

Physics 1

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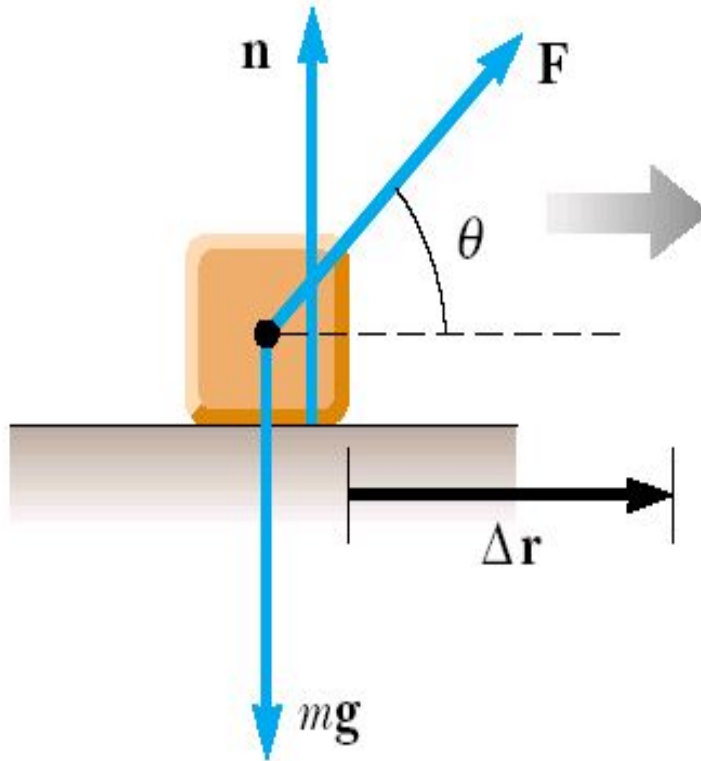
Lecture 3

- Work, energy and power
- Conservation of energy
- Linear momentum.
- Collisions.

Work

- A force acting on an object can do work on the object when the object moves.

$$W \equiv F \Delta r \cos \theta$$

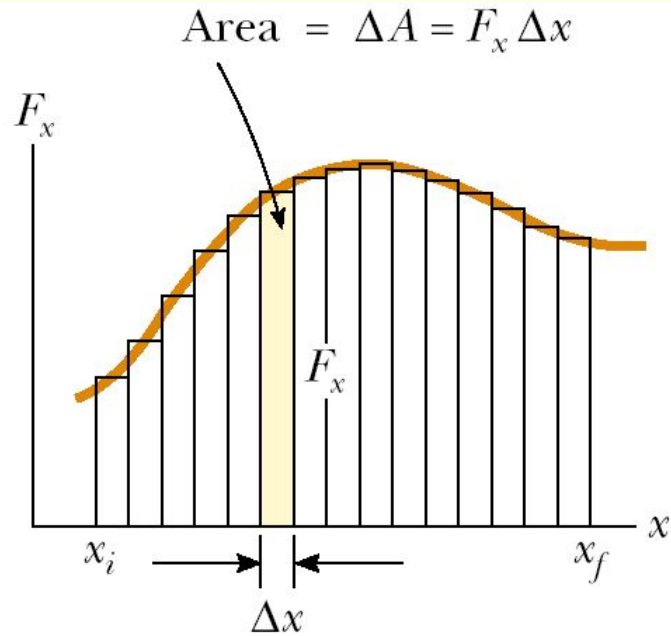


$$W \equiv F \Delta r \cos \theta$$

When an object is displaced on a frictionless, horizontal surface, the normal force n and the gravitational force mg do no work on the object. In the situation shown here, F is the only force doing work on the object.

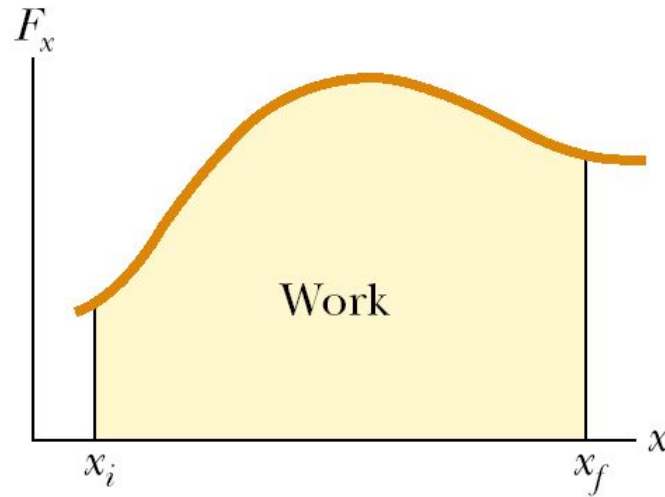
Work Units

Work done by a varying force



$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

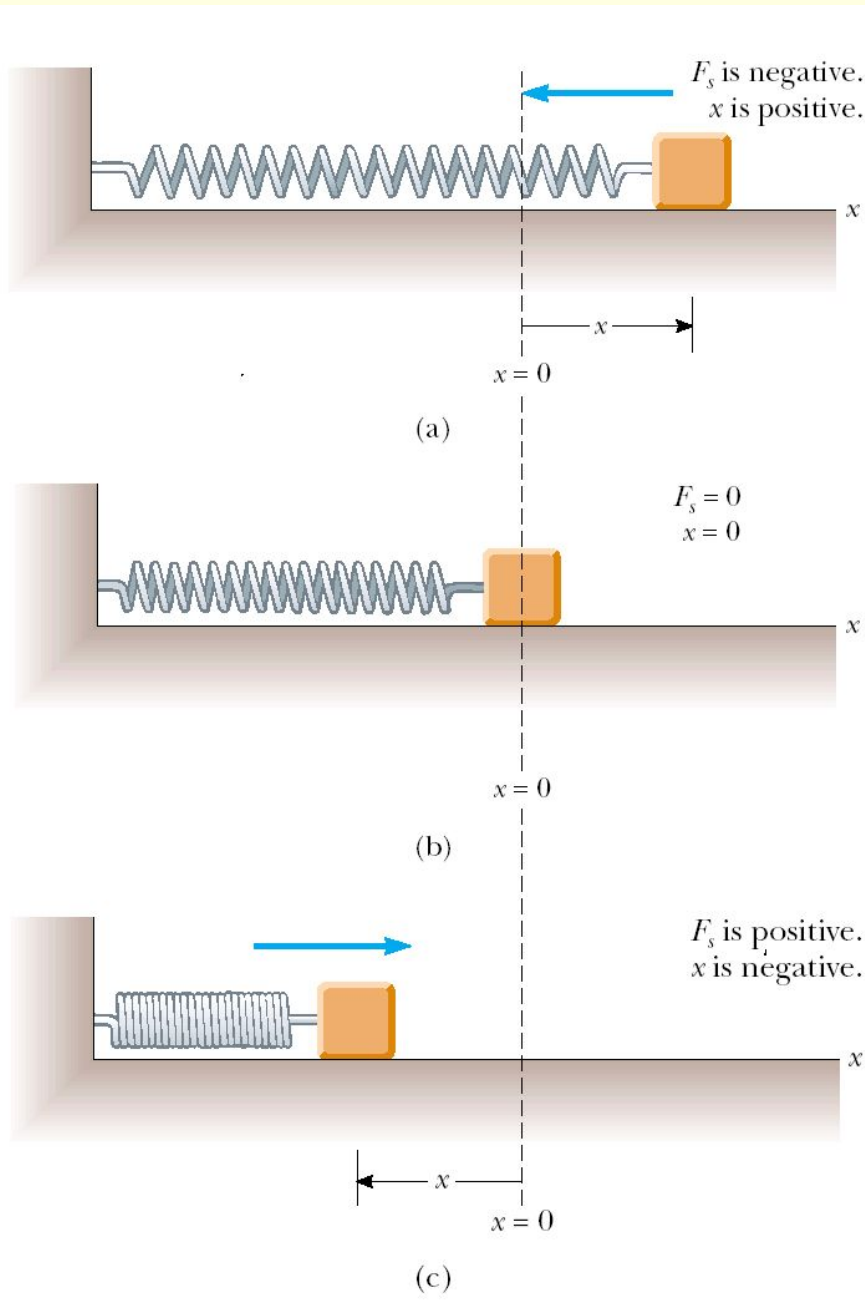


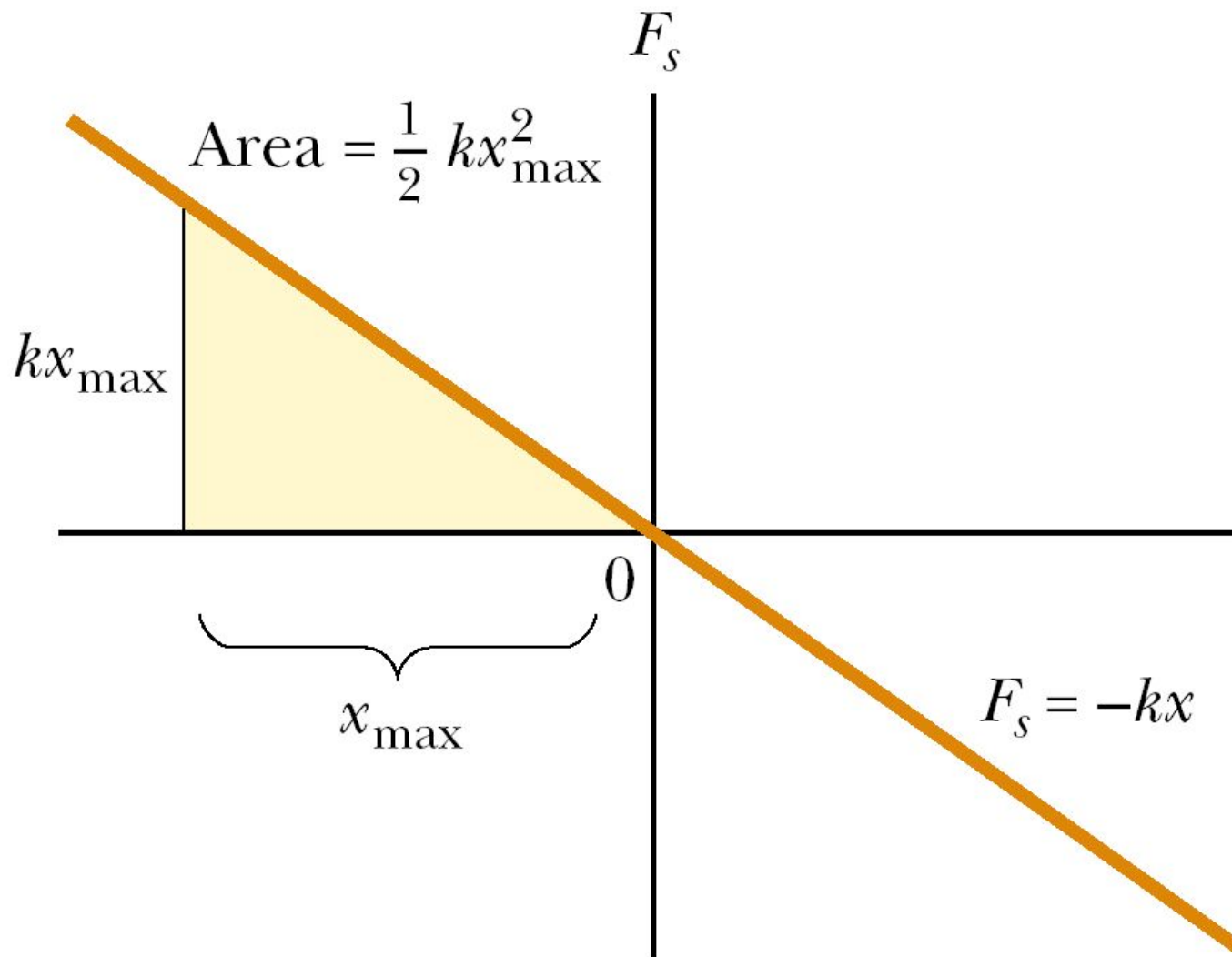
$$W = \int_{x_i}^{x_f} F_x dx$$

Work done by a spring

- If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as

$$F_s = - kx$$





Work of a spring

- So the work done by a spring from one arbitrary position to another is:

$$W_s = \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Kinetic energy

- Work is a mechanism for transferring energy into a system. One of the possible outcomes of doing work on a system is that the system changes its speed.
- Let's take a body and a force acting upon it:

$$\sum W = \int_{x_i}^{x_f} \sum F dx$$

- Using Newton's second law, we can substitute for the magnitude of the net force

$$\sum F = ma$$

- and then perform the following chain-rule manipulations on the integrand:

$$\sum W = \int_{x_i}^{x_f} m a dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{x_i}^{x_f} m \frac{dv}{dx} \frac{dx}{dt} dx = \int_{v_i}^{v_f} m v dv$$

- And finally:

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

- This equation was generated for the specific situation of one-dimensional motion, but it is a general result. It tells us that the work done by the net force on a particle of mass m is equal to the difference between the initial and final values of a quantity

$$K \equiv \frac{1}{2} m v^2$$

Work-energy theorem:

$$\sum W = K_f - K_i = \Delta K$$

Conservative and Nonconservative Forces

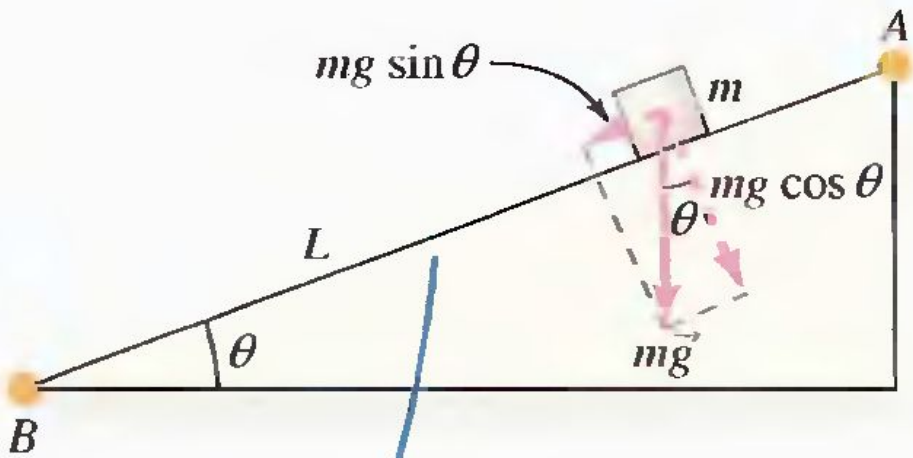
- Forces for which the work is **independent** of the path are called *conservative forces*.
- Forces for which the work **depends** on the path are called *nonconservative forces*
- **The work done by a conservative force in moving an object along any closed path is zero.**

Examples

- Conservative Forces:
 - Spring
 - central forces
 - Gravity
 - Electrostatic forces
- Nonconservative Forces:
 - Various kinds of Friction

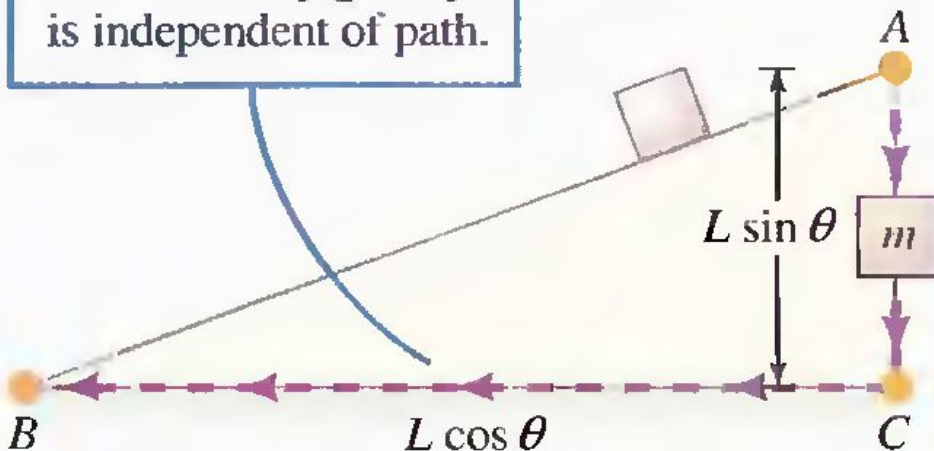
Gravity is a conservative force:

An object moves from point A to point B on an inclined plane under the influence of gravity. Gravity does positive (or negative) work on the object as it moves down (or up) the plane.



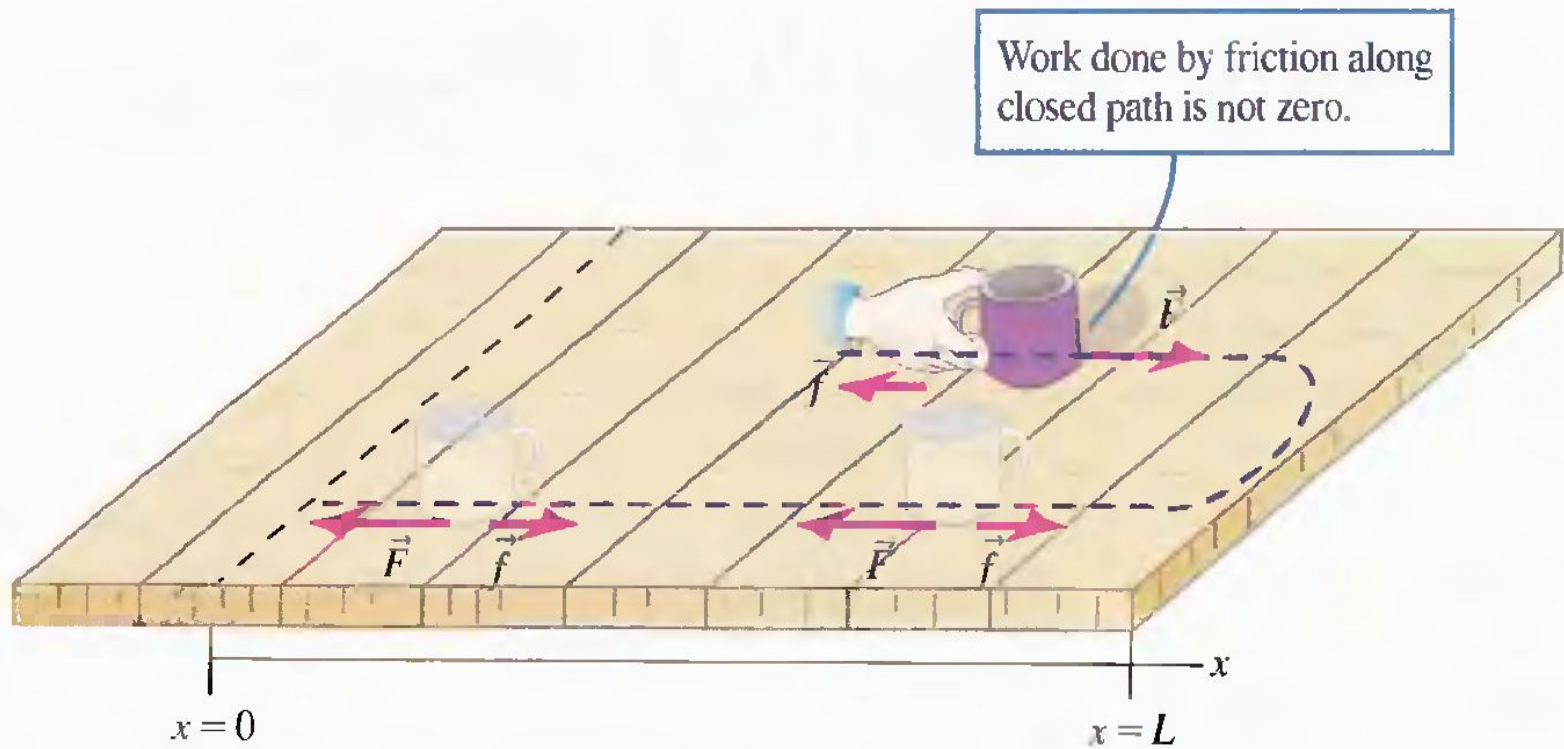
(a)

Work done by gravity is independent of path.



The object now moves from point A to point B by a different path: a vertical motion from point A to point C followed by a horizontal movement from C to B. The work done by gravity is exactly the same as in part (a).

Friction is a nonconservative force:



Power

- Power P is the rate at which work is done:

$$P \equiv \frac{dW}{dt}.$$

$$P = \vec{F} \cdot \frac{d \Delta \vec{r}}{dt} = \vec{F} \cdot \vec{v}.$$

Potential Energy

- Potential energy is the energy possessed by a system by virtue of position or condition.
- We call the particular function U for any given conservative force the **potential energy** for that force.

$$F(x) = -\frac{dU(x)}{dx}$$

- Remember the minus in the formula above.

$$W(x_0, x) = \int_{x_0}^x F(z) dz.$$

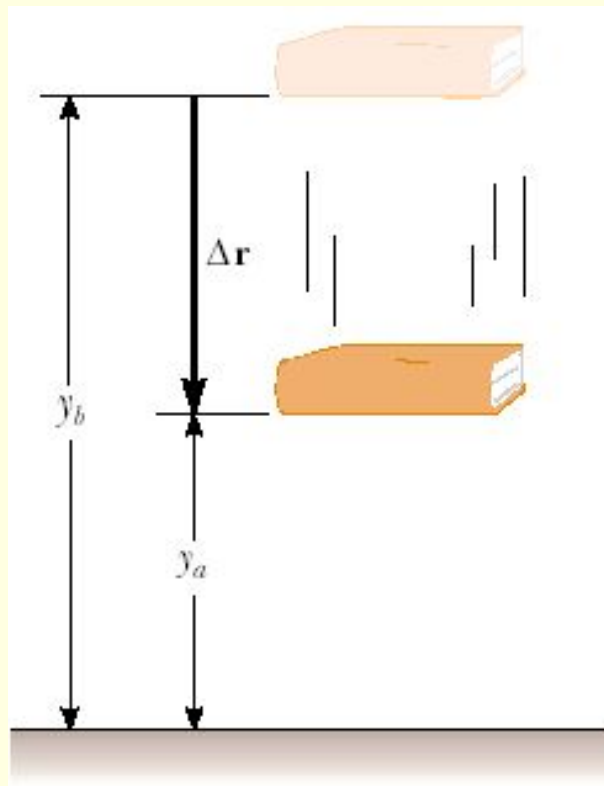
$$U(x) - U(x_0) = -W(x_0, x).$$

$$U(x) - U(x_0) = - \int_{x_0}^x F(z) dz.$$

$$F(x) = - \frac{dU(x)}{dx}$$

Potential Energy of Gravity

$$W_{\text{on book}} = (m\mathbf{g}) \cdot \Delta\mathbf{r} = (-mg\hat{\mathbf{j}}) \cdot [(y_a - y_b)\hat{\mathbf{j}}] = mgy_b - mgy_a$$



$$W_{\text{on book}} = \Delta K_{\text{book}}$$

$$\Delta K_{\text{book}} = mgy_b - mgy_a$$

$$\Delta K = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$

$$E_{\text{mech}} = K + U_g$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Conservation of mechanical energy

- $E = K + U(x) = \frac{1}{2}mv^2 + U(x)$ is called total mechanical energy
- If a system is
 - isolated (no energy transfer across its boundaries)
 - having no nonconservative forces withinthen the mechanical energy of such a system is constant.

Linear momentum

- Let's consider two interacting particles:

$$\mathbf{F}_{21} + \mathbf{F}_{12} = 0$$

and their accelerations are:

$$m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2 = 0$$

using definition of acceleration:

$$m_1 \frac{d\mathbf{v}_1}{dt} + m_2 \frac{d\mathbf{v}_2}{dt} = 0$$

masses are constant:

$$\frac{d(m_1 \mathbf{v}_1)}{dt} + \frac{d(m_2 \mathbf{v}_2)}{dt} = 0$$

$$\frac{d}{dt} (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) = 0$$

- So the total sum of quantities $m\mathbf{v}$ for an isolated system is conserved – independent of time.
- This quantity is called linear momentum.

$$\vec{p} \equiv m\vec{v}.$$

- General form for Newton's second law:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

- It means that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.
- The kinetic energy of an object can also be expressed in terms of the momentum:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m},$$

The law of linear momentum conservation

- The sum of the linear momenta of an isolated system of objects is a constant, no matter what forces act between the objects making up the system.

Impulse-momentum theorem

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F} dt$$

- The impulse of the force \mathbf{F} acting on a particle equals the change in the momentum of the particle.
- Quantity $\mathbf{I} \equiv \int_{t_i}^{t_f} \mathbf{F} dt$ is called the impulse of the force \mathbf{F} .

Collisions

Let's study the following types of collisions:

1. Perfectly elastic collisions:
 1. no mass transfer from one object to another
 2. Kinetic energy conserves (all the kinetic energy before collision goes to the kinetic energy after collision)
2. Perfectly inelastic collisions: two objects merge into one. Maximum kinetic loss.

Perfectly elastic collisions

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2,$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2.$$

$$m_1(v_1 - v'_1) = -m_2(v_2 - v'_2).$$

$$\frac{1}{2} m_1(v_1 - v'_1)(v_1 + v'_1) = -\frac{1}{2} m_2(v_2 - v'_2)(v_2 + v'_2).$$

$$v_1 + v'_1 = v_2 + v'_2.$$

■ Denoting $u_i = v_1 - v_2$ and $u_f = v'_1 - v'_2$.
We can obtain from (5) $u_i = -u_f$.

Here U_i and U_f are initial and final *relative* velocities.

■ So the last equation says that when the collision is elastic, the relative velocity of the colliding objects changes sign but does not change magnitude.

Perfectly inelastic collisions

$$M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2.$$

$$M = m_1 + m_2.$$

$$v = \frac{m_1v_1 + m_2v_2}{M}.$$

Energy loss in perfectly inelastic collisions

$$\begin{aligned}\Delta E &= \frac{1}{2} M v^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \\ &= \frac{1}{2} \frac{M (m_1 v_1 + m_2 v_2)^2}{M^2} - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \\ &= \frac{1}{2} \frac{m_1^2 v_1^2 + 2 m_1 m_2 v_1 v_2 + m_2^2 v_2^2 - M (m_1 v_1^2 + m_2 v_2^2)}{M} \\ &= \frac{1}{2} \frac{m_1 m_2 (-v_1^2 - v_2^2 + 2 v_1 v_2)}{M} = -\frac{1}{2} \frac{m_1 m_2}{M} (v_1 - v_2)^2.\end{aligned}$$

Units in SI

■ Work, Energy W, E $J = N \cdot m = \text{kg} \cdot \text{m}^2 / \text{s}^2$

■ Power P $J / \text{s} = \text{kg} \cdot \text{m}^2 / \text{s}^3$

■ Linear momentum p $\text{kg} \cdot \text{m} / \text{s}$