



Arithmetic fundamentals of number systems

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Outline

- Different number systems
- Why use different ones?
- Binary / Octal / Hexadecimal
- Conversions
- Negative number representation
- Binary arithmetic
- Overflow / Underflow



Number Systems

Four number systems:

- Decimal (10)
- Binary (2)
- Octal (8)
- Hexadecimal (16)



Binary numbers

- Computers work only on two states
 - On
 - Off
- Basic memory elements hold only two states
 - Zero / One
- Thus a number system with two elements
{0,1}
- A binary digit – bit !



Decimal numbers

$$1439 = 1 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$$

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Thousands Hundreds Tens Ones

- Radix = 10



Binary → Decimal

$$\begin{aligned}1101 &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 8 + 4 + 0 + 1\end{aligned}$$

$$(1101)_2 = (13)_{10}$$

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, ...



Decimal \rightarrow Binary

2		13	1	↑	LSB
2		6	0		
2		3	1		
2		1	1		MSB
		0			

$$(13)_{10} = (1101)_2$$



Octal \rightarrow Decimal

$$\begin{aligned} 137 &= 1 \times 8^2 + 3 \times 8^1 + 7 \times 8^0 \\ &= 1 \times 64 + 3 \times 8 + 7 \times 1 \\ &= 64 + 24 + 7 \end{aligned}$$

$$(137)_8 = (95)_{10}$$

- Digits used in Octal number system – 0 to 7



Decimal \rightarrow Octal

8	95	7	↑	LSP
8	11	3		
8	1	1		
	0			MSP

$$(95)_{10} = (137)_8$$



Hex → Decimal

$$\begin{aligned} \text{BAD} &= 11 \times 16^2 + 10 \times 16^1 + 13 \times 16^0 \\ &= 11 \times 256 + 10 \times 16 + 13 \times 1 \\ &= 2816 + 160 + 13 \end{aligned}$$

$$\boxed{(\text{BAD})_{16} = (2989)_{10}}$$

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15



Decimal → Hex

16	2989	13	↑	LSP
16	186	10		
16	11	11		
	0			MSP

$$(2989)_{10} = (\text{BAD})_{16}$$



Why octal or hex?

- Ease of use and conversion
- **Three bits** make one **octal** digit

111 010 110 101

7 2 6 5 => 7265 in octal

- **Four bits** make one **hexadecimal** digit

1110 1011 0101 → 4 bits = nibble

E B 5 => EB5 in hex



Roman Numerals



A Brief History of Roman Numerals

- Roman numerals originated in ancient Rome. This ancient counting system is believed to have started with the ancient Etruscans.
- The symbol for **one** in the roman numeral system probably represented a single tally mark which people would notch into wood or dirt to keep track of items or events they were counting. It would also be easy to write on a wax tablet.



Arabic numbers → Roman numerals conversion

- Roman numerals are written as combinations of **seven letters**.

I = 1 V = 5 X = 10 L = 50

C = 100 D = 500 M = 1000

- The letters can be written as capital (XVI) or lower-case letters (xvi).



As a general guide

- Roman Numerals are made up by adding or subtracting numbers like this:-
- $11 = 10 + 1 = XI$ $9 = 10 - 1 = IX$
- $40 = 50 - 10 = XL$
- If you want to say **1,100** in Roman Numerals, you would say **M** for **1000** and then put a **C** after it for **100**; $1,100 = MC$
- $900 = 1000 - 100$ so the **C** comes before **M** = **CM**



Some more examples:

- VIII = $5+3 = 8$
- XIX = $10+ 9 = 19$
- (Remember 9 is always = IX (1 less than 10))
- XL = $50-10 = 40$
- XC = $100-10 = 90$
- Try these on whiteboards:
7 = 12 = 15 = 20 =



Check your answers.

7 = VII

12 = XII

15 = XV

20 = XX



Can you convert these numbers to Roman Numerals?

• 17 = 22 = 26 = 29 = 30 =

• 32 = 35 = 50 = 40 =

• 44 = 49 = 58 = 60 =



Were you correct?

- 17=XVII 22=XXII 26=XXVI 29=XXIX
- 30=XXX 32=XXXII 35=XXXV
- 50=L 40=XL
- 44=XLIV 49=XLIX 58=LVIII 60=LX



Some more large numbers to try:

- $600 =$ $700 =$ $800 =$

- $1000 =$ $900 =$

- $1600 =$ $1700 =$ $1900 =$

- $2000 =$



Check your answers.

- 600 = DC 700 = DCC 800 = DCCC
- 1000 = M 900 = CM
- 1600 = MDC 1700 = MDCC
- 1900 = MCM 2000 = MM



The last one

- Can you convert 2017?
- MMXVII

Now try to write today's date.

Day / Month / Year

- Well done. You are a Roman Numeral Converter!



Binary Arithmetic

- Addition
- Subtraction



Addition

Like normal decimal addition

B

A

+	0	1
0	0	1
1	1	10

$$\begin{array}{r} 0101 (5) \\ + 1001 (9) \\ \hline 1110 (14) \end{array}$$

The carry out of the MSB is neglected



Subtraction

Like normal decimal subtraction

B

A

-	0	1
0	0	11
1	1	0

1001 (9)

- 0101 (5)

0100 (4)

A borrow (shown in red) from the MSB implies a negative