

Simplification

$$YX + X = X$$

$$XY + X\bar{Y} = X$$

Absorption

$$Y + X\bar{Y} = X + Y$$

2nd Distributive

$$(X + A)(X + B) = X + AB$$

$$(X + A)(X + B)(X + C) = X + ABC$$

Most Common Stupid Errors

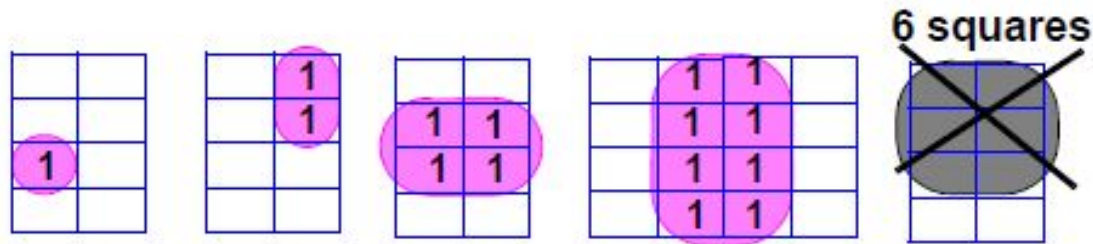
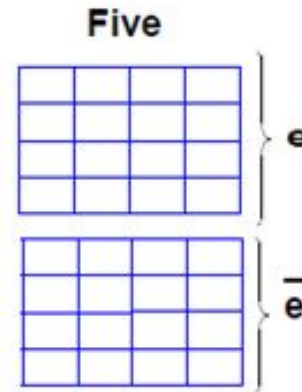
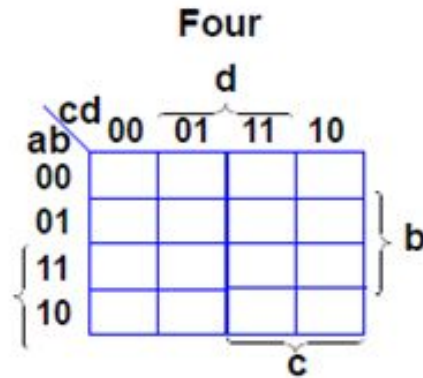
~~$$\bar{X} \cdot \bar{Y} = \overline{XY}$$~~

~~$$X + 1 = X$$~~

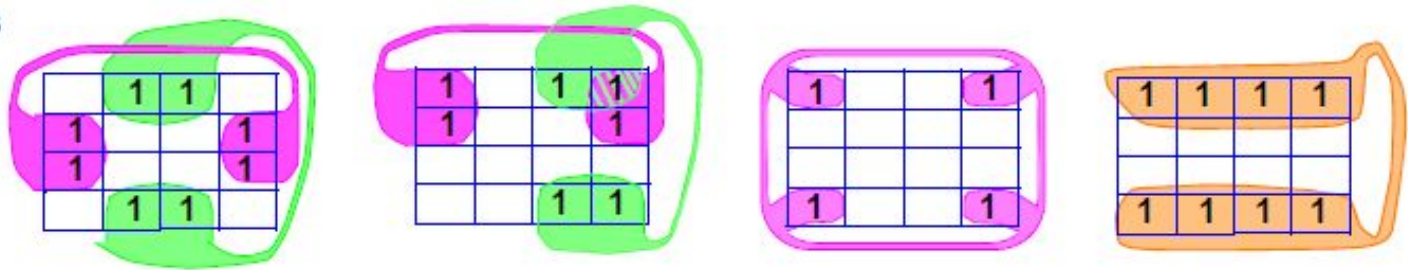
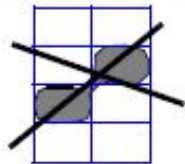
Lecture 4: Multiple Output Maps

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Review

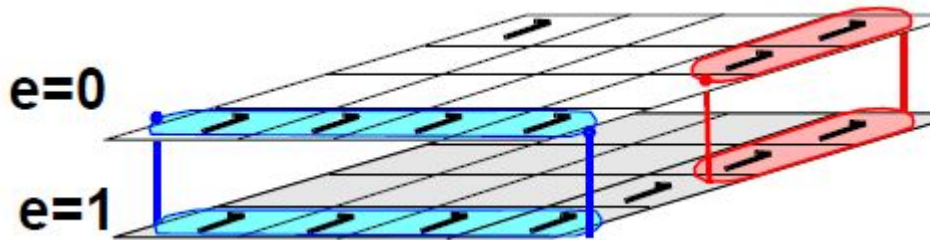


Diagonal squares
not adjacent

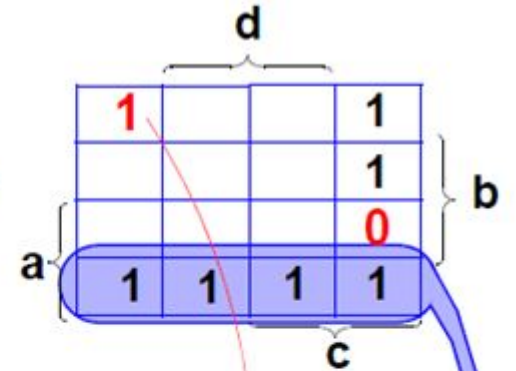


Five-Variable Karnaugh Maps

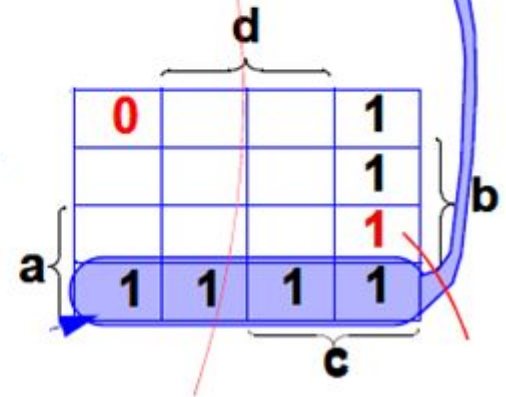
Think of two maps on top of each other



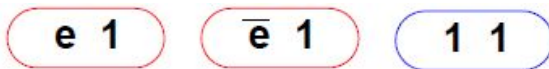
This map for \bar{e}



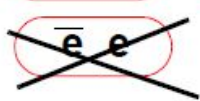
This map for e



Can loop

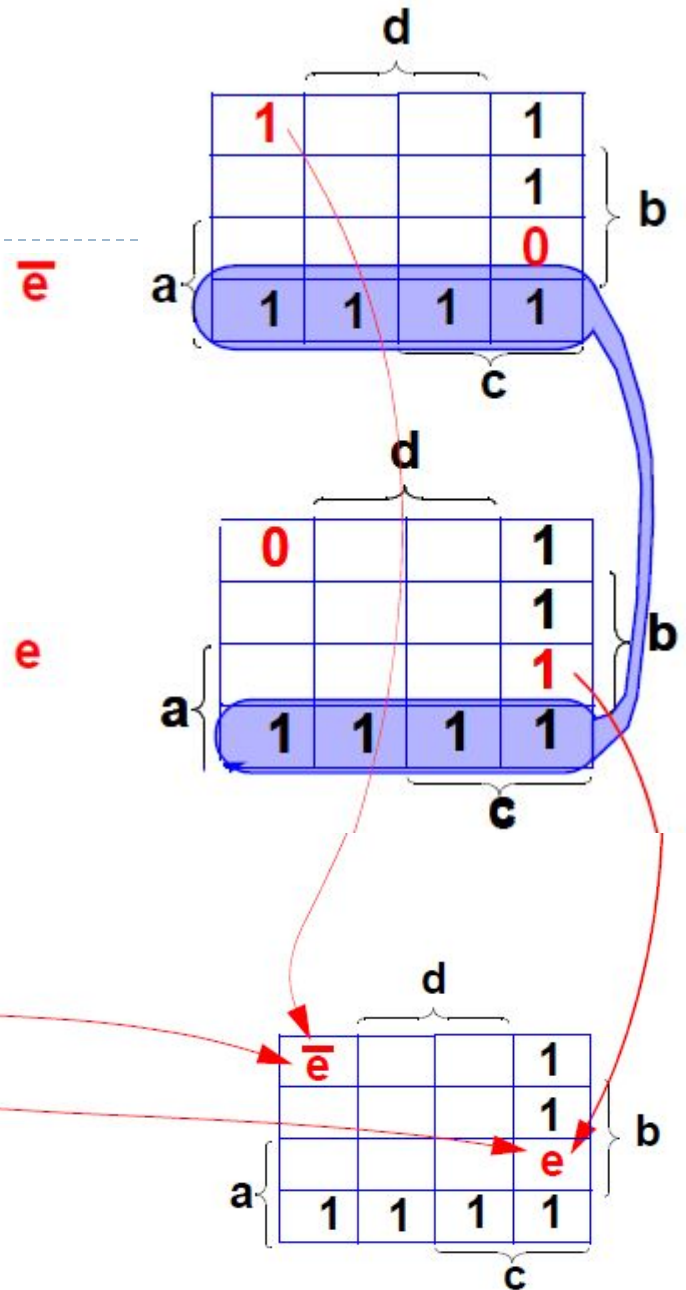


Cannot loop



“ e ” for “1”s on top layer

“ \bar{e} ” for “1”s on bottom layer.



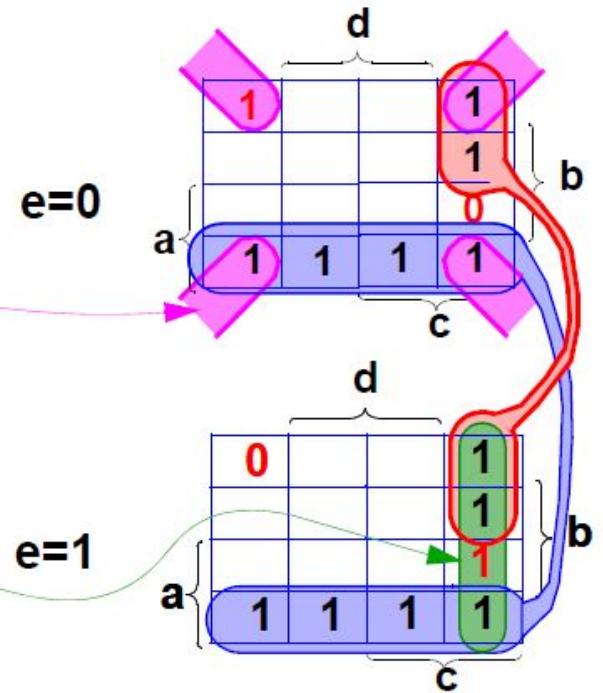
Equations From Five Variable Maps

e=0 (top) map are ANDed with \bar{e}

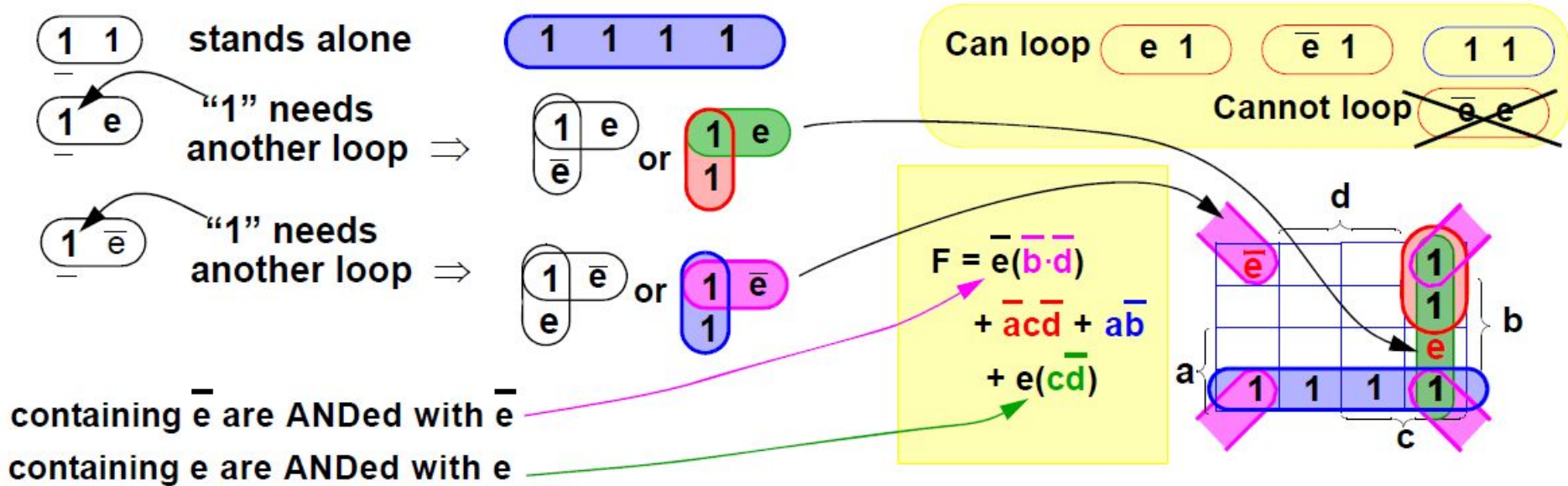
$$F = \bar{e}(\bar{b}\bar{d}) + \bar{a}c\bar{d} + a\bar{b}$$

e=1 (bottom) map are ANDed with e

$$+ e(c\bar{d})$$



Equations From Five Variable Maps

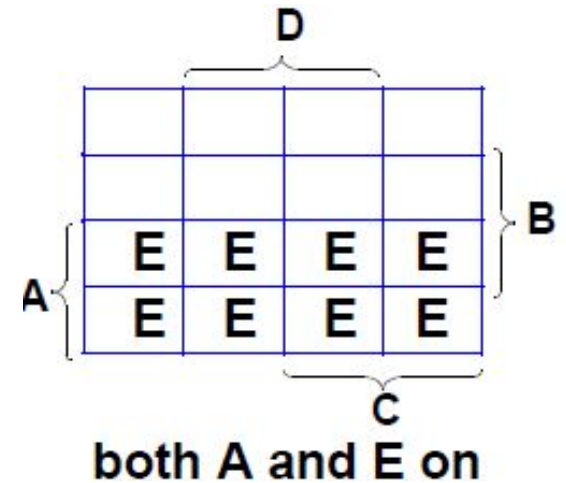
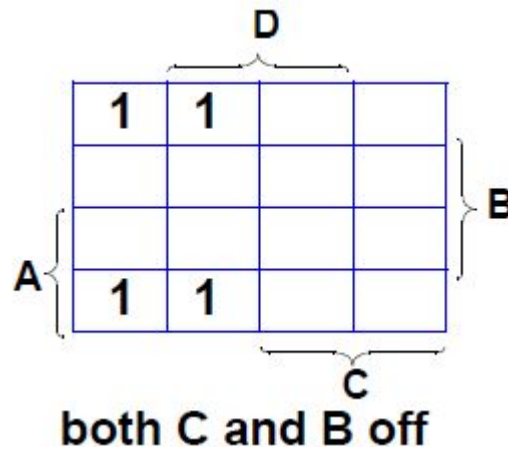
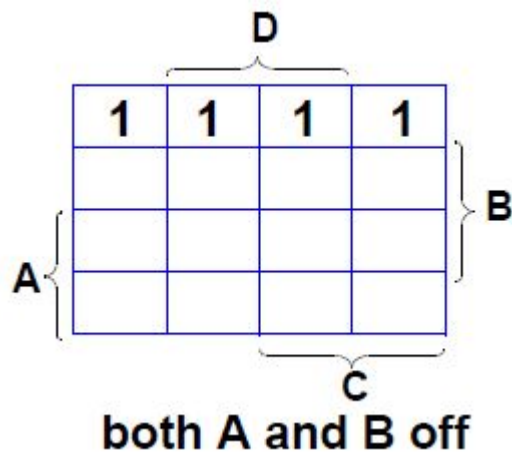


Example: F is true when:

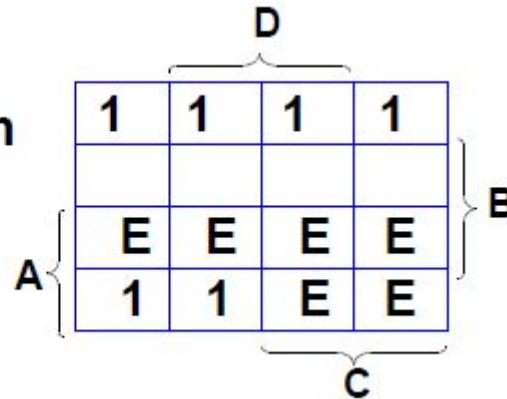
both A and B are off

or both C and B are off

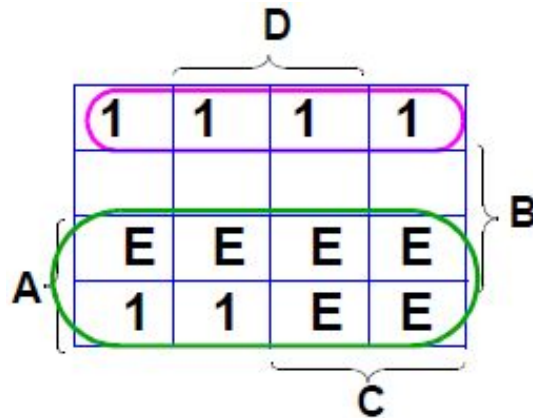
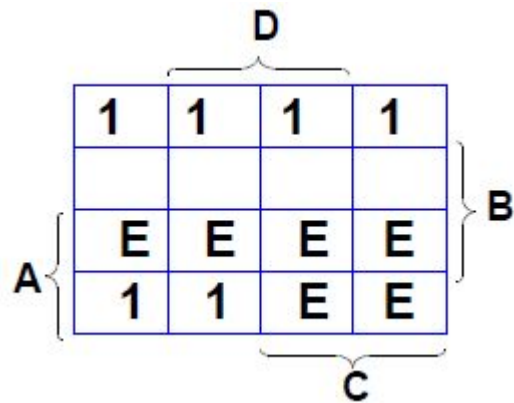
or both A and E are on



Combine on
one map



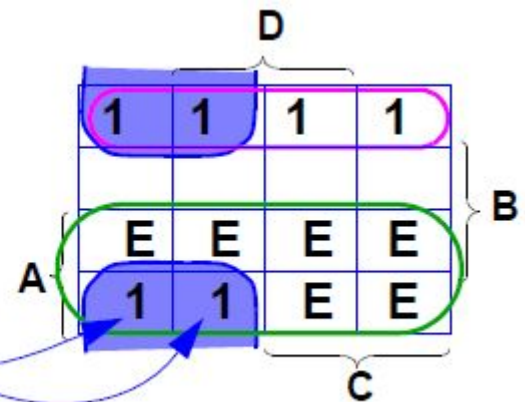
Example: F is true when:
 both A and B are off
 or both C and B are off
 or both A and E are on



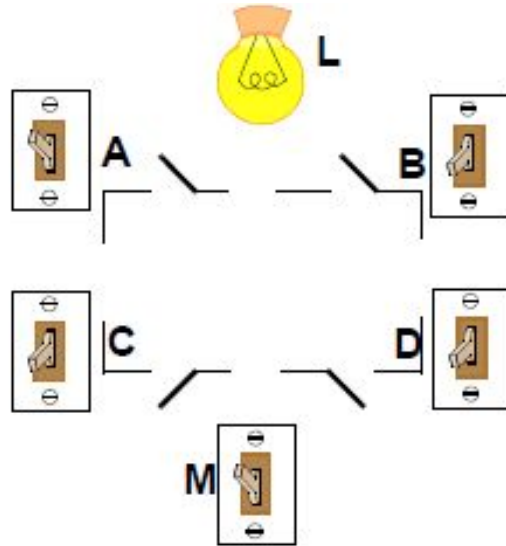
These squares not looped
 in \bar{e} plane

$$F = EA + \bar{A}\bar{B} + \bar{C}\bar{B} + 0$$

{ e plane
 { both planes
 { \bar{e} plane

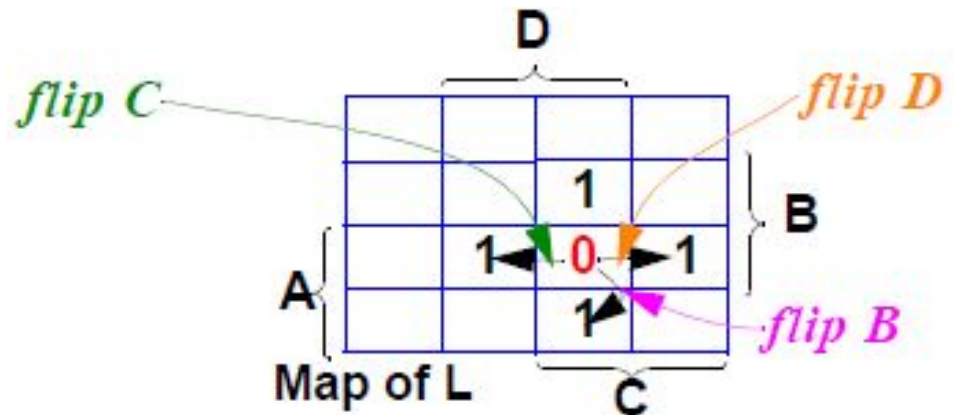


Example: Light Control

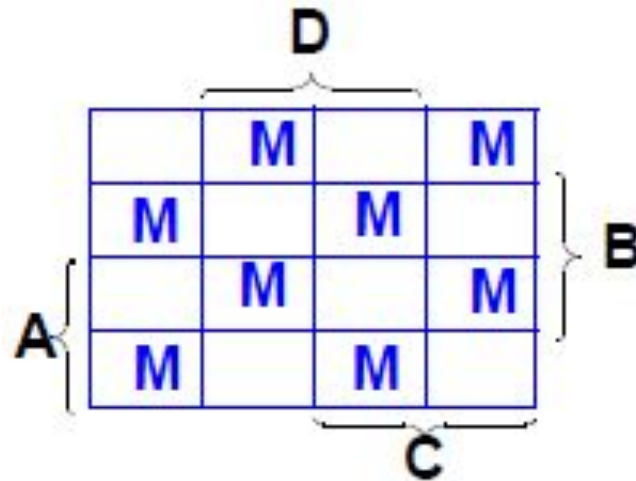
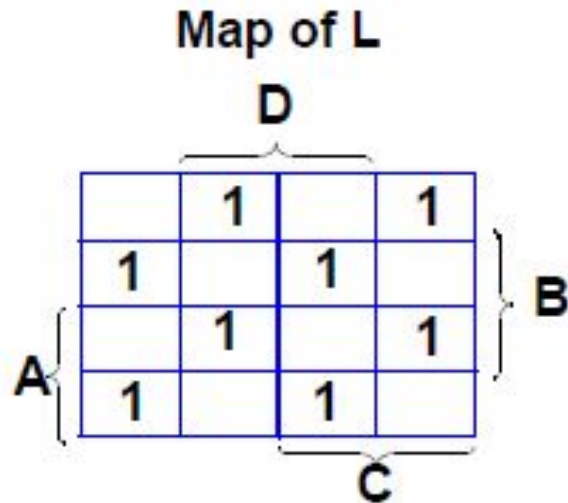


Flicking A,B,C or D
-turns on light if it is **off**
-turns **off** light if it is on
M turns **off** all lights

Start at ABCD=1111; L= 0 (off)
Flick any one switch,
one bit will change.
Move one square, get L=1



If $M = 0$, light is always off.
All "1"s become "0"s.



$$L = M(A \oplus B \oplus C \oplus D)$$

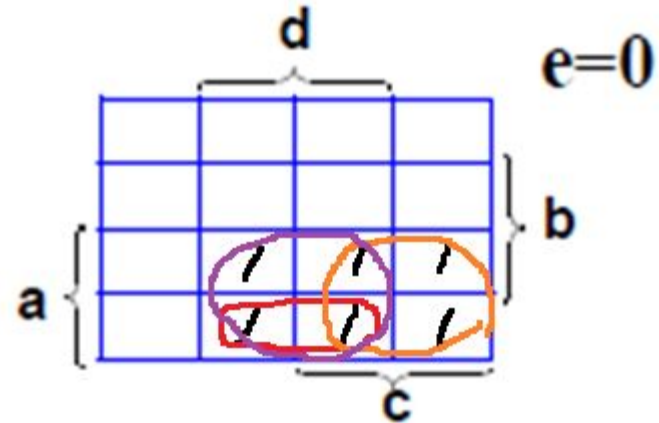
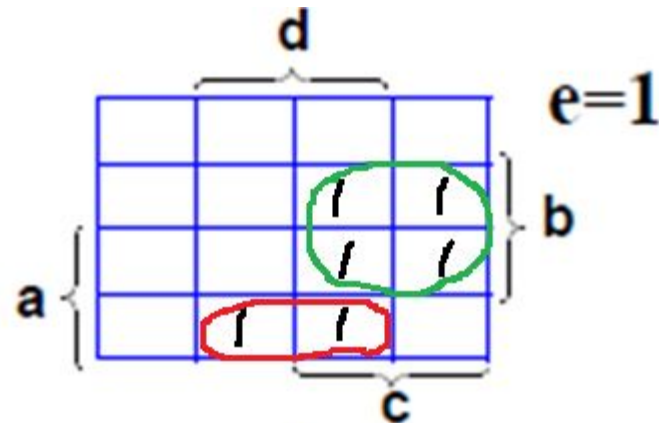
Changing any one bit
changes L

4-variable map is
alternate "1"s and "0"s.

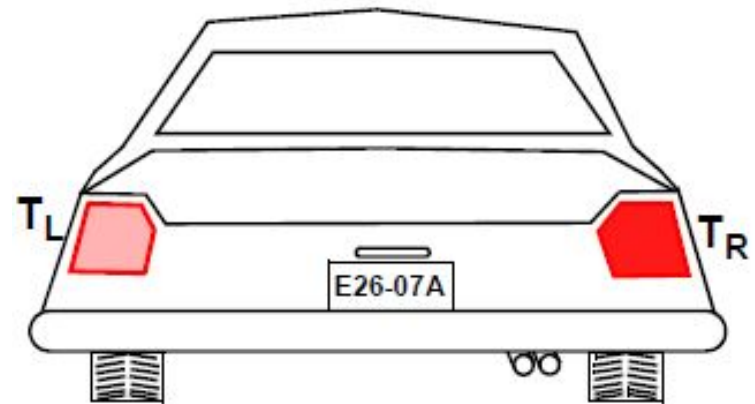
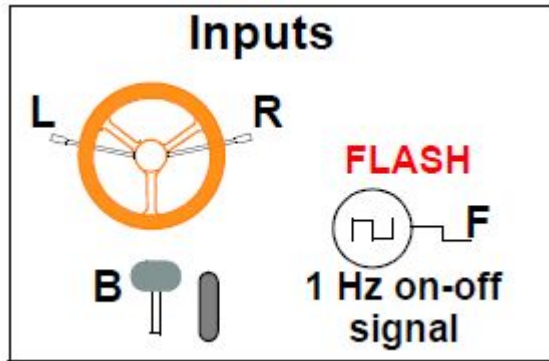
$$L = A \oplus B \oplus C \oplus D$$



Example: $F = (abc + a\bar{b}d + cb)e + (a\bar{b}d + ac + adb)\bar{e}$
 $= (abc + cb)e + (ac + adb)\bar{e} + a\bar{b}d$

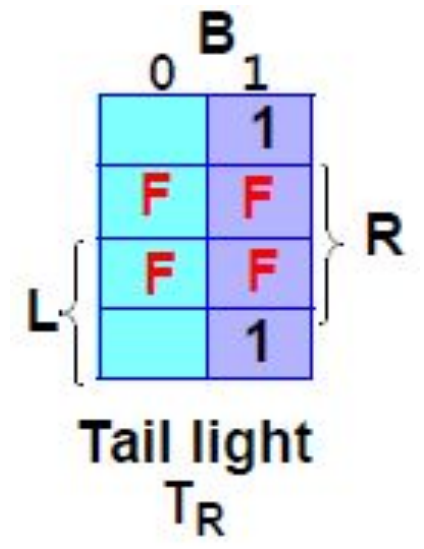
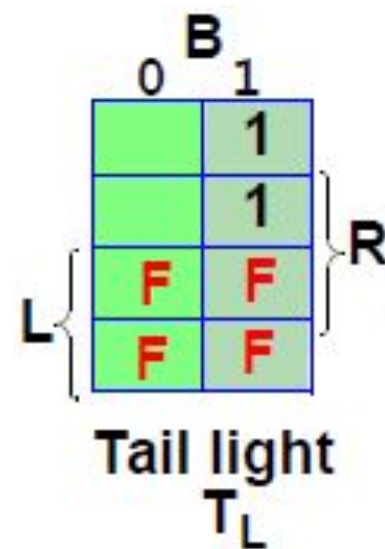


Example: Tail Light Control



Inputs	Tail light T_L	Tail light T_R
LR B		
--- 00 0	0	0
-R- 01 0	0	Flash
LR- 11 0	Flash	Flash
L-- 10 0	Flash	0
--B 00 1	Solid	Solid
-RB 01 1	Solid	Flash
LRB 11 1	Flash	Flash
L-B 10 1	Flash	Solid

LR B	T _L	T _R
00 0	0	0
01 0	0	Flash
11 0	Flash	Flash
10 0	Flash	0
00 1	Solid	Solid
01 1	Solid	Flash
11 1	Flash	Flash
10 1	Flash	Solid

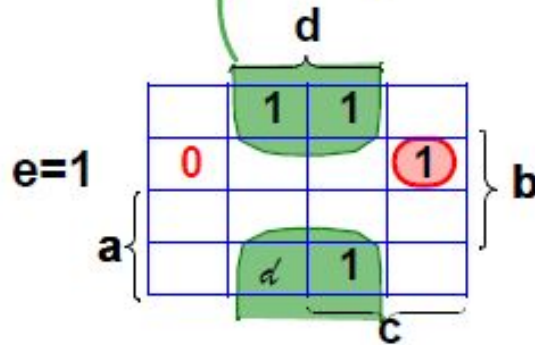
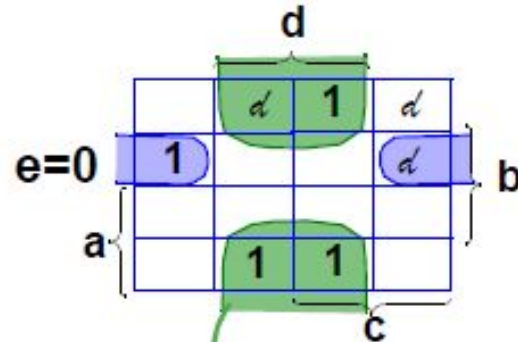


e=0

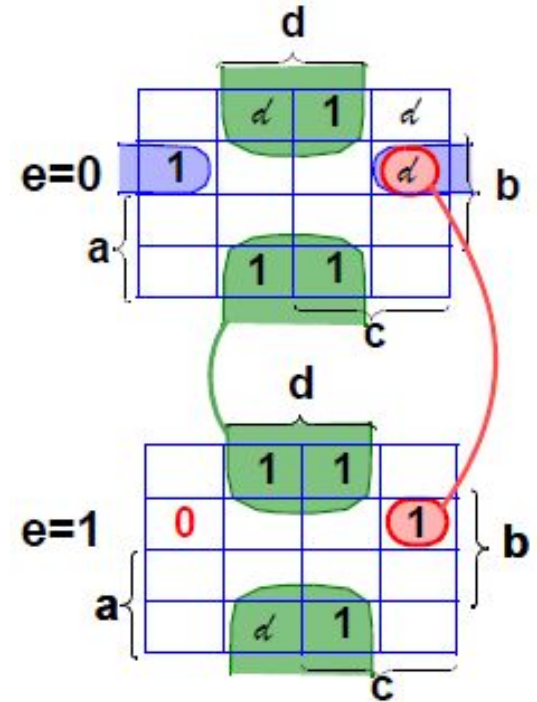
	d	1	d
1			d
	1	1	

e=1

	1	1	0
0			1
	d	1	



$$F = \bar{e}(\bar{a}\bar{b}\bar{d}) + \bar{b}d + e(\bar{a}bc\bar{d})$$

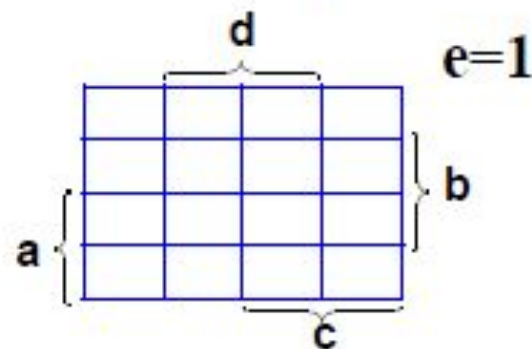
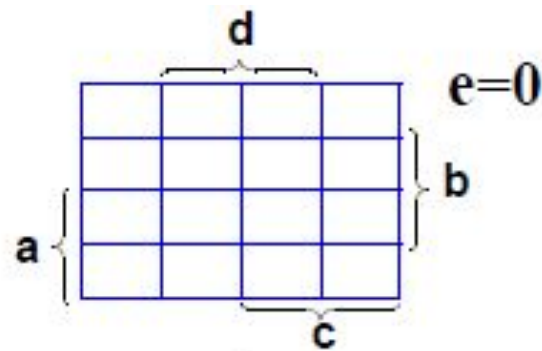


$$F = \bar{e}(\bar{a}\bar{b}\bar{d}) + \bar{b}d + \bar{a}bc\bar{d}$$

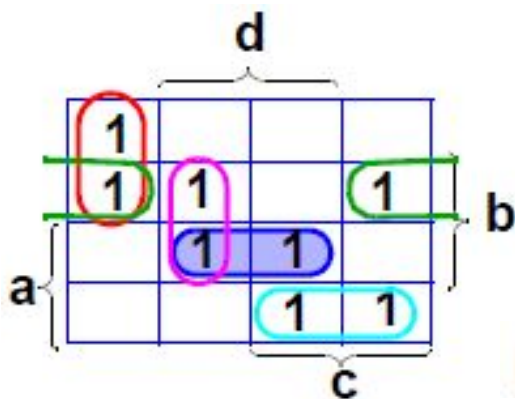
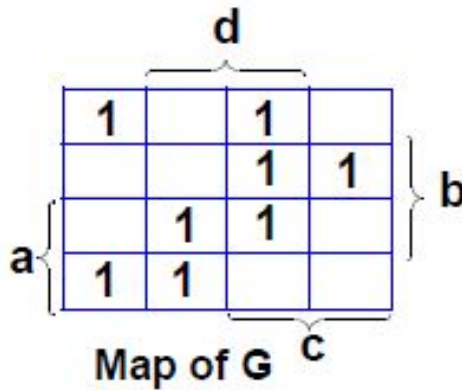
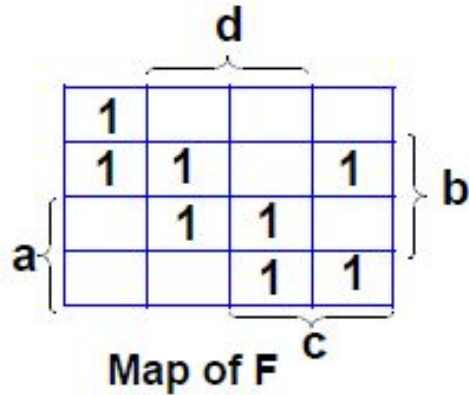


Example: $F = a\bar{b}d + \bar{b}cd + \bar{a}\bar{b}c\bar{d}\bar{e} + \bar{a}bc\bar{d}e$

Further a,c,d,e can never take on the values 0,1,0,0 ($\bar{a}\bar{c}\bar{d}\bar{e}$)
and a,b,c,d can never take on the values 0,0,0,1 ($\bar{a}\bar{b}\bar{c}d$)

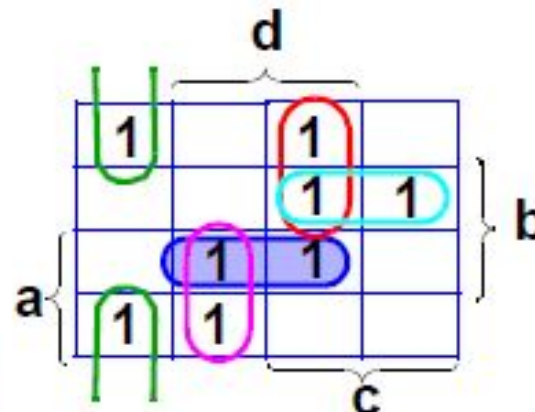


Example:

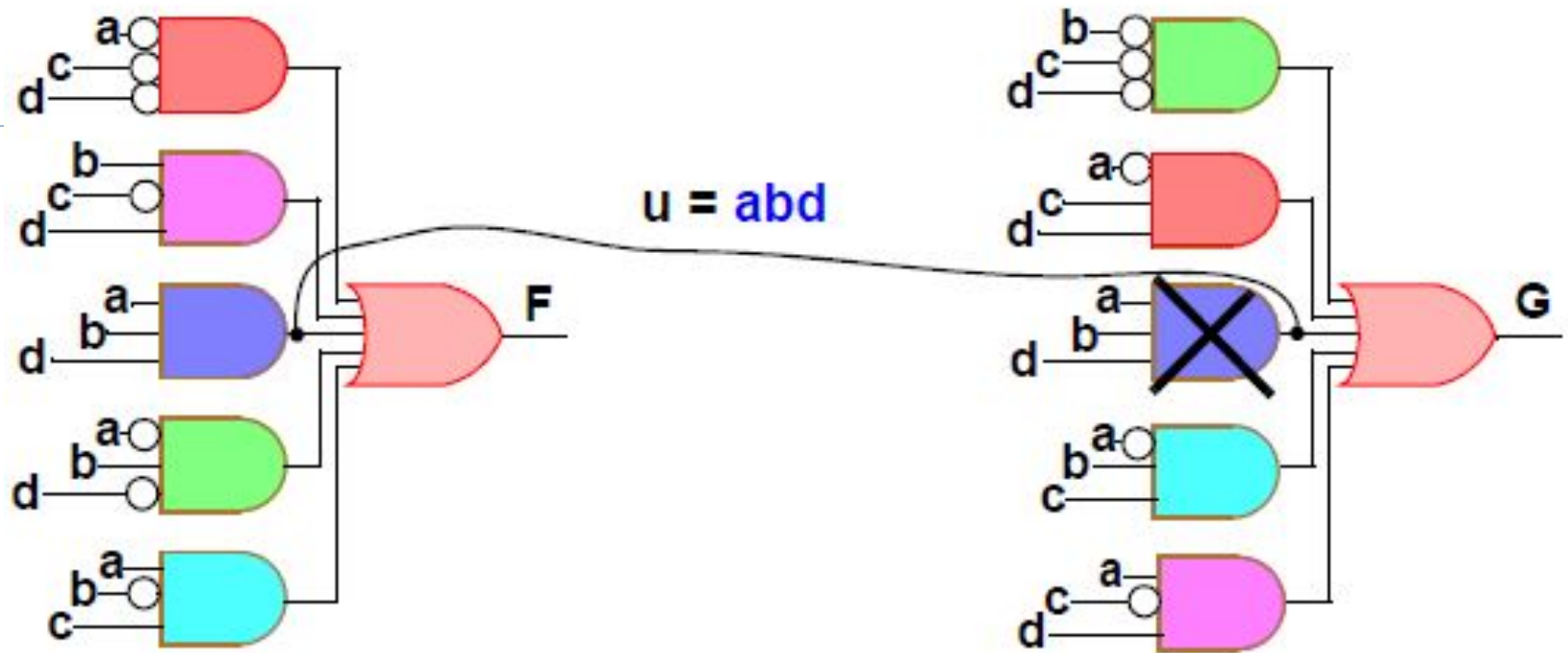


$$u = abd$$

$$F = \bar{a} \cdot \bar{c} \cdot \bar{d} + \bar{b} \bar{c} d + u + \bar{a} b \bar{d} + \bar{a} b c$$



$$G = \bar{b} \cdot \bar{c} \cdot \bar{d} + \bar{a} c d + u + \bar{a} b c + \bar{a} c \bar{d}$$

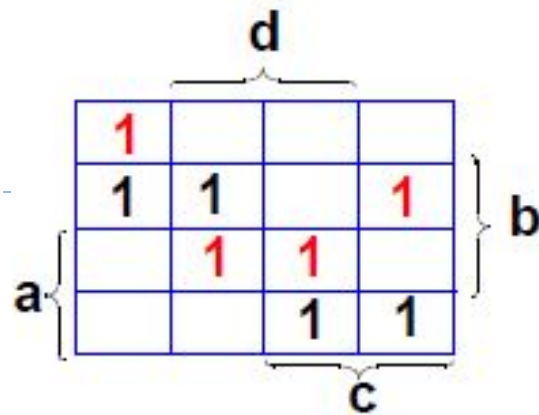


circuit size estimates

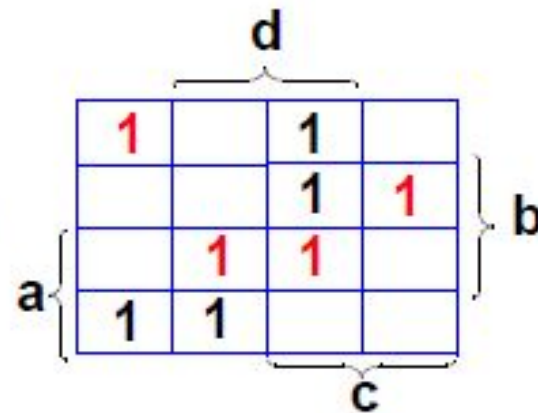
29 letters (literals) 13+13+3

37 gate inputs 15+5+12+5

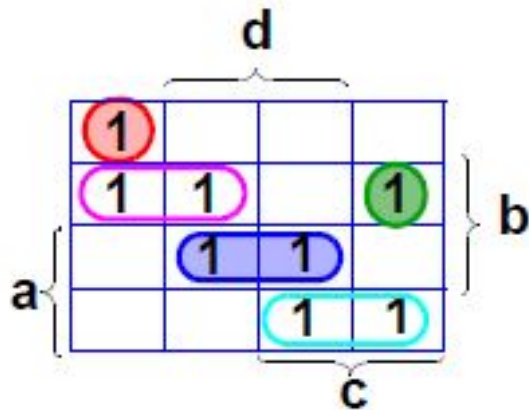
11 gates



Map of F



Map of G



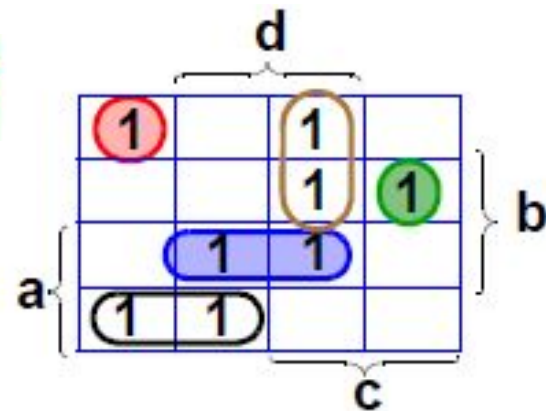
$$F = v + w + u$$

$$+ \overline{a}bc + a\overline{b}c$$

$$v = \overline{a} \cdot b \cdot c \cdot d$$

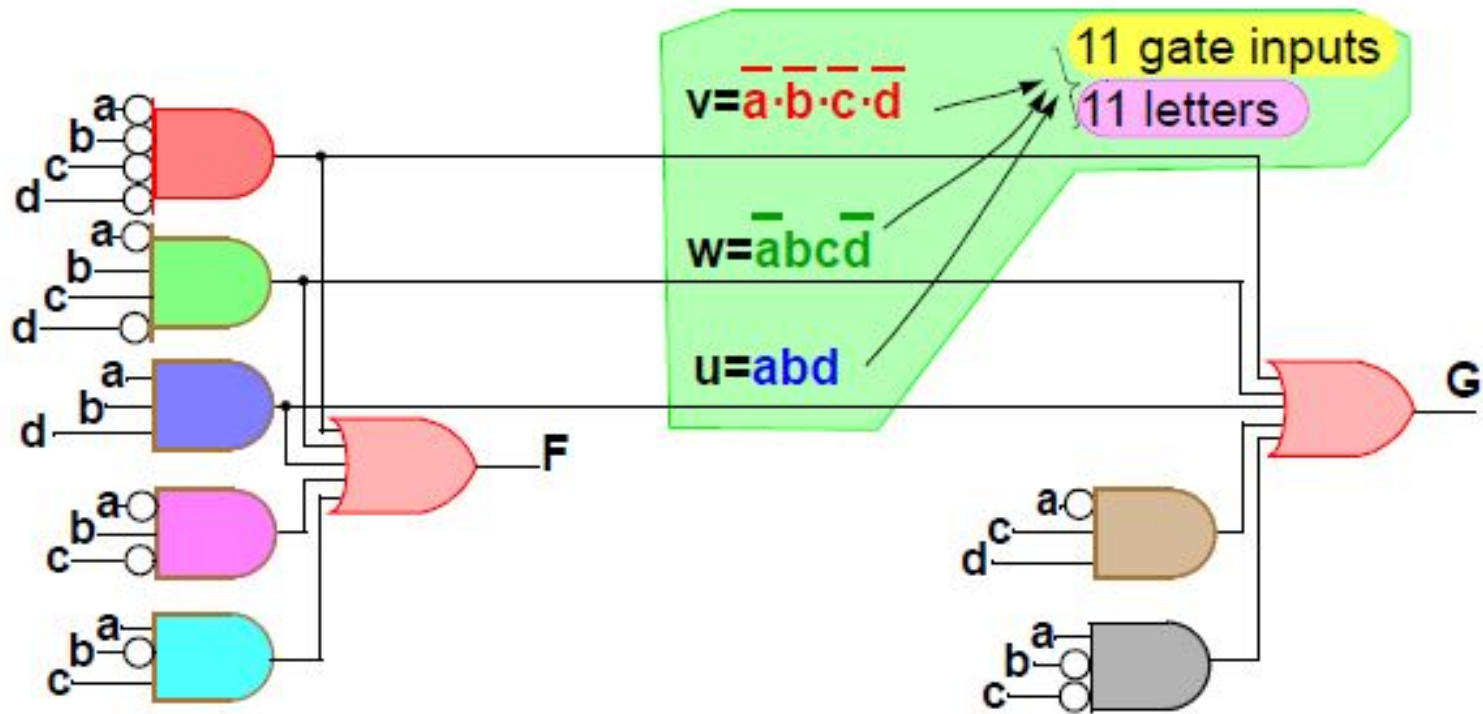
$$w = \overline{a}bc\overline{d}$$

$$u = abd$$



$$G = v + w + u$$

$$+ \overline{a}cd + a\overline{b}\overline{c}$$



size measures

Prev slide	This slide
29	29 letters (literals)
37	33 gate inputs
11	9 gates