

УПРАЖНЕНИЯ ПЛАНИМЕТРИИ НА ГОТОВЫХ ЧЕРТЕЖАХ

IX КЛАСС



.

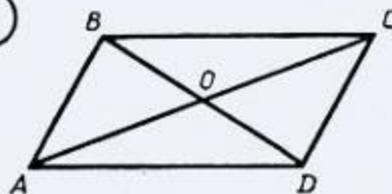
Содержание

1. Векторы на плоскости
2. Решение треугольников
3. Площадь треугольника
4. Площадь четырехугольника
5. Правильные многоугольники
6. Прямая, отрезок, полупрямая
7. Углы
8. Равенство треугольников
9. Параллелограмм
10. Вписанные и описанные окружности
11. Векторы

ВЕКТОРЫ НА ПЛОСКОСТИ

Равные векторы

1



$ABCD$ – параллелограмм
Укажите равные векторы

2

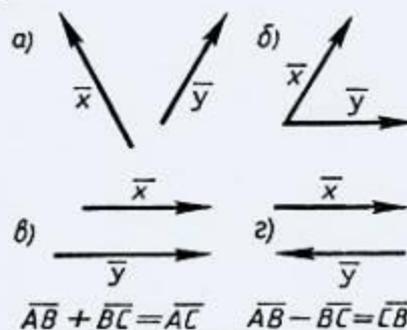
Дано: точки $A(-5, 2)$ и $B(2, 6)$.

- Найдите координаты векторов \vec{AB} и \vec{BA} .
- Найдите $|\vec{AB}|$.
- Отложите вектор, равный \vec{AB} : а) от точки $C(-2, 7)$, б) от точки $O(0, 0)$

Сложение векторов

Найдите $\vec{x} + \vec{y}$ и $\vec{x} - \vec{y}$.

3



4

Дано:

- $\vec{x}(-1, 4)$, $\vec{y}(5, -7)$;
- $\vec{x}(0, 4)$, $\vec{y}(5, -5)$;
- $\vec{x}(4, 0)$, $\vec{y}(0, 7)$.

Отложите данные и полученные векторы от начала координат

Умножение вектора на число

5



Дано: векторы \vec{a} и \vec{b} .

Постройте: $2\vec{a}$, $-3\vec{b}$, $-\frac{1}{2}\vec{b}$.

$\frac{1}{2}\vec{a}$, $\frac{2}{3}\vec{b}$, $2\vec{a} - 3\vec{b}$, $\frac{1}{2}\vec{a} + \frac{1}{4}\vec{b}$

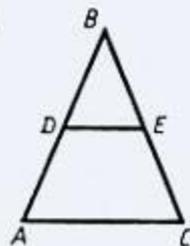
6

Дано: векторы $\vec{a}(-2, 5)$ и $\vec{b}(1, -4)$.

Найдите: $|2\vec{a} - 3\vec{b}|$, $|3\vec{a} + \vec{b}|$, $|5\vec{b} - \vec{a}|$

ВЕКТОРЫ НА ПЛОСКОСТИ

1



Дано: DE — средняя линия.
Докажите: а) $DE \parallel AC$,
б) $DE = \frac{1}{2} AC$

3

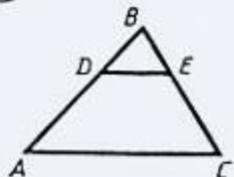
а) Докажите, что векторы $\vec{a}(2, 4)$ и $\vec{b}(-1, -2)$ коллинеарны.

б) Найдите m , если известно, что $\vec{a}(2, 5)$ и $\vec{b}(m, 9)$ коллинеарны. Запишите соотношение между \vec{a} и \vec{b} в виде $\vec{a} = \lambda \vec{b}$.

в) Даны точки $A(-2, 2)$, $B(5, 6)$, $C(10, 5)$, $D(3, 1)$.

Докажите, что $ABCD$ — параллелограмм

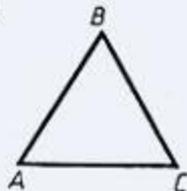
2



Дано: $BD = \frac{1}{3} AB$, $BE = \frac{1}{3} BC$
Докажите: $DE \parallel AC$,
 $DE = \frac{1}{3} AC$

Скалярное произведение векторов

4



Дано: $AB = BC = AC = 2$.
Найдите:
а) $\vec{AB} \cdot \vec{AC}$,
б) $\vec{AB} \cdot \vec{BC}$

6

а) Найдите угол между $\vec{a}(1, 3)$ и $\vec{b}(-1, 5)$.

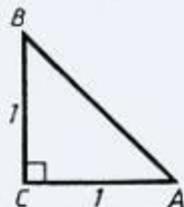
б) Даны точки $A(2, 2)$, $B(4, 6)$, $C(0, 8)$, $D(-2, 4)$.

Докажите, что $ABCD$ — прямоугольник.

в) Даны точки $A(-2, 2)$, $B(-8, -1)$, $C(-8, -5)$.

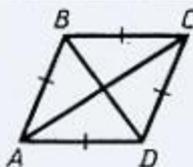
Найдите косинусы углов $\triangle ABC$

5



Найдите:
а) $\vec{AB} \cdot \vec{AC}$,
б) $\vec{AB} \cdot \vec{CA}$,
в) $\vec{CA} \cdot \vec{CB}$

7



Докажите, что $AC \perp BD$

8

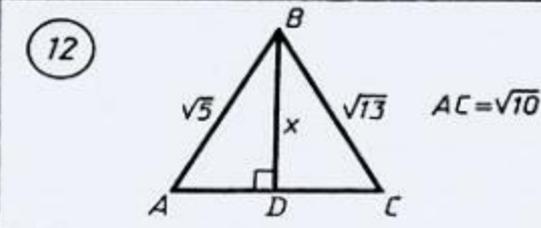
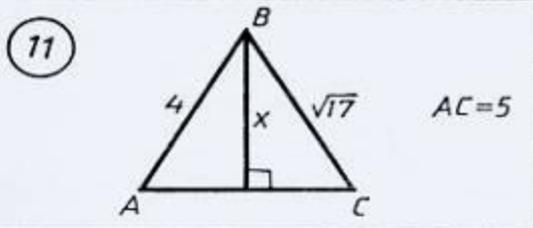
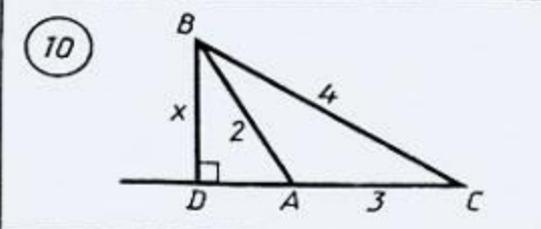
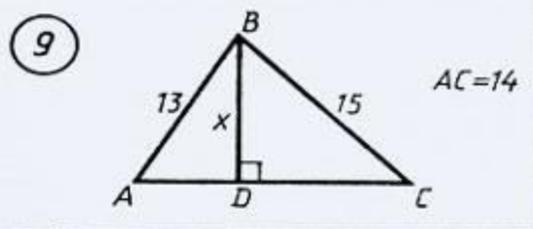
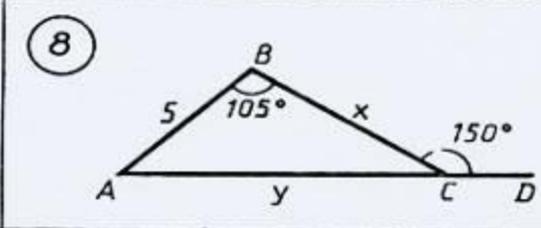
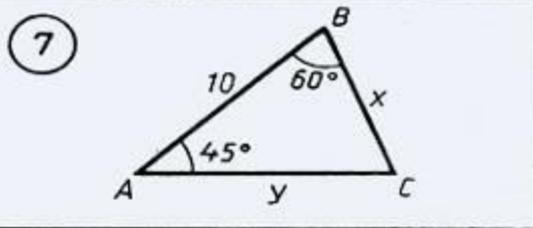
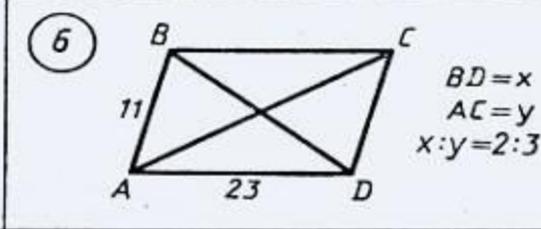
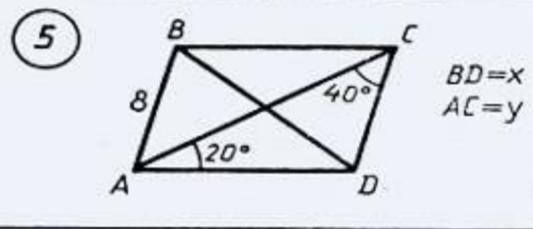
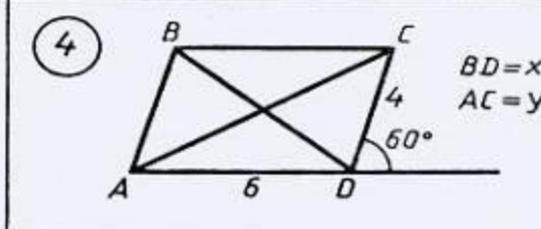
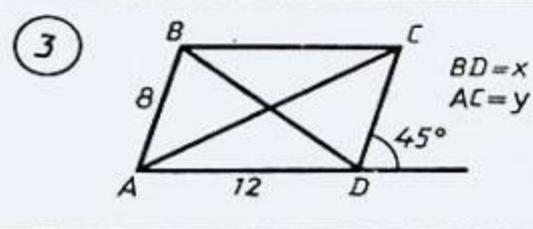
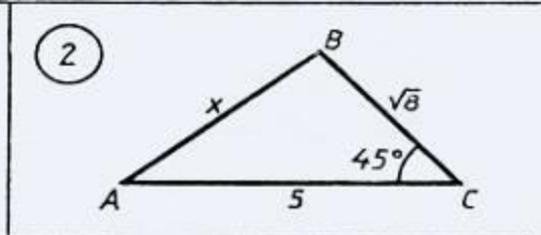
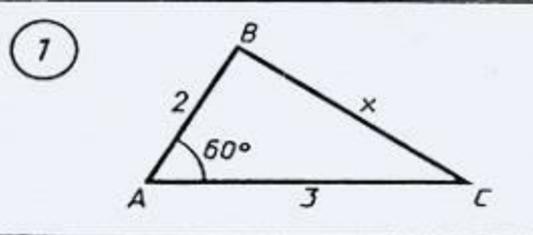
Даны точки $A(-3, 4)$, $B(0, 8)$, $C(5, 8)$, $D(2, 4)$.

Докажите, что $ABCD$ — ромб



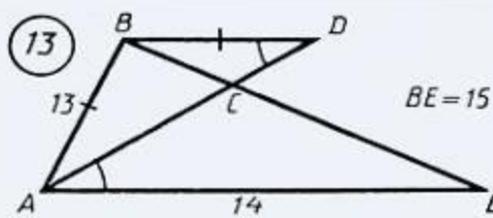
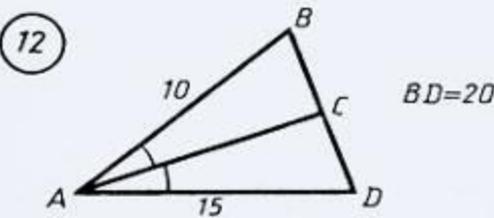
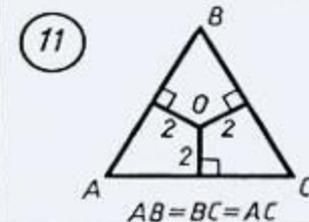
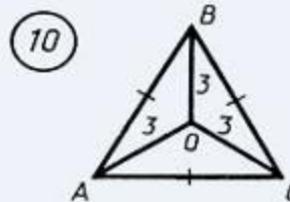
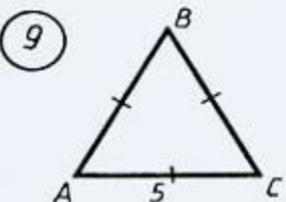
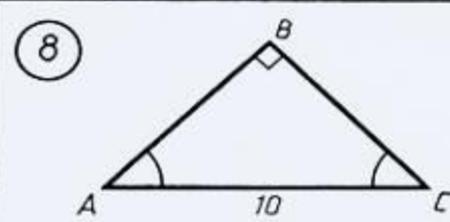
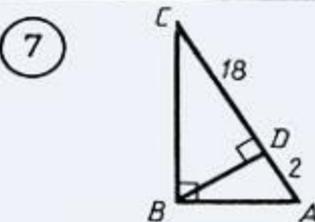
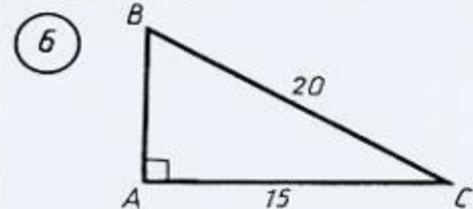
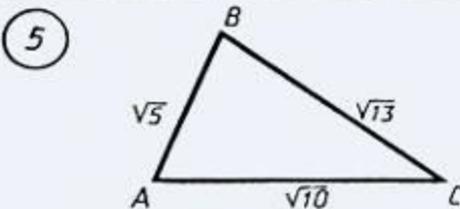
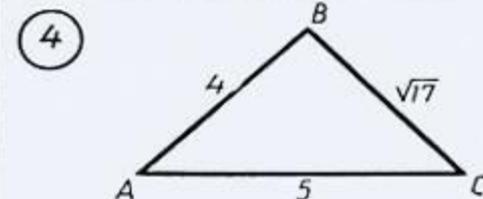
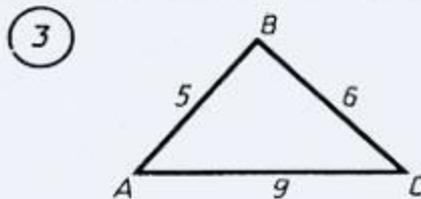
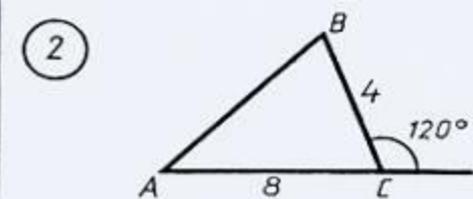
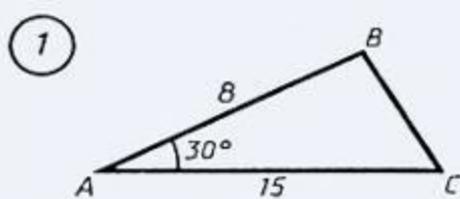
РЕШЕНИЕ ТРЕУГОЛЬНИКОВ

Четырехугольник $ABCD$ – параллелограмм. Найдите значения x и y .



ПЛОЩАДЬ ТРЕУГОЛЬНИКА

Вычислите площадь
треугольника ABC .

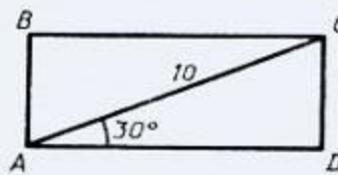


ПЛОЩАДЬ ЧЕТЫРЕХУГОЛЬНИКА

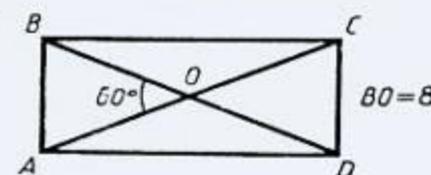
Найдите площадь $ABCD$.

$ABCD$ — прямоугольник

1

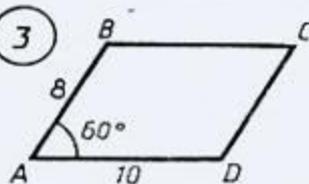


2

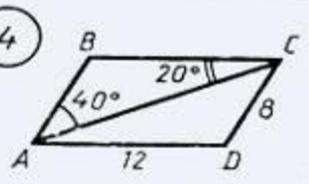


$ABCD$ — параллелограмм

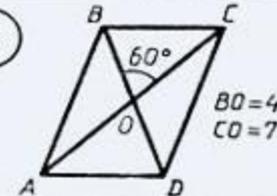
3



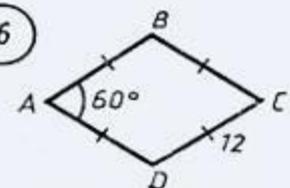
4



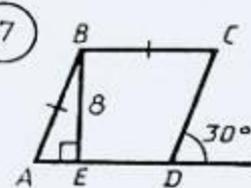
5



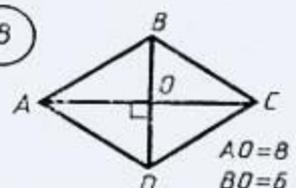
6



7

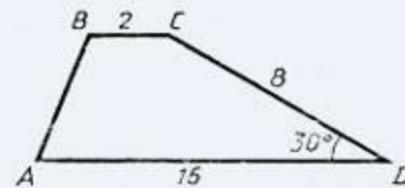


8

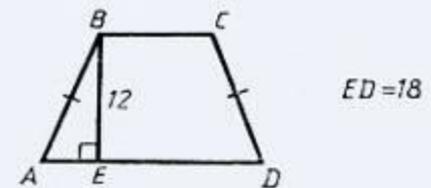


$ABCD$ — трапеция

9

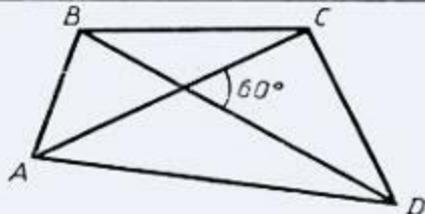


10



$ABCD$ — произвольный четырехугольник

11



$BD = 18$
 $AC = 12$



ПРАВЕЛЬНЫЕ МНОГОУГОЛЬНИКИ

Обозначения: a - сторона многоугольника, $R(r)$ – радиус описанной (вписанной) окружности, S – площадь многоугольника.

На рисунках 1 найдите R, r, S .

На рисунках 2 и 3 - a и S .

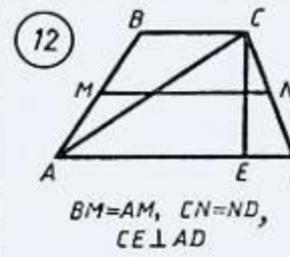
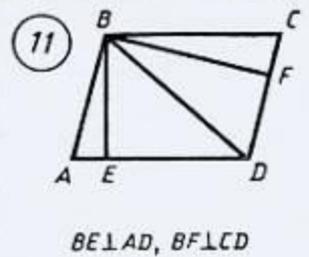
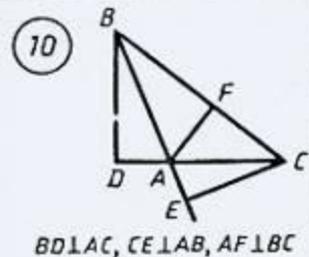
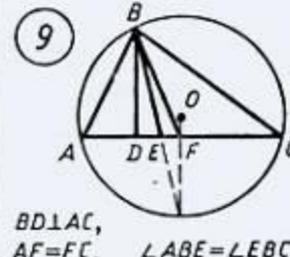
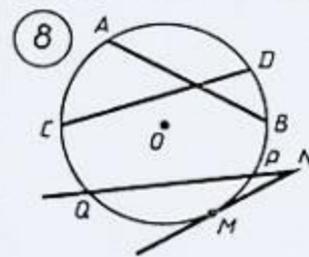
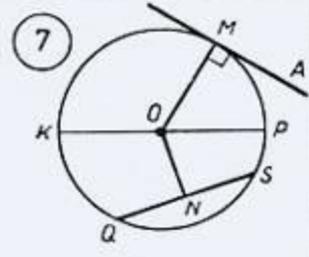
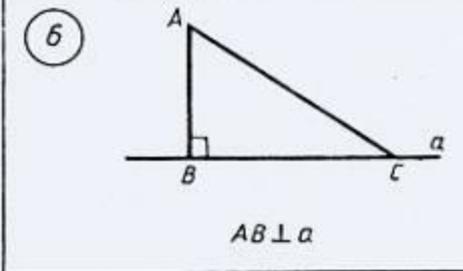
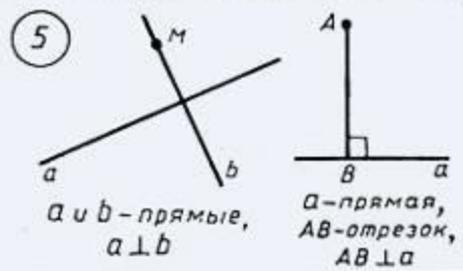
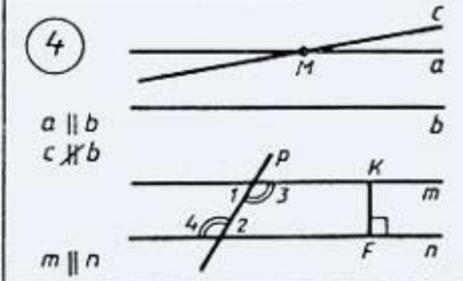
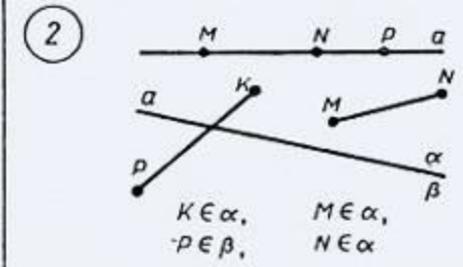
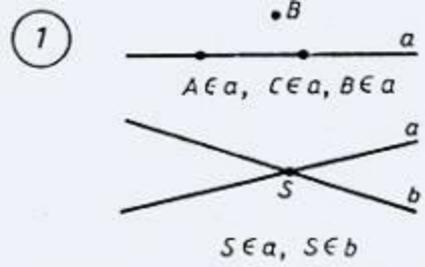
На рисунках 4 найдите r/R .

На рисунках 5 найдите R/r .

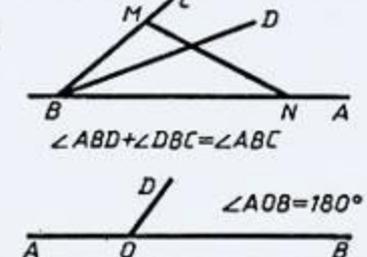
n -угольник	Треугольник	Четырехугольник	Шестиугольник
<p>1</p>			
<p>2</p>			
<p>3</p>			
<p>4</p>			
<p>5</p>			



ПРЯМАЯ, ОТРЕЗОК, ПОЛУПРЯМАЯ

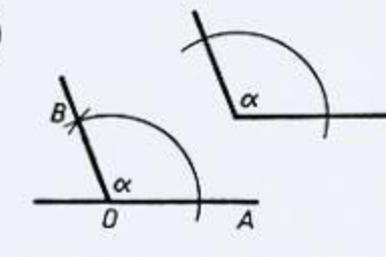


УГЛЫ

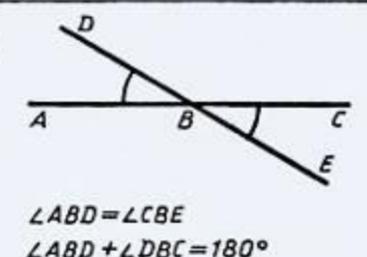
1 

$$\angle ABD + \angle DBC = \angle ABC$$

$$\angle AOB = 180^\circ$$

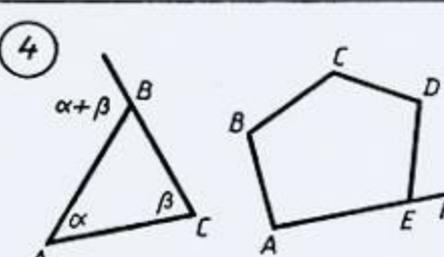
2 

$$\alpha$$

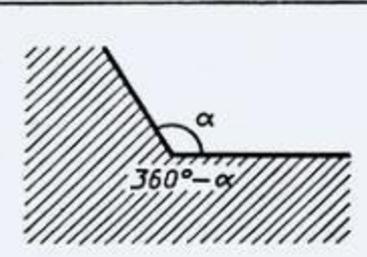
3 

$$\angle ABD = \angle CBE$$

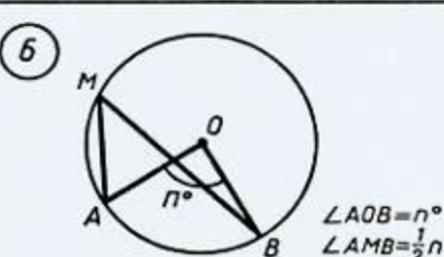
$$\angle ABD + \angle DBC = 180^\circ$$

4 

$$\alpha + \beta$$

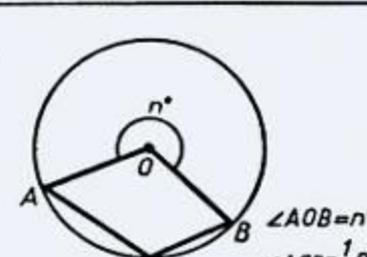
5 

$$360^\circ - \alpha$$

6 

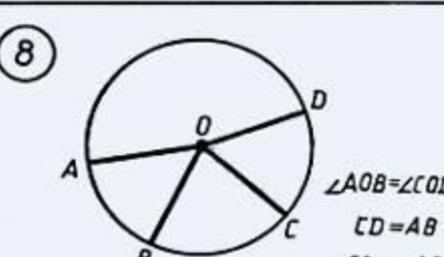
$$\angle AOB = n^\circ$$

$$\angle AMB = \frac{1}{2}n^\circ$$

7 

$$\angle AOB = n^\circ$$

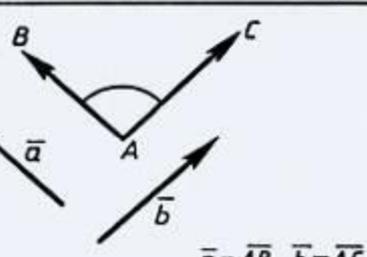
$$\angle ACB = \frac{1}{2}n^\circ$$

8 

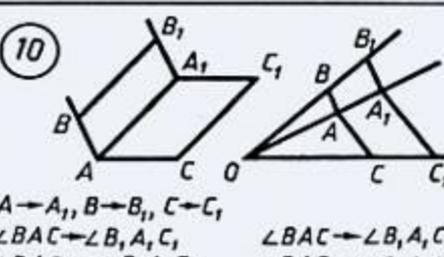
$$\angle AOB = \angle COD$$

$$CD = AB$$

$$\overset{\frown}{CB} = \overset{\frown}{AD}$$

9 

$$\vec{a} = \overline{AB}, \vec{b} = \overline{AC}$$

10 

$$A \rightarrow A_1, B \rightarrow B_1, C \rightarrow C_1$$

$$\angle BAC \rightarrow \angle B_1 A_1 C_1$$

$$\angle BAC = \angle B_1 A_1 C_1$$

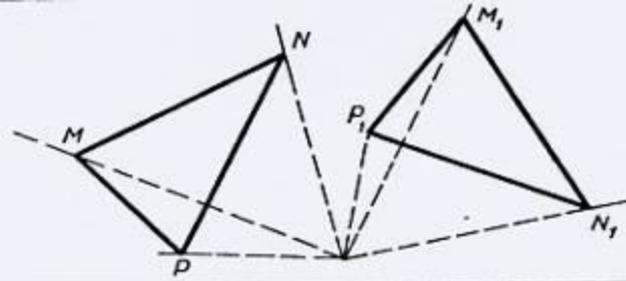
$$\angle BAC \rightarrow \angle B_2 A_2 C_2$$

$$\angle BAC = \angle B_2 A_2 C_2$$

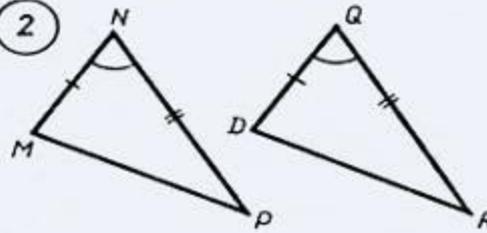


РАВЕНСТВО ТРЕУГОЛЬНИКОВ

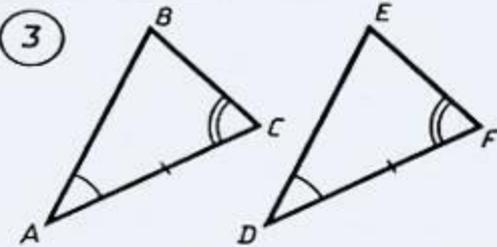
1



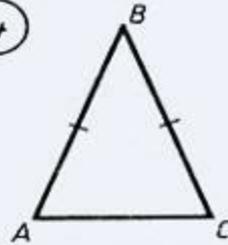
2



3

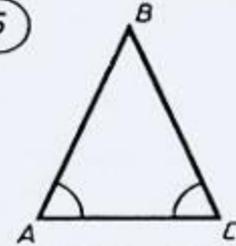


4



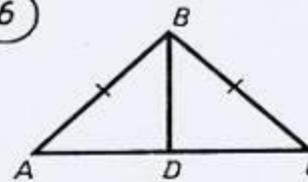
Если $AB=BC$,
то $\angle A = \angle C$

5

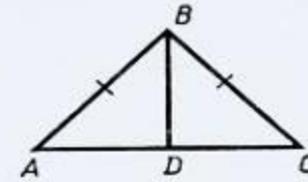


Если $\angle A = \angle C$,
то $AB=BC$

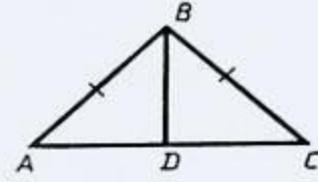
6



$AD=DC$

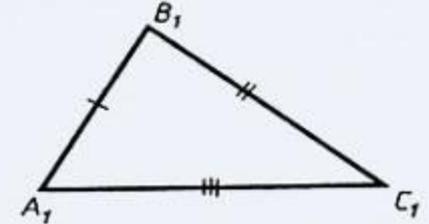
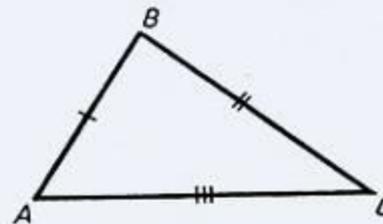


$\angle ABD = \angle CBD$

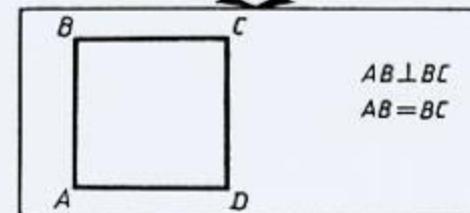
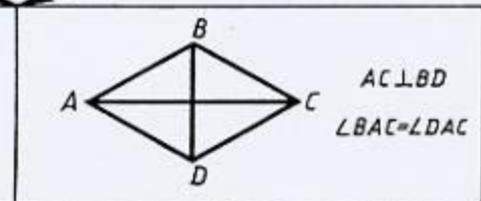
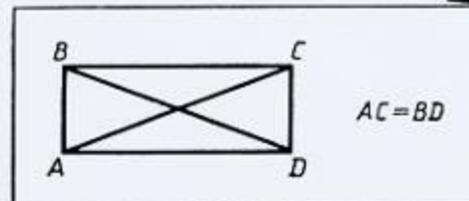
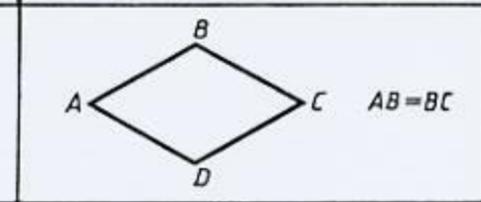
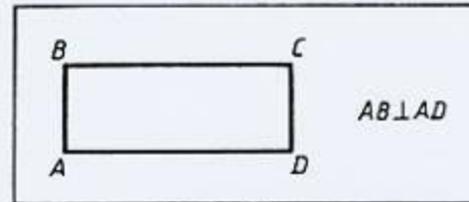
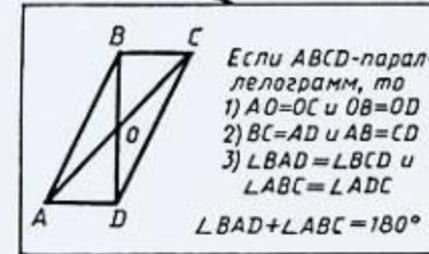
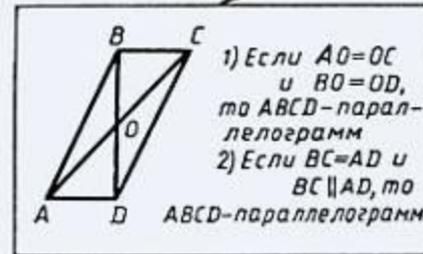
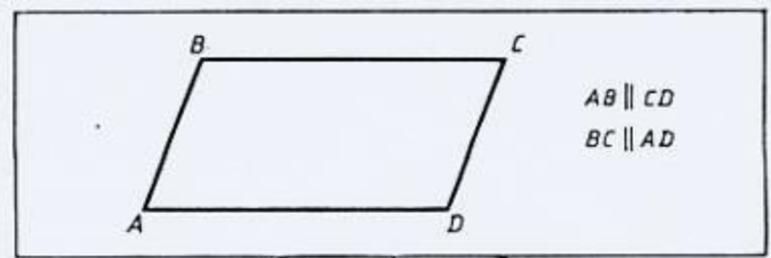


$BD \perp AC$

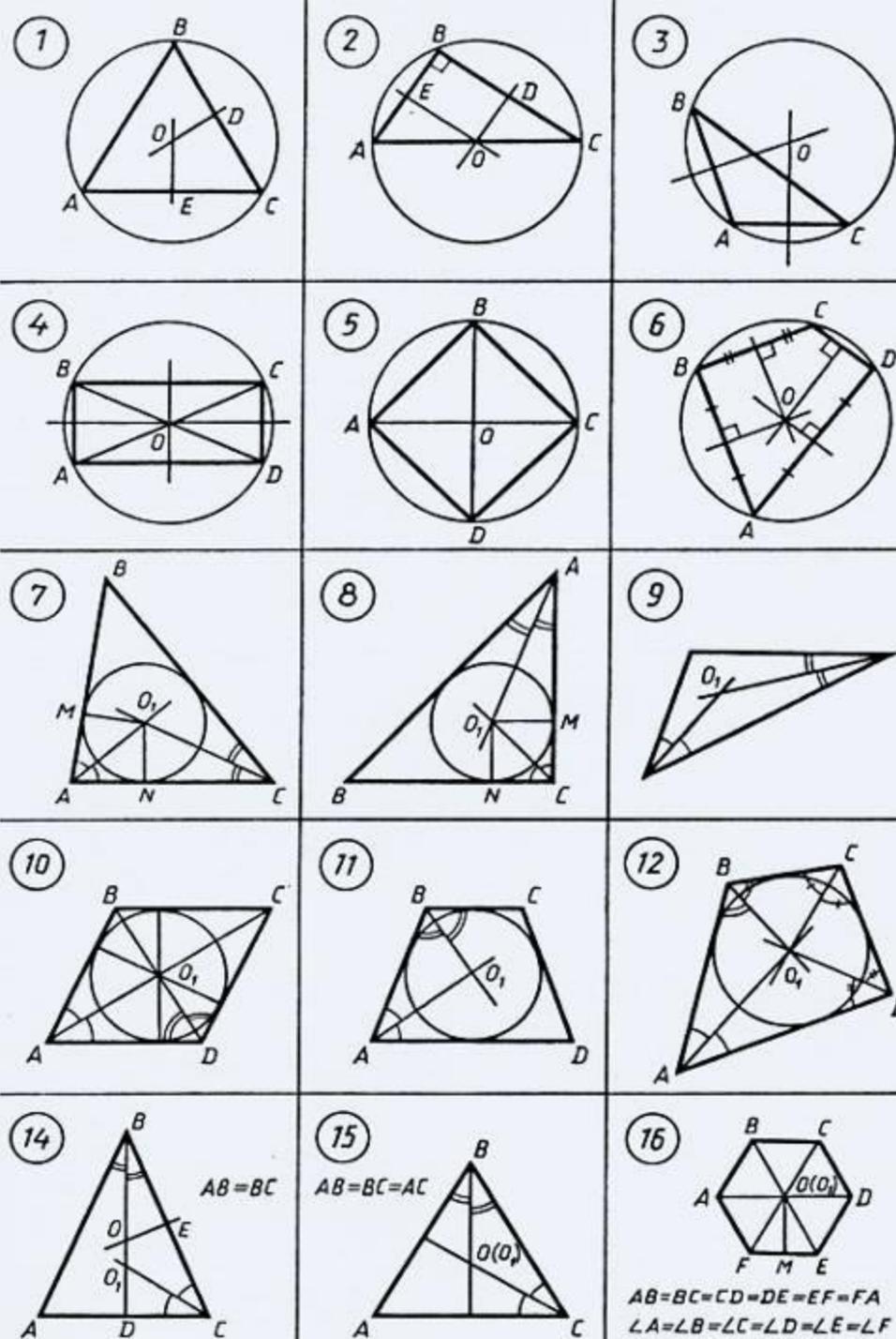
7



ПАРАЛЛЕЛОГРАММ

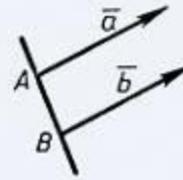


ВПИСАННЫЕ И ОПИСАННЫЕ ОКРУЖНОСТИ

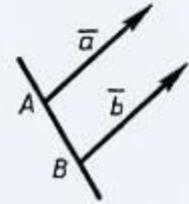


ВЕКТОРЫ

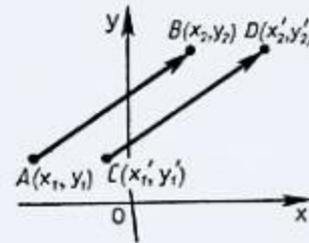
1 Если $|\vec{a}| = |\vec{b}|$ и $\vec{a} \uparrow \uparrow \vec{b}$, то $\vec{a} = \vec{b}$.



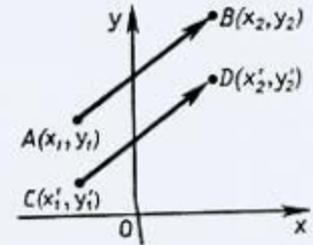
2 Если $\vec{a} = \vec{b}$, то $|\vec{a}| = |\vec{b}|$ и $\vec{a} \uparrow \uparrow \vec{b}$.



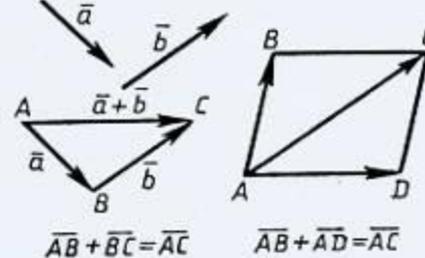
3 Если $x_2 - x_1 = x'_2 - x'_1$ и $y_2 - y_1 = y'_2 - y'_1$, то $\vec{AB} = \vec{CD}$.



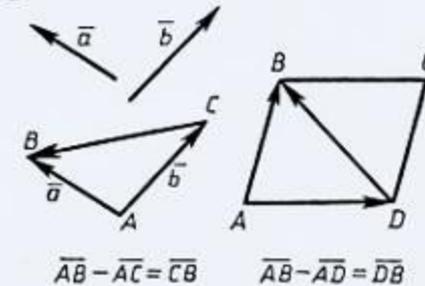
4 Если $\vec{AB} = \vec{CD}$, то $x_2 - x_1 = x'_2 - x'_1$ и $y_2 - y_1 = y'_2 - y'_1$.



5



6



7 $\vec{a}(a_1, a_2) + \vec{b}(b_1, b_2) = \vec{c}(a_1 + b_1, a_2 + b_2)$
 $\vec{a}(a_1, a_2) - \vec{b}(b_1, b_2) = \vec{c}(a_1 - b_1, a_2 - b_2)$
 $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
 $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

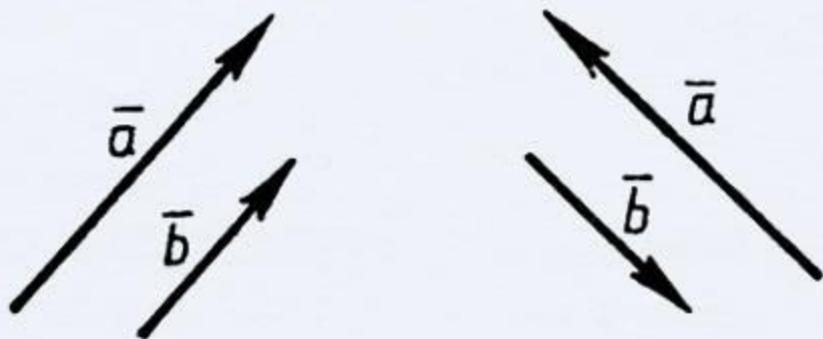
8 $\lambda \vec{a}(a_1, a_2) = \vec{c}(\lambda a_1, \lambda a_2)$
 $(\lambda + \mu) \vec{a} = \lambda \vec{a} + \mu \vec{a}$
 $\lambda(\vec{a} + \vec{b}) = \lambda \vec{a} + \lambda \vec{b}$



ВЕКТОРЫ (продолжение)

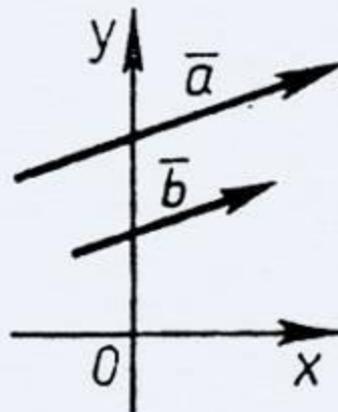
9

$$\vec{a} = \lambda \vec{b} \ (\lambda > 0) \quad \vec{a} = \lambda \vec{b} \ (\lambda < 0)$$
$$|\vec{a}| = \lambda |\vec{b}| \quad |\vec{a}| = -\lambda |\vec{b}|$$



10

$$\vec{a} (a_1, a_2), \vec{b} (b_1, b_2)$$
$$\vec{a} = \lambda \vec{b}$$
$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$



11

$$\vec{a} (a_1, a_2) \cdot \vec{b} (b_1, b_2) = a_1 b_1 + a_2 b_2$$

12

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos \varphi$$

