Lecture №2

Kinds of algorithmic systems

Plan of lecture

1. Post machine.
2. Turing machine.
3. Algorithmically solvable and unsolvable problems.
4. Markov system.

Keywords

Post machine

Turing machine

Markov system

Algorithmically solvable

Algorithmically unsolvable

Instruction

Tape

Ordered

Marked

Non-marked

Final word

Coincide

1.Post machine

For this machine inbox and outbox information is given in alphabet $X = \{0,1\}$, algorithm is in the kind of finite ordered collection of rules (commands), which are called instructions (directions, operations). The information is written on the tape which is divided into cells, one next to the other. In every cell of the tape only one letter can be put: 0 or 1. Cells with 1-s are called marked. Other ones called non-marked. There is also "empty" symbol #, which fills all empty cells of tape.

1.Post machine

The instructions, which consist of algorithm, belong to one of 6 types.

1. To mark the active cell (to write down 1 in it) and to pass to the execution of the i^{th} instruction.

2. To erase the mark of the active cell (to write down 0 in it) and to pass to the execution of the j^{th} instruction.

3. To move an active cell to one step to the right and to pass to the execution of the i^{th} instruction.

1.Post machine

4. To move an active cell on the one step to the left and to pass to the execution of the i^{th} instruction.

5. If an active cell is marked (there is 1 in it), then to pass to the execution of the j^{th} instruction, otherwise you should pass to the execution of the i^{th} instruction.

6. Stop and the end of the algorithm's work.

1.Post machine

The ***Post machine*** is a 4-tuple $(X, a_0, \#, P)$, where:

$X = \{0,1\}$ is the alphabet of the machine;

a_0 is the starting active cell of the machine;

is an "empty" symbol;

P is the algorithm of the machine function, which consists of the finite number of the instructions, that were given above.

1.Post machine

The work of Post machine is the execution of directions (instructions) of algorithm function of Post's machine.

Machine is stopped if and only when the last instruction, done by the machine, is the instruction of the 6^{th} *type and the* ***result of its work is the word q****, which is written on the tape and which is called the* ***final*** *one.*

1.Post machine

Theorem. The class of all algorithms, which are equivalent to the Post algorithms, coincides with the class of all partial recursive functions.

2. Turing machine

Turing machine consists of:

– The tape , which is divided into cells, one next to the other. Each cell contains a symbol from some finite alphabet. Every sell is used for the record of only one symbol, which can be looked about by the head.

– The head that can read and write symbols on the tape and move the tape to the left and to the right only to one cell.

2. Turing machine

The work of the Turing machine is done with the help of program P.The program of Turing machine's work is a finite set of 5-tuples $x_i q_i x_k q_r s_p$, each of them is called a command. The execution of the command $x_i q_i x_k q_r s_p$ means that if the head of Turing machine is in the state q_i and reads the current tape symbol x_i, then the head writes on the place of x_i a new symbol x_k (which can be the same as preceding state) and the head moves along the tape with the value $s_p = 0, \pm 1$.

2. Turing machine

Turing machine T is a 5-tuple $(\boldsymbol{X}, \boldsymbol{Q}, \boldsymbol{q_0}, \#, \boldsymbol{P})$, where:

$\boldsymbol{X}$ is the alphabet of the machine;

$\boldsymbol{Q}$ is a finite, non-empty set of states, including a starting state q_0 $(X \cap Q = \emptyset)$;

$\boldsymbol{q_0}$ is a starting state $(q_0 \in Q)$;

is an "empty" symbol, $\# \notin X \cup Q$;

$\boldsymbol{P}$ is the program of machine's work.

2. Turing machine

The main steps of Turing machine work are:

1) Reading of a symbol of the tape, which is looked by the head;

2) The search a needed command, i.e. search the command $x_i q_i x_k q_r s_p$, for which q_i is a state of the head at the present moment, x_i is a symbol in the cell, which is looked by the head being in the state q_i.

3) Doing the found command.

2. Turing machine

Turing machine is stopped if and only when it is impossible to use any command of its program.

The result of Turing machine work *after its stop* ***is the word, which is written on the tape.***

4. Markov Algorithmic System

X – finite alphabet;
$F(X)$ – a set of words of this alphabet;
e – empty word, $e \in F(X)$;
$p \to q$ and $p \to . q$ – formulas of substitution in X, where $p, q \in F(X)$; $\to$ and $. \notin X$.

Formula $p \to q$ is called ***elementary***.
Formula $p \to . q$ is a ***final*** formula.

4. Markov Algorithmic System

Rewriting system is a following system:

$$R: \begin{cases} p_1 \to [.]q_1 \\ p_2 \to [.]q_2 \\ \dots \quad \dots \quad \dots \\ p_n \to [.]q_n \end{cases} \qquad \text{where} \quad p \to [.]q = \begin{cases} p \to q \\ p \to .q \end{cases}$$

4. Markov Algorithmic System

Markov normal algoriphm in X is called function $f: F(X) \to F(X)$, if the following conditions are executed:

Rules:

1) If any word of words p_i $(i = \overline{1,n})$ is not a subword of p, then p is left without changes and is a result of rewriting (denoted as $R{:}p!$);
2) $\exists p_i$ $(i = \overline{1,n})$, that are subwords of p $\exists$minimal number $m, 1 \le m \le n$, and p_m is a subword of p. Word p' is a result of substitution from the left side of word p_m by word q_m in word p (denoted $R{:}p \vdash p'$).

Conditions of the work stop:

1) If formula $p_n \to [.]q_n$ is a final formula.
2) If $R{:}p \vDash q$ means that $\exists\, r_0, r_1, \dots, r_k \in F(X)$ that $p = r_0$, $q = r_k$ and $R{:}r_i \vdash r_{i+1} (i = \overline{1, k-2})$ and or $R{:}r_k!$, or $R{:}r_{k-1} \to .r_k$.

Thank you for your attention!!!

Hometask

- To work out the material of the lecture.