

МБОУ «Кижингинская средняя школа им. Хоца Намсараева»

\* «Решу задачи  
планиметрии!»»

*Система тренировочных упражнений  
для учащихся 10-11 классов*

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учитель математики

\* Проблема:

Задачи **стереометрии**



Задачи **планиметрии**



## *Цели и задачи:*

- Устранить у обучающихся «пробелы» в знаниях по планиметрии.
- Формировать специальные математические навыки решения планиметрических задач.



# Содержание

I. Подобные треугольники.

II. Соотношения между сторонами и углами прямоугольного треугольника.

III. Площади фигур.

IV. Правильные многоугольники.

- **Справочная таблица;**
- **Набор тренировочных упражнений;**
- **Проверочная работа;**

# \* I. ПОДОБНЫЕ ТРЕУГОЛЬНИКИ

## 1. Справочная таблица

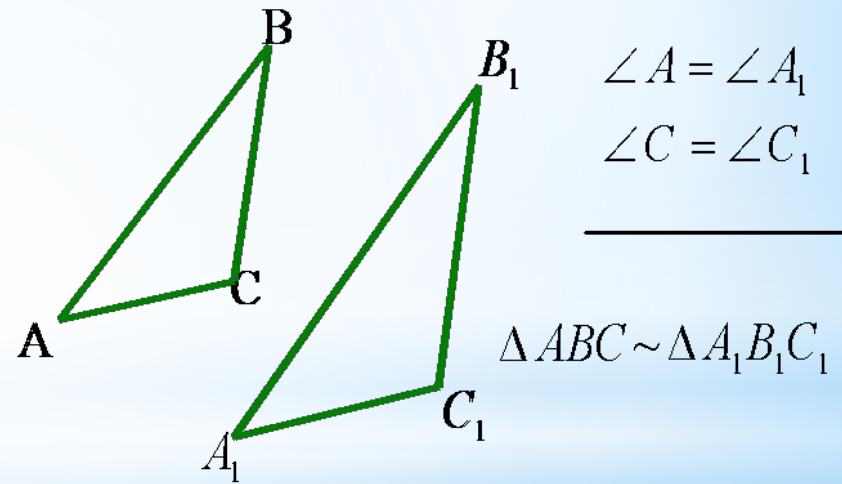
### 1. Определение подобных треугольников

$$\angle A = \angle A_1, \angle B = \angle B_1, \angle C = \angle C_1$$

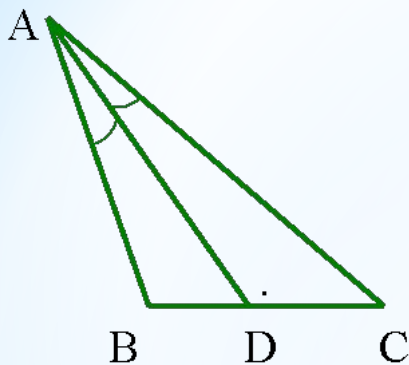
$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1} = k$$

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = k^2$$

### 2. Первый признак подобия треугольников

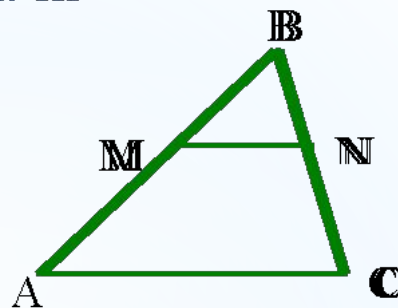


### 3.Задачи



**AD- биссектриса**

$$\frac{BD}{AB} = \frac{DC}{AC}$$



$$MN \parallel AC$$

$$\frac{MB}{MA} = \frac{2}{3}$$

$$\frac{MA}{AB} = ?$$

**Решение:**

$$\frac{MB}{MA} = \frac{2}{3}, \text{ значит, } MB = 2k, MA = 3k,$$

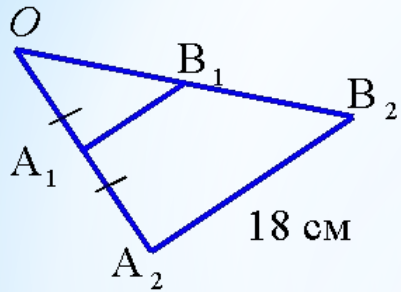
$$AB = 5k \text{ и } \frac{MA}{AB} = \frac{3k}{5k} = \frac{3}{5}.$$

$$\text{Ответ: } \frac{MA}{AB} = \frac{3}{5}.$$

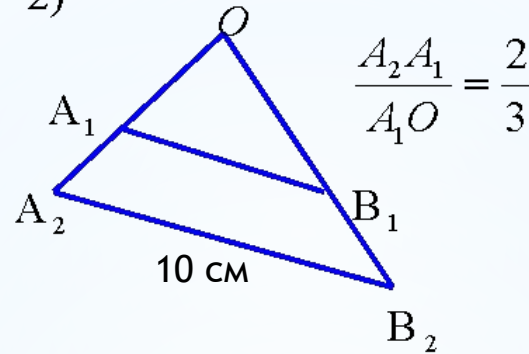
## 2. Тренировочные упражнения

Найти  $A_1B_1$ , если  $A_1B_1 \parallel A_2B_2$

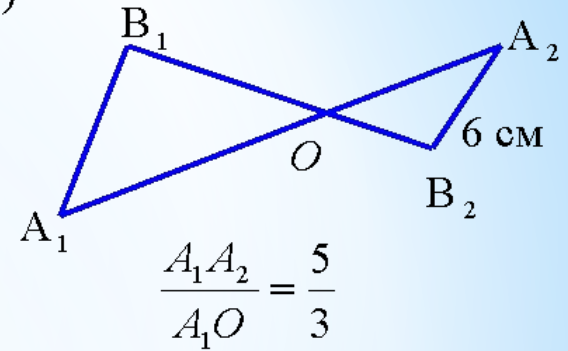
1)



2)

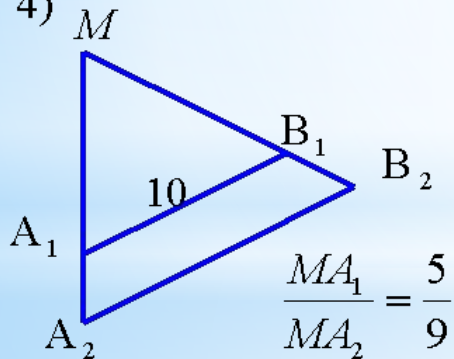


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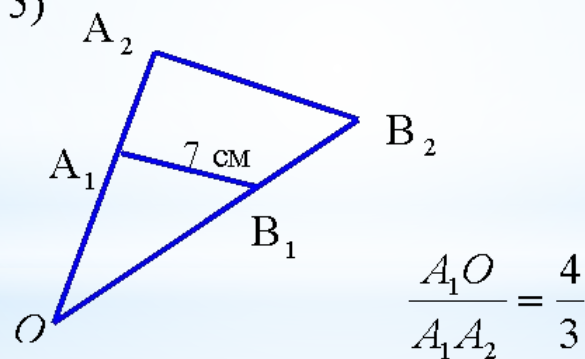


Найти  $A_2B_2$ , если  $A_1B_1 \parallel A_2B_2$

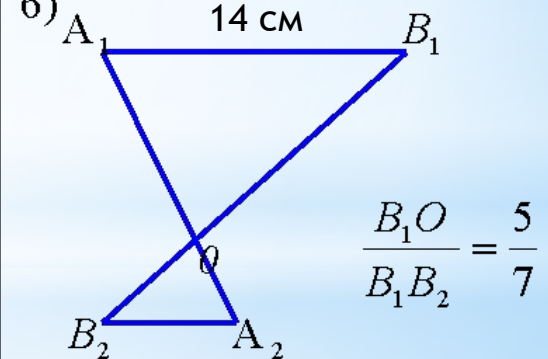
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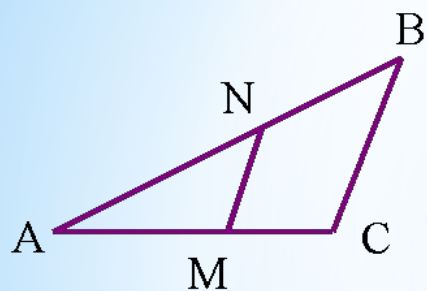


6)



### 3. Проверочная работа

1)

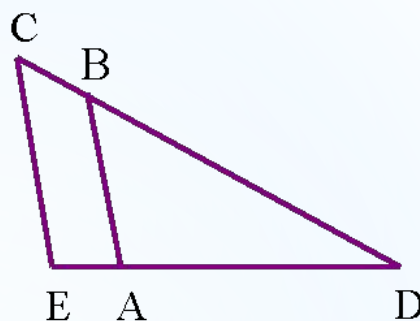


**V-I**

Дано:  $MN \parallel BC$   
 $AN : NB = 7 : 2$   
 $BC = 2,7 \text{ см.}$

Найти:  $MN$

1)

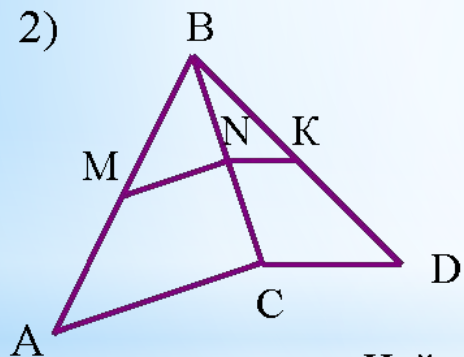


**V-II**

Дано:  $BA \parallel CE$   
 $DB : BC = 5 : 2$   
 $AB = 4,5 \text{ см}$

Найти:  $CE$

2)



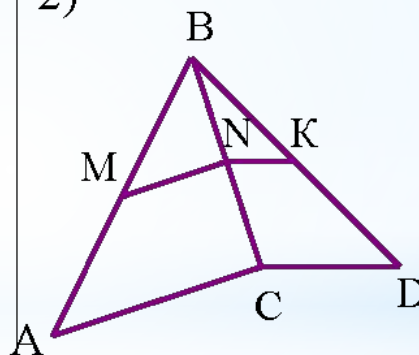
Дано:  $MN \parallel AC$   
 $NK \parallel CD$

$$\frac{AM}{MB} = \frac{5}{3}$$

$$NK = 1,8 \text{ см}$$

Найти:  $CD$

2)



Дано:  $MN \parallel AC$   
 $NK \parallel CD$

$$\frac{AM}{MB} = \frac{2}{3}$$

$$CD = 2,1 \text{ дм}$$

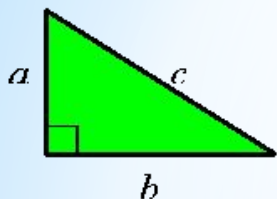
Найти:  $NK$



# \* II. СООТНОШЕНИЯ МЕЖДУ СТОРОНАМИ И УГЛАМИ ПРЯМОУГОЛЬНОГО ТРЕУГОЛЬНИКА

## 1. Справочная таблица

### 1. Теорема Пифагора

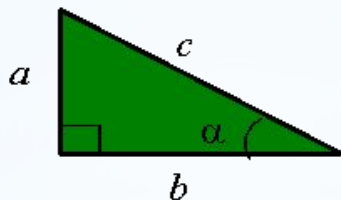


$$c^2 = a^2 + b^2, \quad c = \sqrt{a^2 + b^2};$$

$$a^2 = c^2 - b^2, \quad a = \sqrt{c^2 - b^2};$$

$$b^2 = c^2 - a^2, \quad b = \sqrt{c^2 - a^2}$$

### 2. Определение синуса, косинуса и тангенса острого угла и следствия из них

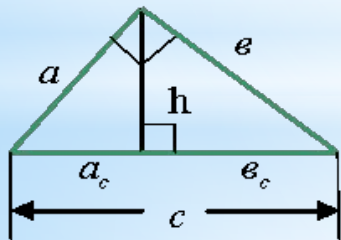


$$\sin \alpha = \frac{a}{c}, \quad a = c \cdot \sin \alpha, \quad c = \frac{a}{\sin \alpha};$$

$$\cos \alpha = \frac{b}{c}, \quad b = c \cdot \cos \alpha, \quad c = \frac{b}{\cos \alpha};$$

$$\operatorname{tg} \alpha = \frac{a}{b}, \quad a = b \cdot \operatorname{tg} \alpha, \quad b = \frac{a}{\operatorname{tg} \alpha}.$$

### 3. Теоремы о среднем пропорциональном

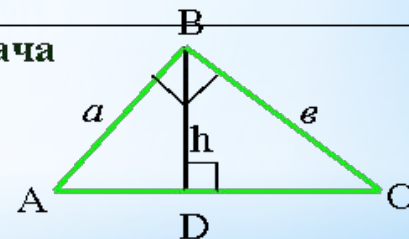


$$h^2 = a_c \cdot b_c, \quad h = \sqrt{a_c \cdot b_c};$$

$$a^2 = c \cdot a_c, \quad a = \sqrt{c \cdot a_c};$$

$$b^2 = c \cdot b_c, \quad b = \sqrt{c \cdot b_c}.$$

### Задача



Найти: BD

Решение:

$$\sin A = \frac{BD}{a}; \quad \sin A = \frac{b}{AC};$$

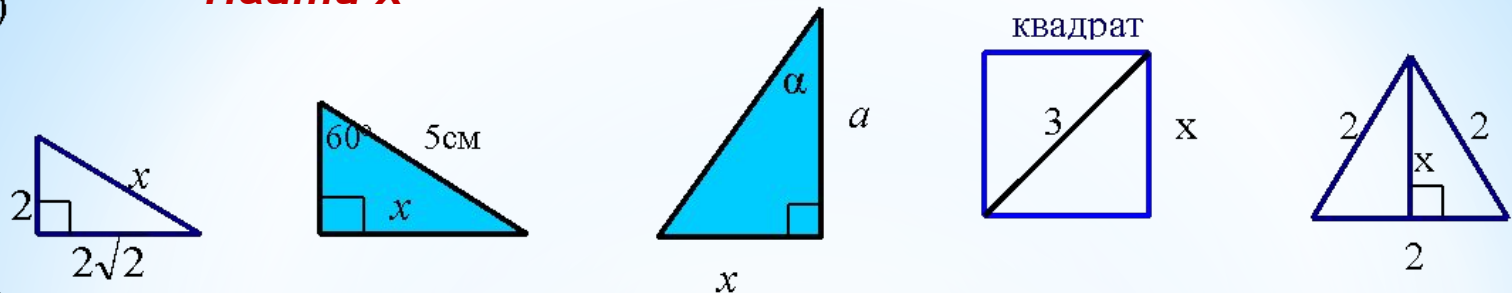
$$\therefore$$

$$BD = \frac{ab}{\sqrt{a^2 + b^2}}.$$

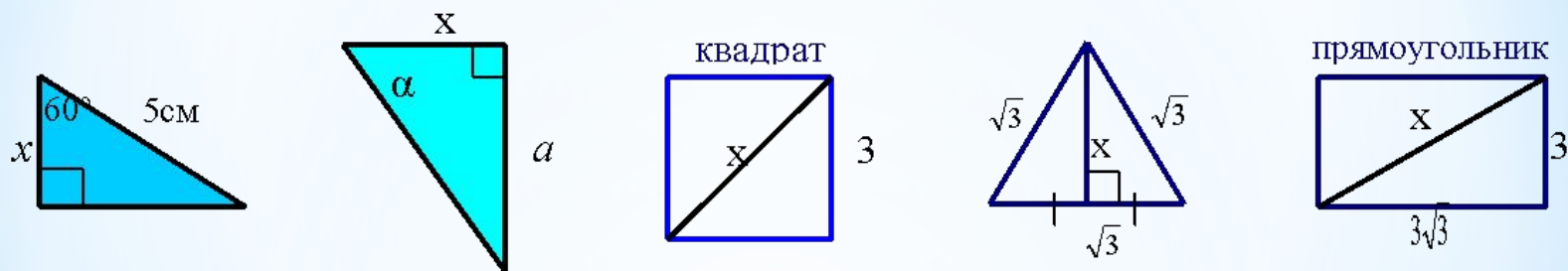
## 2. Тренировочные упражнения

Найти  $x$

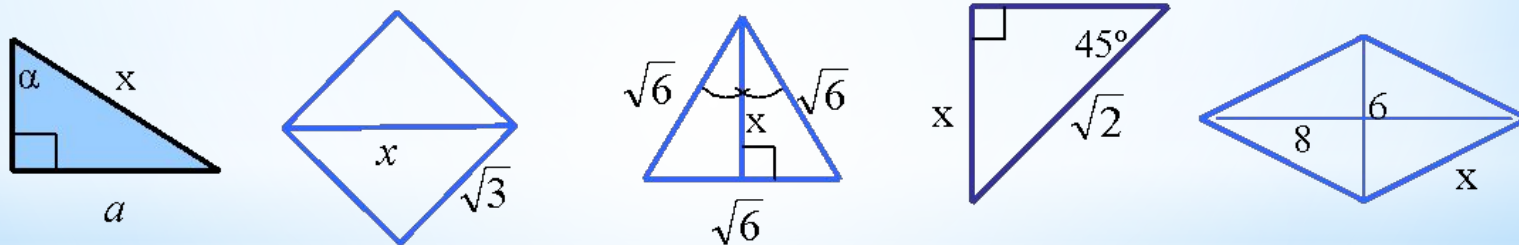
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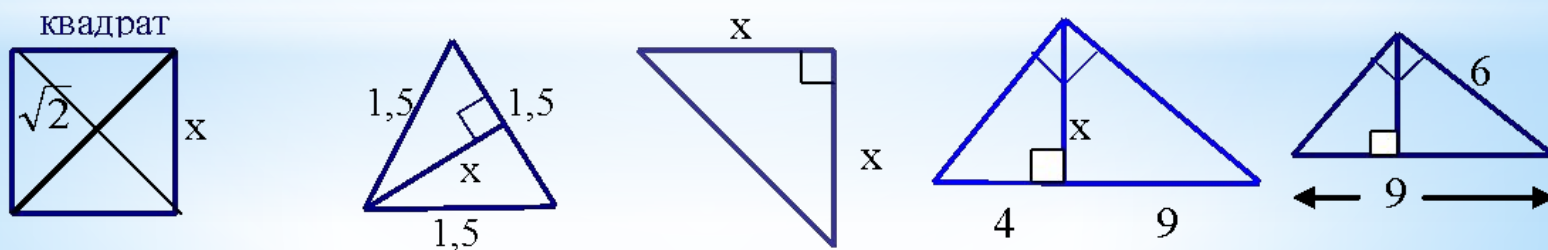
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3)

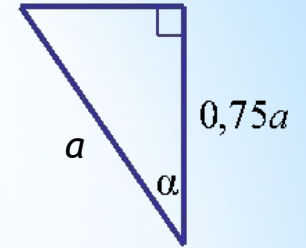
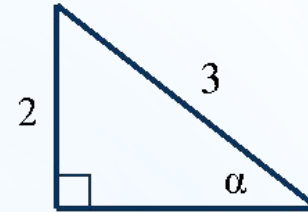
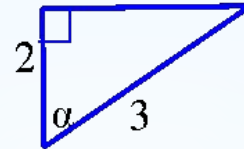
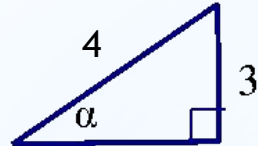
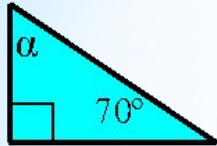


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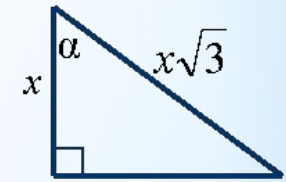
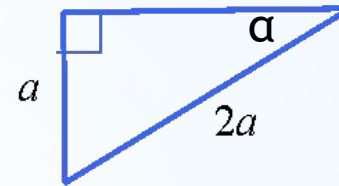
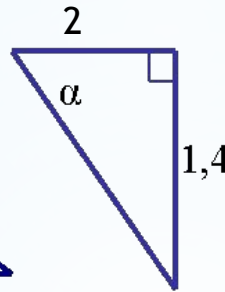
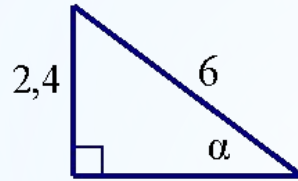
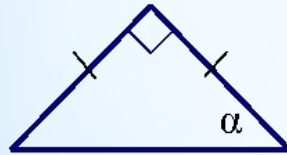


## Найти угол $\alpha$

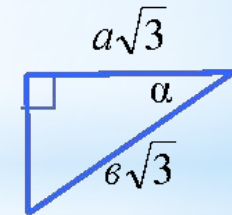
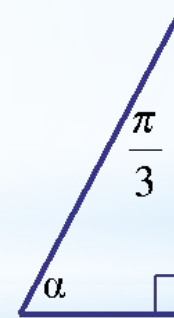
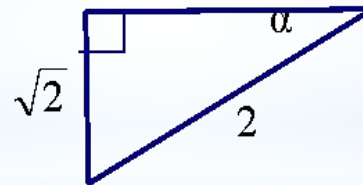
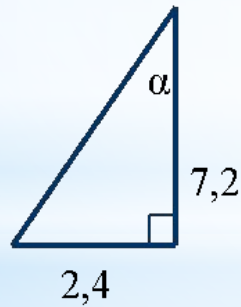
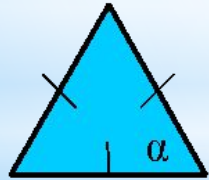
5)



6)



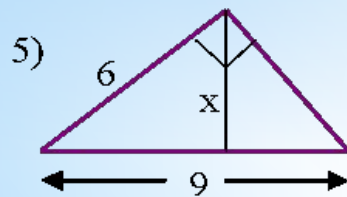
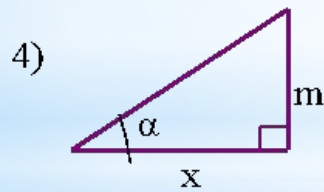
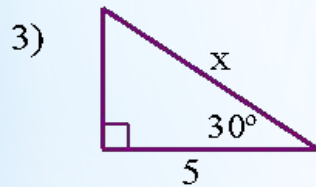
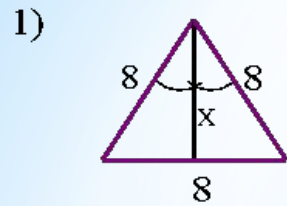
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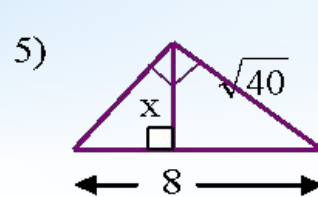
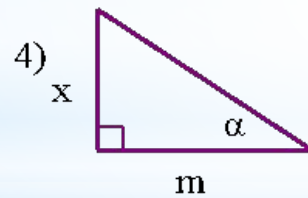
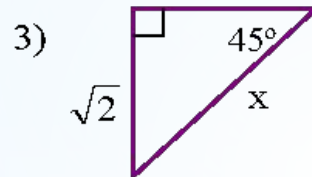
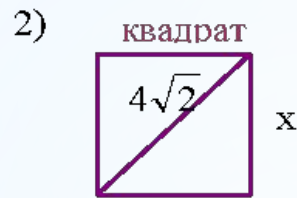
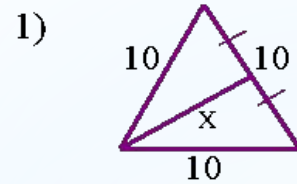
# Проверочная работа №1

## Найти $x$

### Вариант 1



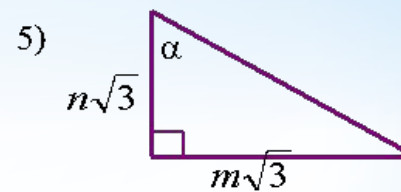
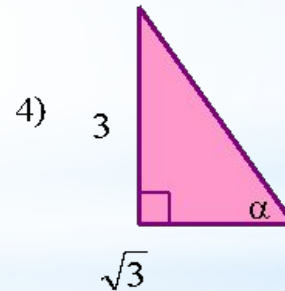
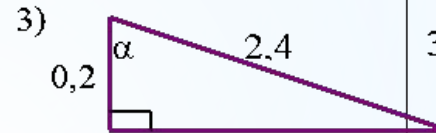
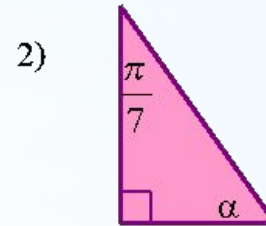
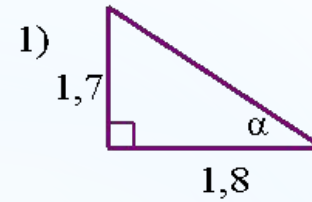
### Вариант 2



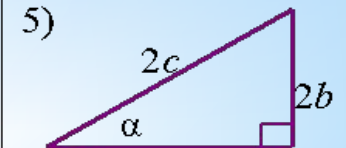
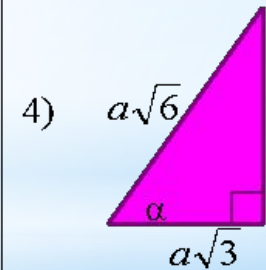
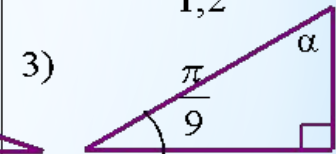
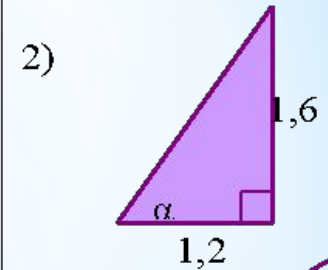
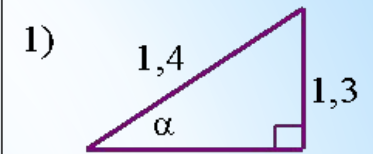
# Проверочная работа №2

## Найти угол $\alpha$

### Вариант 1

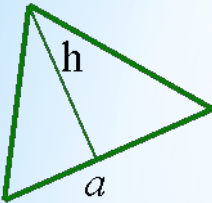
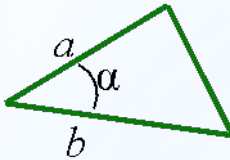
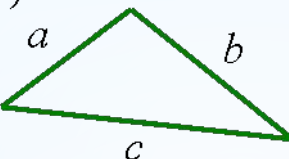
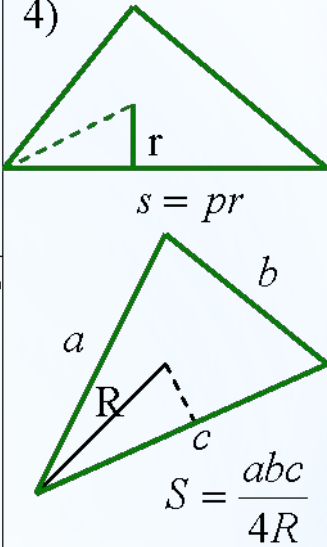
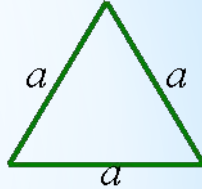
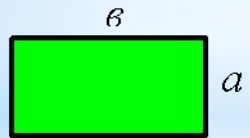
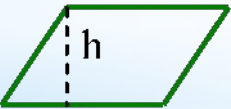
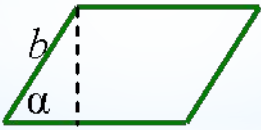
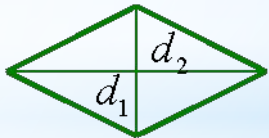
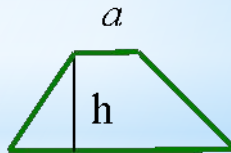


### Вариант 2



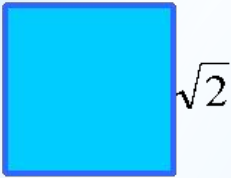
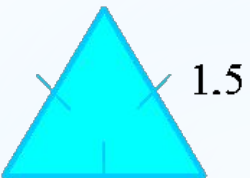
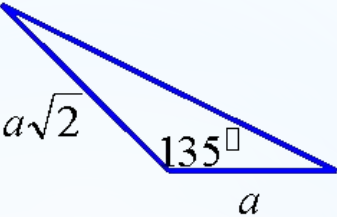
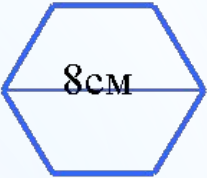
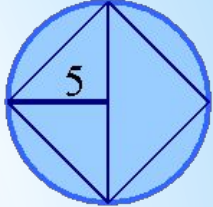
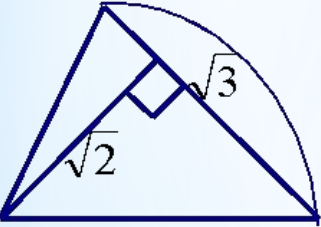
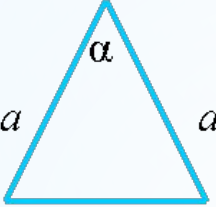
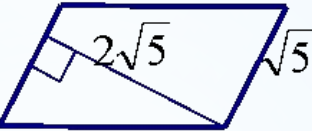
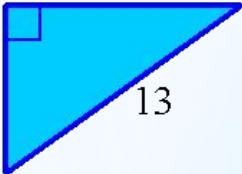
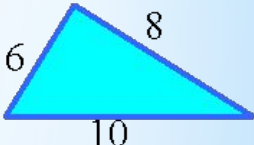
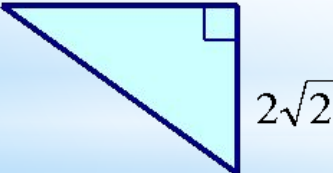
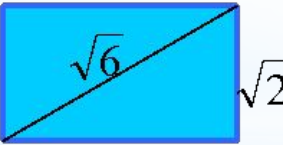
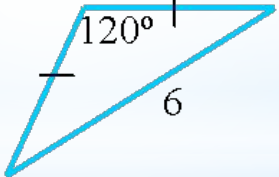
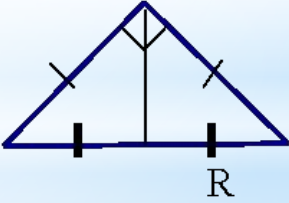
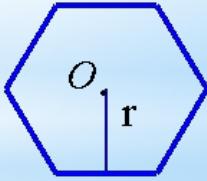
# \* III. ФОРМУЛЫ ПЛОЩАДЕЙ

## 1.Справочная таблица

<p>1)</p>  $S = \frac{1}{2}ah$	<p>2)</p>  $S = \frac{1}{2}ab \sin \alpha$	<p>3)</p>  $S = \sqrt{p(p-a)(p-b)(p-c)}$ $p = \frac{a+b+c}{2}$	<p>4)</p>  $s = pr$ $S = \frac{abc}{4R}$	<p>5)</p>  $S = \frac{a^2 \sqrt{3}}{4}$
 $S = ab$	 $S = ah$	 $S = ab \sin \alpha$	 $S = \frac{1}{2}d_1 d_2$	 $S = \frac{1}{2}(a+b) \cdot h$

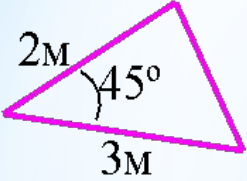
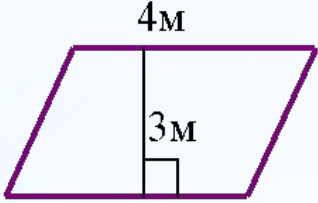
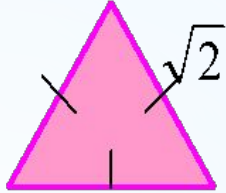
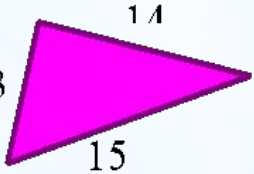
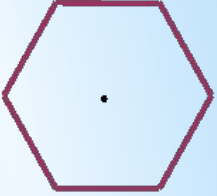

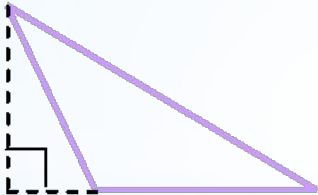
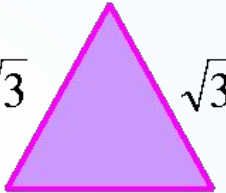
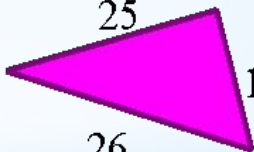
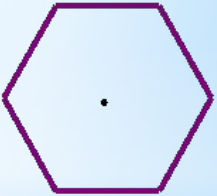
## 2. Тренировочные упражнения

Вычислить площадь фигуры

1.					
2.					
3.					

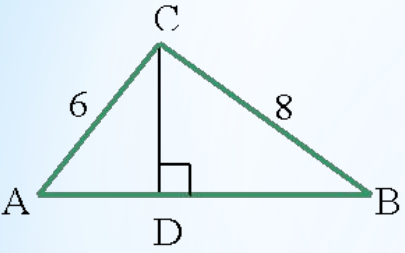
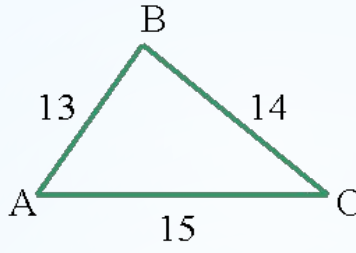
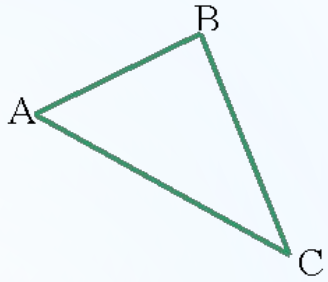
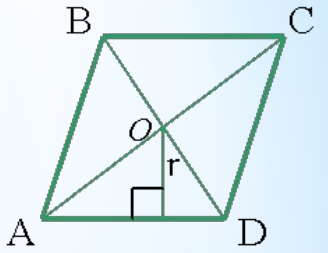
### 3. Проверочная работа

Вычислить площадь фигуры:

В-I	 <p>Triangle with sides <math>2m</math> and <math>3m</math>, and an angle of <math>45^\circ</math>.</p>	 <p>Parallelogram with base <math>4m</math> and height <math>3m</math>.</p>	 <p>Equilateral triangle with side length <math>\sqrt{2}</math>.</p>	 <p>Triangle with sides <math>13</math> and <math>15</math>, and an angle of <math>11^\circ</math>.</p>	 <p>Hexagon with side length <math>4\text{cm}</math>.</p>
В-II	 <p>Parallelogram with sides <math>2m</math> and <math>3m</math>, and an angle of <math>60^\circ</math>.</p>	 <p>Triangle with base <math>4m</math> and height <math>3m</math>.</p>	 <p>Equilateral triangle with side length <math>\sqrt{3}</math>.</p>	 <p>Triangle with sides <math>25</math> and <math>17</math>, and an angle of <math>26^\circ</math>.</p>	 <p>Hexagon with side length <math>3\text{cm}</math>.</p>

**Вычисление некоторых линейных элементов  $[h, r, R]$   
 фигуры с использованием формулы площади**

**1.Справочная таблица**

<i>Задачи с решениями</i>			
<i>Найти <math>h</math></i>	<i>Найти <math>r</math></i>	<i>Найти <math>R</math></i>	<i>Найти <math>r</math></i>
 <p style="margin-top: 10px;">Дано : <math>\angle C = 90^\circ</math>,  <math>CD \perp AB</math>, <math>AC = 6</math>, <math>CB = 8</math>.                      Найти : <math>CD</math>                      Решение : <math>2S_{ABC} = AC \cdot BC</math>,  <math>2S_{ABC} = AB \cdot CD</math>,  <math>CD = \frac{AC \cdot BC}{AB}</math>;  <math>AB = \sqrt{6^2 + 8^2} = 10</math>,  <math>CD = \frac{6 \cdot 8}{10} = 4,8</math>                      Ответ : <math>CD = 4,8</math></p>	 <p style="margin-top: 10px;">Найти : <math>r</math>                      Решение :  <math>S = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84</math>  <math>S = p \cdot r</math>  <math>r = \frac{s}{p} = \frac{84}{21} = 4</math>                      Ответ : <math>r = 4</math>.</p>	 <p style="margin-top: 10px;">Найти : <math>R</math>                      Решение :  <math>R = \frac{abc}{4S}</math>,  <math>R = \frac{13 \cdot 14 \cdot 15}{4 \cdot 84} = 8 \frac{1}{8}</math>.                      Ответ : <math>R = 8 \frac{1}{8}</math>.</p>	 <p style="margin-top: 10px;">Дано : <math>ABCD</math> –                      ромб,  <math>AC = 20</math>, <math>BD = 15</math>.                      Найти : <math>r</math>                      Решение :  <math>S = \frac{1}{2} AC \cdot BD = 150</math>  <math>BC = 12,5</math>  <math>r = \frac{S}{p} = \frac{150}{25} = 6</math>.                      Ответ : <math>r = 6</math>.</p>

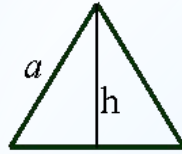
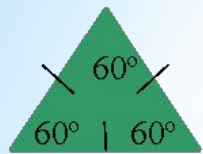


## 2. Тренировочные упражнения

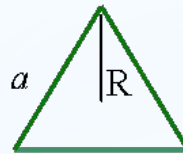
	<i>Найти <math>h</math></i>			<i>Найти <math>r</math> и <math>R</math></i>
1.				
2.				

# \* IV. ПРАВИЛЬНЫЕ МНОГОУГОЛЬНИКИ

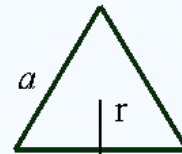
## 1.Справочная таблица



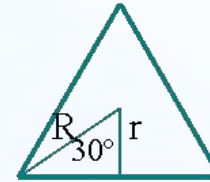
$$h = \frac{a\sqrt{3}}{2}$$



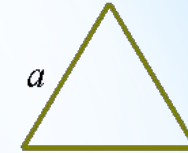
$$R = \frac{a}{\sqrt{3}}; a = R\sqrt{3}$$



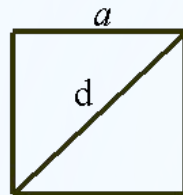
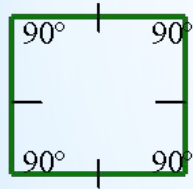
$$r = \frac{a}{2\sqrt{3}}$$



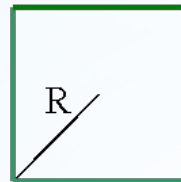
$$\frac{r}{R} = \frac{1}{2}$$



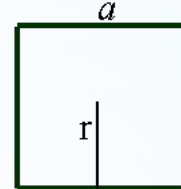
$$S = \frac{a^2\sqrt{3}}{4}$$



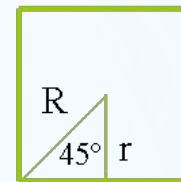
$$d = a\sqrt{2}$$



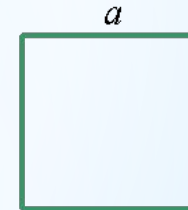
$$R = \frac{a}{\sqrt{2}}; a = R\sqrt{2}$$



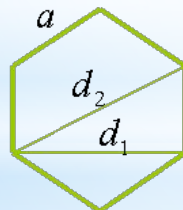
$$r = \frac{a}{2}$$



$$\frac{r}{R} = \frac{\sqrt{3}}{2}$$

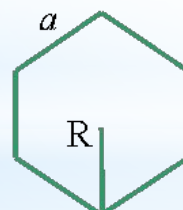


$$S = a^2$$

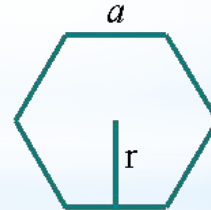


$$d_1 = a\sqrt{3}$$

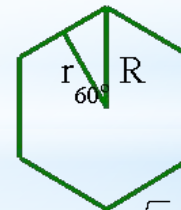
$$d_2 = 2a$$



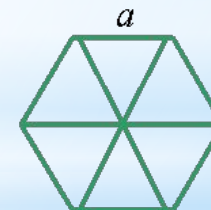
$$R = a$$



$$r = \frac{a\sqrt{3}}{2}$$

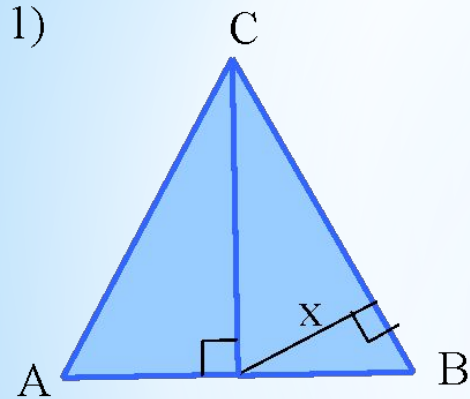


$$\frac{r}{R} = \frac{\sqrt{3}}{2}$$



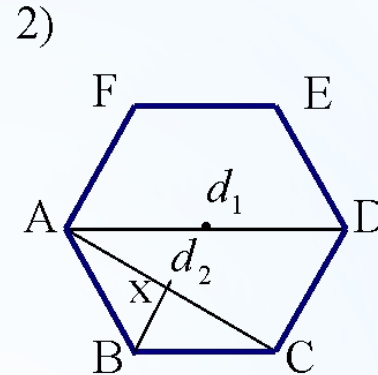
$$S_{\Delta} = \frac{3a^2\sqrt{3}}{2}$$

## 2. Тренировочные упражнения



Дано:  $\triangle ABC$  –  
равносторонний,  
 $AB = 3$ .

Найти:  $h, r, R,$   
 $S, x$ .



Дано:  $ABCDEF$  –  
правильный  
шестиугольник,  
 $AB = \sqrt{5}$ .

Найти:  $d_1, d_2, r, s, x$ .

### 3. Проверочная работа .

#### Дан правильный треугольник ABC

##### I вариант

1.  $AB = m$ .  
Найти медиану  $BD$ .
2.  $BD \perp AC$ ,  $BD = 1$ . Найти  $AB$ .
3.  $O$ - центр  $\triangle ABC$ ,  $AO = 3$ .  
Найти  $AB$ .
4.  $O$ - центр,  $OM \perp AB$ ,  $OM = 2$ .  
Найти  $AB$ .
5.  $S_{\triangle ABC} = 6\sqrt{3}$ . Найти  $AB$ .

##### II вариант

1.  $AB = n$ . Найти биссектрису  $AD$ .
2.  $AD \perp BC$ ,  $AD = k$ .  
Найти  $AC$ .
3.  $O$ - центр  $\triangle ABC$ ,  $OK \perp AC$ ,  
 $OK = 3$ . Найти  $AC$ .
4.  $O$ - центр  $\triangle ABC$ ,  $OC = 2$ .  
Найти  $AC$ .
5.  $S_{\triangle ABC} = 8\sqrt{3}$ . Найти  $AC$ .

**\*Спасибо за внимание!**