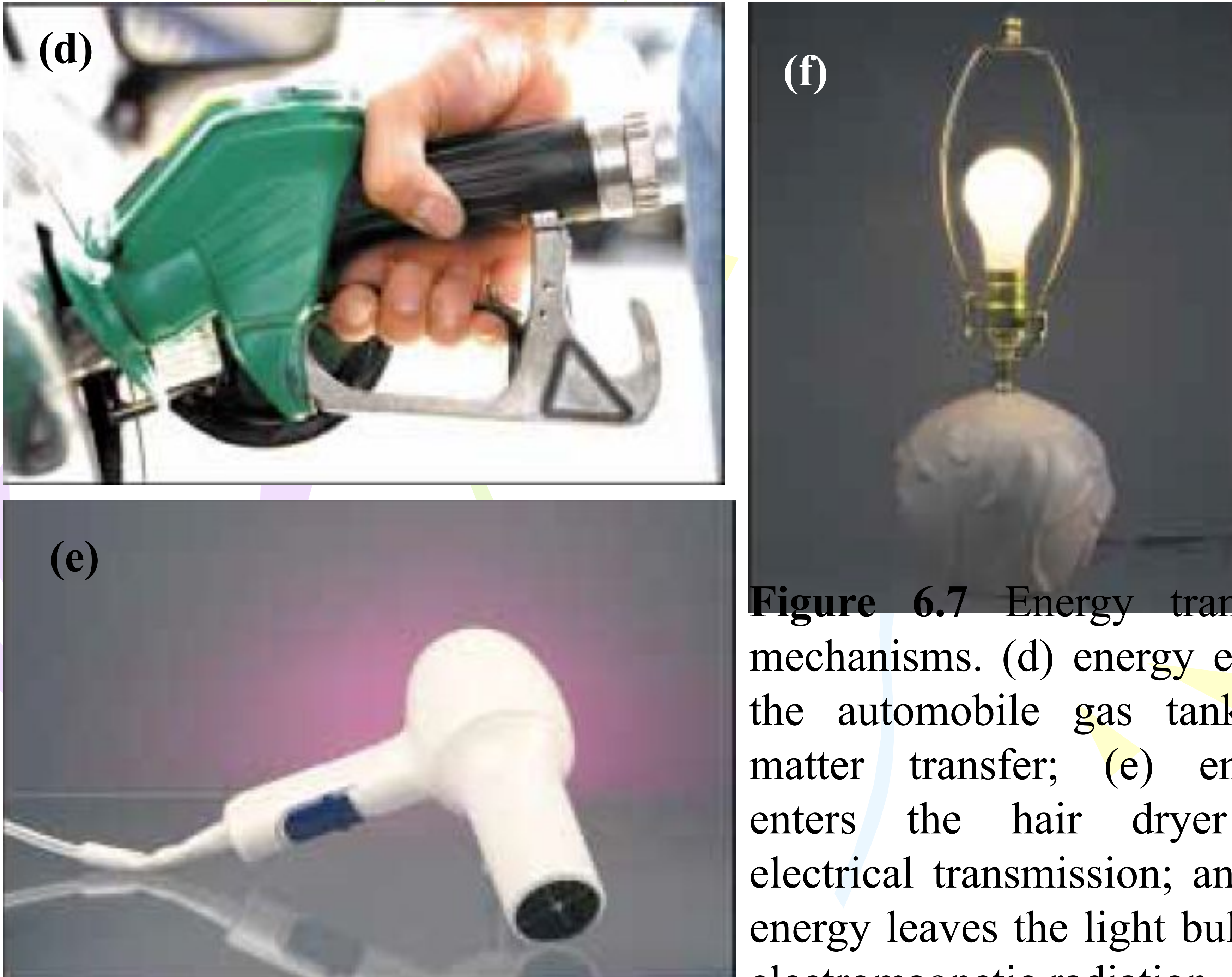


Figure 6.7 Energy transfer mechanisms. (d) energy enters the automobile gas tank by matter transfer; (e) energy enters the hair dryer by electrical transmission; and (f) energy leaves the light bulb by electromagnetic radiation.

One of the central features of the energy approach is the notion that **we can neither create nor destroy energy—energy is always *conserved***. Thus, **if the total amount of energy in a system changes, it can *only* be due to the fact that energy has crossed the boundary of the system by a transfer mechanism such as one of the methods listed above.** This is a general statement of the principle of **conservation of energy**. We can describe this idea mathematically as follows:

$$\boxed{\Delta E_{\text{system}} = \sum T} \qquad \textbf{(6.8)}$$

Power

The time rate of energy transfer is called power. If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the **average power** during this interval is defined as

$$\boxed{\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}}$$

In a manner similar to the way we approached the definition of velocity and acceleration, we define the instantaneous power as the limiting value of the average power as Δt approaches zero:

$$\boxed{\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}}$$

$$dW = \mathbf{F} \cdot d\mathbf{r}$$

$$\boxed{\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}} \quad \textbf{(6.9)}$$

In general, power is defined for any type of energy transfer. Therefore, the most general expression for power is

$$\boxed{\mathcal{P} = \frac{dE}{dt}} \qquad \textbf{(6.10)}$$

The SI unit of power is joules per second (J/s), also called the **watt** (W) (after James Watt):

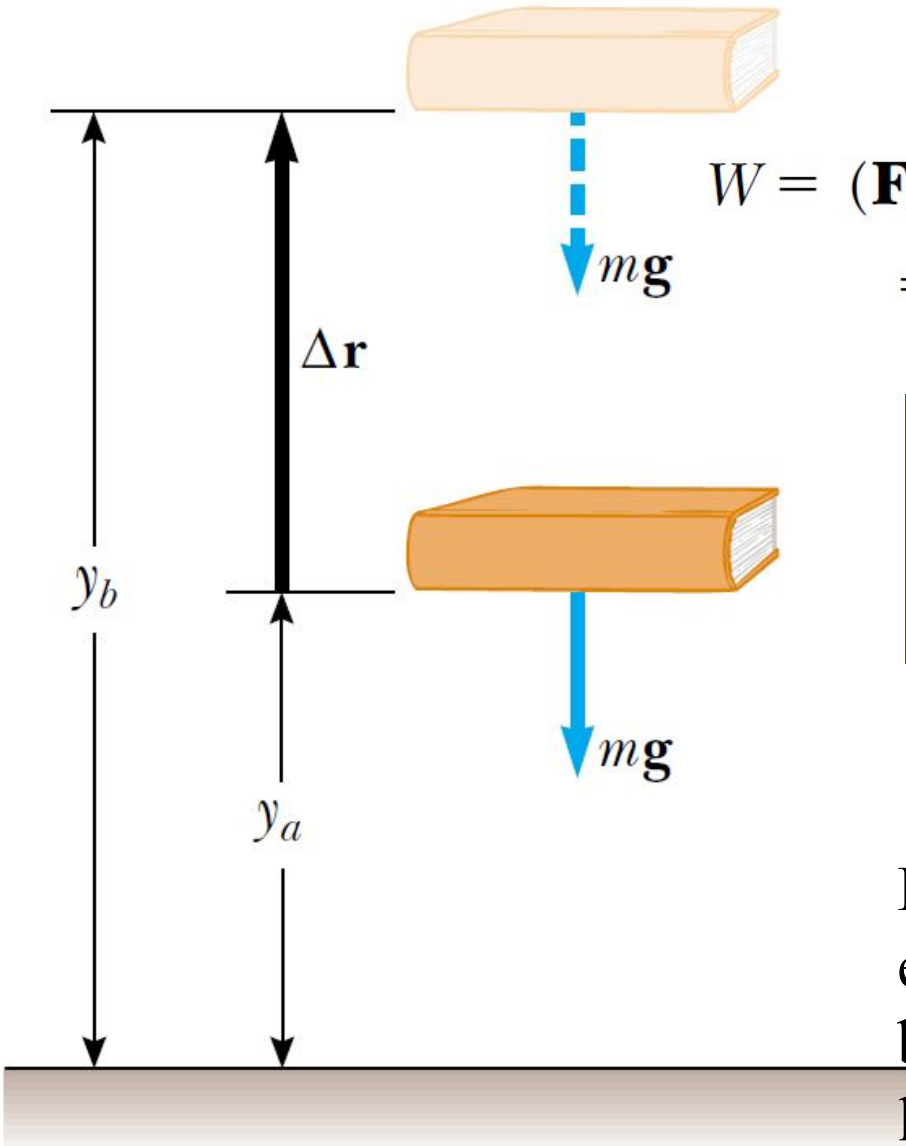
$$1\ \mathrm{W} = 1\ \mathrm{J/s} = 1\ \mathrm{kg \cdot m^2/s^3}$$

A unit of power in the U.S. customary system is the **horsepower** (hp):

$$1\ \mathrm{hp} = 746\ \mathrm{W}$$

$$1\ \mathrm{kWh} = (10^3\ \mathrm{W})(3\,600\ \mathrm{s}) = 3.60 \times 10^6\ \mathrm{J}$$

Potential Energy of a System

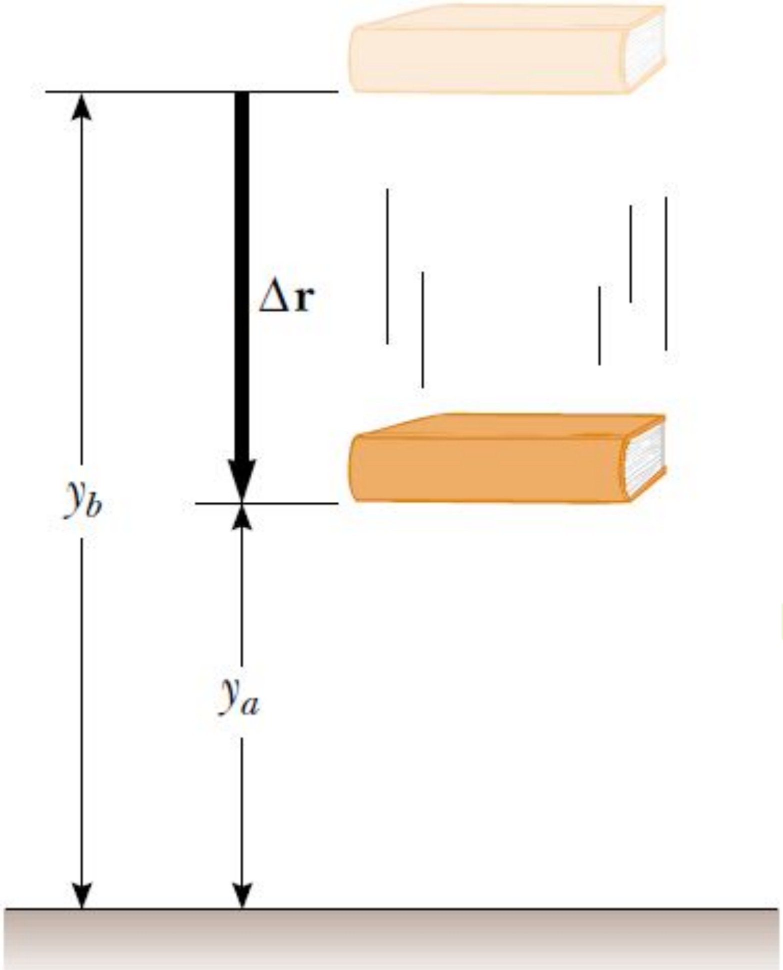


$$W = (\mathbf{F}_{\text{app}}) \cdot \Delta\mathbf{r} = (mg\hat{\mathbf{j}}) \cdot [(y_b - y_a)\hat{\mathbf{j}}] =$$
$$= mgy_b - mgy_a$$

$$\boxed{U_g \equiv mgy} \quad \textbf{(6.11)}$$

Figure 6.8 The work done by an external agent on the system of the book and the Earth as the book is lifted from a height y_a to a height y_b is equal to mgy_b - mgy_a.

The Isolated System–Conservation of Mechanical Energy



$$W_{\text{on book}} = (m\mathbf{g}) \cdot \Delta\mathbf{r} =$$

$$= (-mg\hat{\mathbf{j}}) \cdot [(y_a - y_b)\hat{\mathbf{j}}] =$$

$$= mgy_b - mgy_a$$

$$W_{\text{on book}} = \Delta K_{\text{book}}$$

Figure 6.9 The work done by the gravitational force on the book as the book falls from y_b to a height y_a is equal to $mgy_b - mgy_a$.

Therefore, equating these two expressions for the work done on the book,

$$\boxed{\Delta K_{\text{book}} = mgy_b - mgy_a} \quad \textbf{(6.12)}$$

Now, let us relate each side of this equation to the *system* of the book and the Earth. For the right-hand side,

$$mgy_b - mgy_a = -(mgy_a - mgy_b) =$$

$$= -(U_f - U_i) = -\Delta U_g$$

$$\boxed{\Delta K = -\Delta U_g} \qquad \boxed{\Delta K + \Delta U_g = 0}$$

(6.13) **(6.14)**

We define the sum of kinetic and potential energies as mechanical energy:

$$\boxed{E_{mech} = K + U_g} \qquad \textbf{(6.15)}$$

We will encounter other types of potential energy besides gravitational later in the text, so we can write the general form of the definition for mechanical energy without a subscript on *U:*

$$\boxed{E_{mech} \equiv K + U} \qquad \textbf{(6.16)}$$

$$\boxed{(K_f - K_i) + (U_f - U_i) = 0} \qquad \textbf{(6.17)}$$

$$K_f + U_f = K_i + U_i$$

(6.18)

Equation 6.18 is a statement of conservation of mechanical energy for an isolated system. An isolated system is one for which there are no energy transfers across the boundary. The energy in such a system is conserved—the sum of the kinetic and potential energies remains constant.

Conservative and Nonconservative Forces

Conservative Forces

Nonconservative Forces

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

Nonconservative Forces

A force is nonconservative if it does not satisfy properties 1 and 2 for conservative forces. Nonconservative forces acting within a system cause a change in the mechanical energy E_{mech} of the system. We have defined mechanical energy as the sum of the kinetic and all potential energies.

Changes in Mechanical Energy for Nonconservative Forces

$$\boxed{\Delta K = -f_k d} \qquad \textbf{(6.19)}$$

$$\boxed{\Delta E_{\text{mech}} = \Delta K + \Delta U_g = -f_k d}$$

$$\boxed{\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k d}$$

(6.20)

Relationship Between Conservative Forces and Potential Energy

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U \qquad \textbf{(6.21)}$$

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \qquad \textbf{(6.22)}$$

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i \qquad \textbf{(6.22)}$$

Relationship Between Conservative Forces and Potential Energy

$$\boxed{dU = -F_x\, dx}$$

$$\boxed{F_x = -\frac{dU}{dx}}$$

That is, the x component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to x.

Quick Quiz 1 A block of mass *m* is projected across a horizontal surface with an initial speed *v*. It slides until it stops due to the friction force between the block and the surface. The same block is now projected across the horizontal surface with an initial speed 2*v*. When the block has come to rest, how does the distance from the projection point compare to that in the first case? (a) It is the same. (b) It is twice as large. (c) It is four times as large. (d) The relationship cannot be determined.

Quick Quiz 2