



OKAN ÜNİVERSİTESİ
İSTANBUL

BBA182 Applied Statistics

Week 9 (1)

Calculating the probability of a continuous random variable – Normal Distribution

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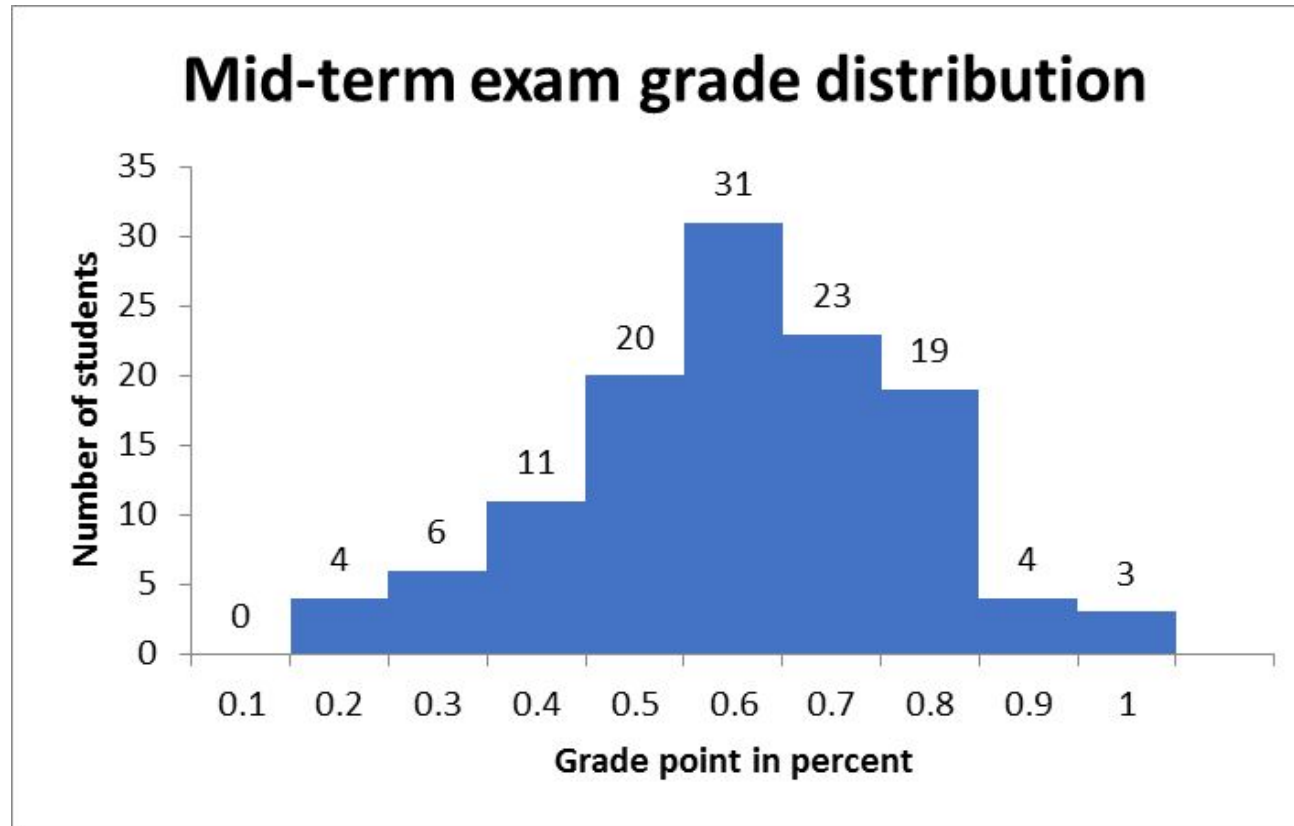
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Mid-term exam statistics

<i>Mid-term statistics</i>	
Mean	0.563554
Median	0.57
Mode	0.61
Standard Deviation	0.173872
Sample Variance	0.030231
Kurtosis	0.080928
Skewness	-0.28804
Range	0.885
Minimum	0.115
Maximum	1
Sum	68.19
Count	121



Mid-term exam statistics



Continuous random variable

A **continuous random variable** can assume any value in an interval on the real line or in a collection of intervals.

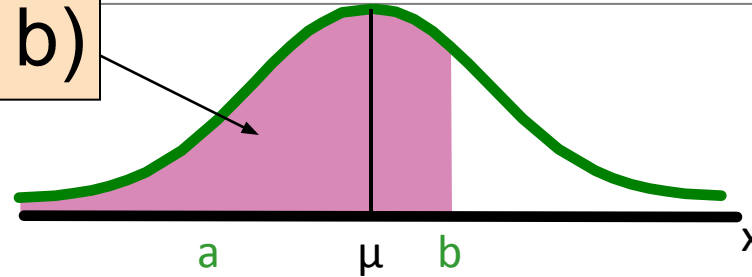
It is not possible to talk about the probability of the random variable assuming a particular value, because the probability will **be close to 0**.

Instead, we talk about the probability of the random variable assuming a value **within a given interval**.

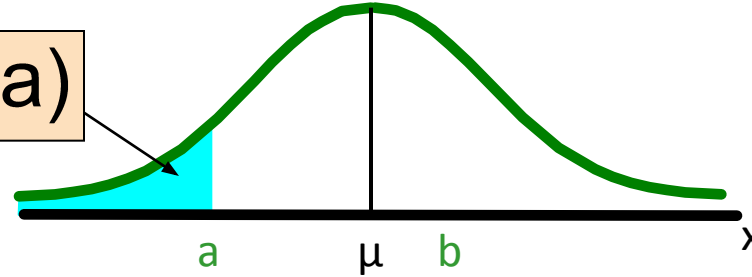


Calculating probabilities of continuous random variables

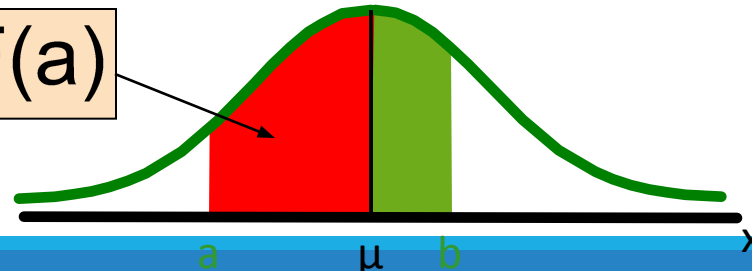
$$F(b) = P(X < b)$$



$$F(a) = P(X < a)$$



$$P(a < X < b) = F(b) - F(a)$$

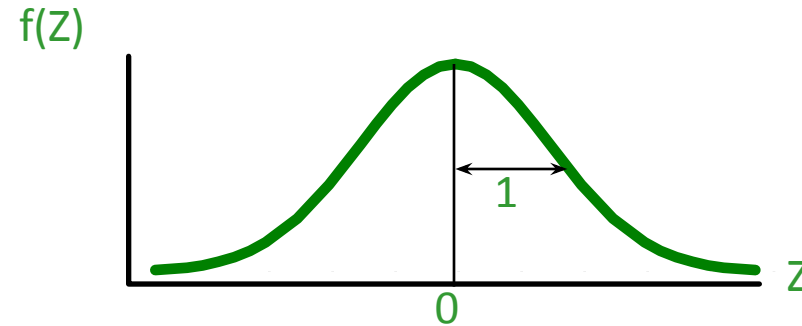




The Standard Normal Distribution – -values

Any normal distribution, $F(x)$ (with any mean and standard deviation combination) can be transformed into the **standardized normal distribution** $F(z)$, with **mean 0 and standard deviation 1**

$$Z \sim N(0,1)$$



We say that Z follows the standard normal distribution.



procedure for calculating the probability of x using the **Standard Normal Table**

For $\mu = 100$, $\sigma = 15$, find the probability that X is less than 130 = $P(x < 130)$

Transforming x - random variable into a z - standard random variable:

$$Z = \frac{X - \mu}{\sigma} = \frac{130 - 100}{15}$$
$$= \frac{30}{15} = 2 \text{ std dev}$$

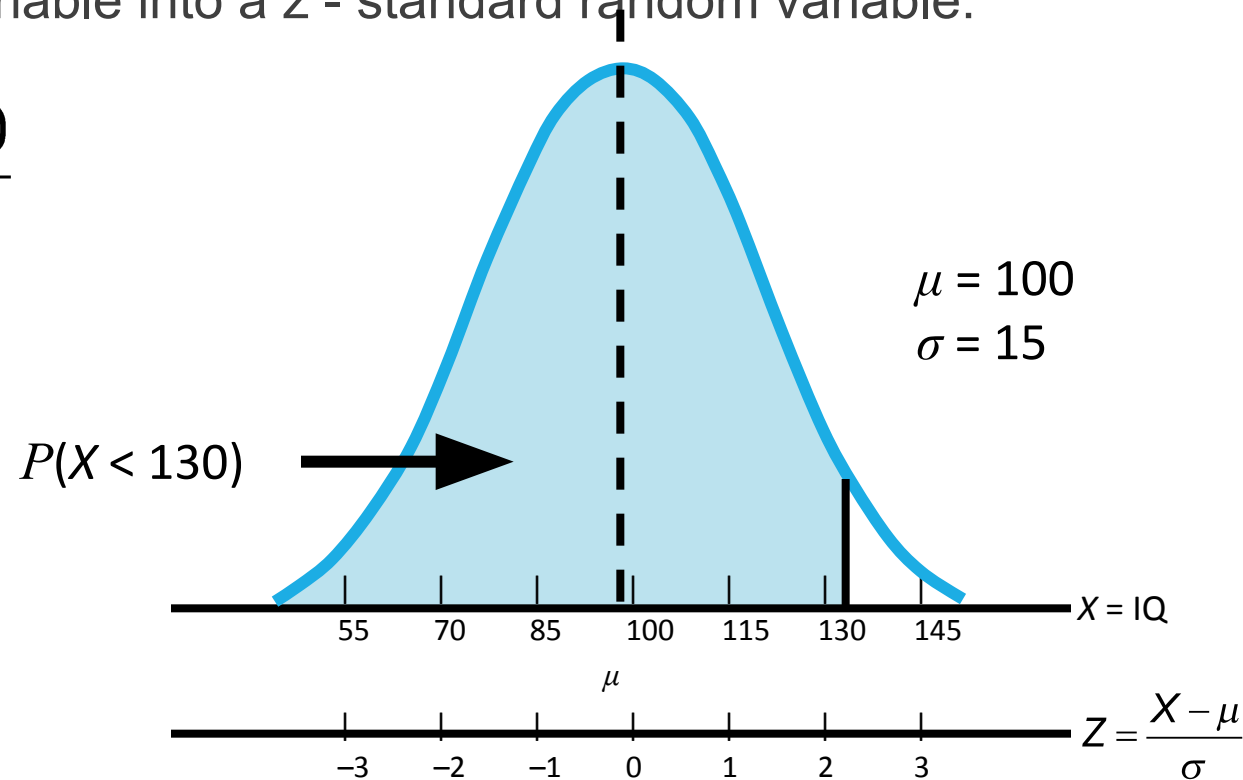


FIGURE 2.9
- Normal Distribution

Procedure for calculating the probability of x using the **Standard Normal Table** (continued)

Step 2

- Look up the probability from the table of normal curve areas
- Column on the left is Z value
- Row at the top has second decimal places for Z values



Using the Standard Normal Table

TABLE 2.10 – Standardized Normal Distribution (partial)

AREA UNDER THE NORMAL CURVE				
Z	0.00	0.01	0.02	0.03
1.8	0.96407	0.96485	0.96562	0.96638
1.9	0.97128	0.97193	0.97257	0.97320
2.0	0.97725	0.97784	0.97831	0.97882
2.1	0.98214	0.98257	0.98300	0.98341
2.2	0.98610	0.98645	0.98679	0.98713

For $Z = 2.00$

$$P(X < 130) = P(Z < 2.00) = 0.97725$$

$$\begin{aligned} P(X > 130) &= 1 - P(X \leq 130) = 1 - P(Z \leq 2) \\ &= 1 - 0.97725 = 0.02275 \end{aligned}$$

$$P(z < + 2) = P(z > -2) = .9772$$

In probability terms, a z-score of -2.0 and +2.0 has the same probability, because they are mirror images of each other.

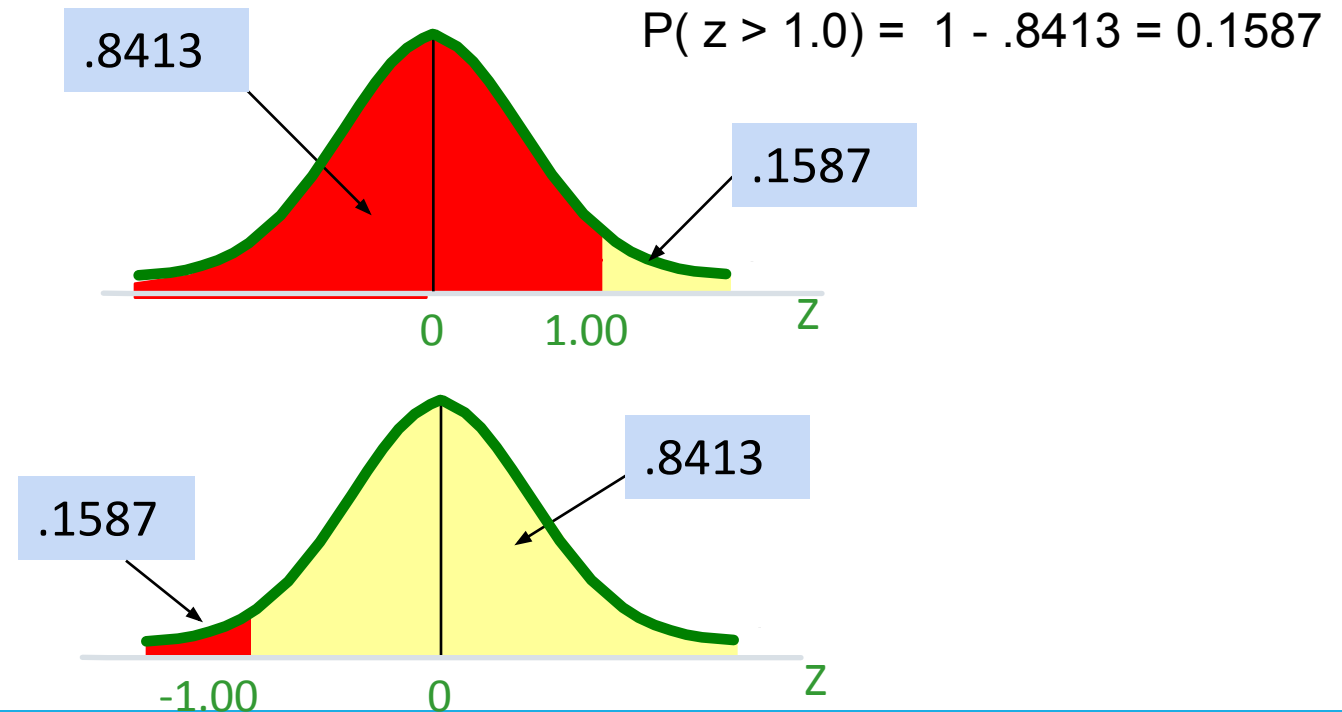
If we look for the **z-score 2.0** in the table we find a value of **9772**.



The Standard Normal Table

To find the probability of: $P(z > 1)$ and $P(z < -1)$ we will use the **complement rule**:

$$P(z < -1.0) = 1 - .8413 = 0.1587$$





Finding the probability of z-scores with two decimals and graph the probability

$$P (z < + 0.55) = \mathbf{0.7088 \quad \text{or } 70.88 \%}$$

$$P (z > + .55) = \mathbf{1.0 - 0.7088 = 0.2912 \quad \text{or } 29.12\%}$$

$$P (z > - 0.55) = \mathbf{0.7088 \quad \text{or } 70.88 \%}$$

$$P (z < - 0.55) = \mathbf{1.0 - .7088 = 0.2912 \quad \text{or } 29.12 \%}$$

$$P (z < + 1.65) = \mathbf{0.9505 \quad \text{or } 95.05 \%}$$

$$P (z > + 1.65) = \mathbf{1.0 - 0.9505 = 0.0495 \quad \text{or } 4.96 \%}$$

$$P (z > - 2.36) = \mathbf{.9909 \quad \text{or } 99.09 \%}$$

$$P (z < + 2.36) = \mathbf{.9909 \quad \text{or } 99.09 \%}$$

Determine for shampoo filling machine 1 the proportion of bottles that:

$$\mu = 500 \text{ ml } \sigma = 10 \text{ ml}$$

Contain less than 510 ml $P(x < 510)$

Contain more than 515 ml $P(x > 515)$

Contains more than 480 ml $P(x > 480)$

Contain less than 490 ml $P(x < 490)$

Contain more than 505 ml $P(x > 505)$



Solution: Contain more than 515 ml $P(x > 515\text{ml})$

1. Draw the graph to see which area we are looking for:

$$2. Z \text{ -score} = \frac{515-500}{10} = 1.5 = P(z > 1.5)$$

3. We can find $P(z < 1.5) = .9332$ directly from the table

$$P(z > 1.5) = 1 - .9332 = .0668$$

6.68% of the shampoo bottles contain more than 515 ml.

Solution: Contain more than 505 ml

$P(x > 505) ?$

1. Draw the curve so you see which probability area we are looking for.

$$2. Z \text{ -score} = \frac{505-500}{10} = 0.5 = P(z < .5) = .6915$$

$$3. P(z > 0.5) = 1 - .6915 = .3085$$

30.85 % of the shampoo bottles contain more than 505ml shampoo.

Draw a graph of the below probabilities and find the probability of z in the standard normal table with $\mu = 0, \sigma = 1$

$$P (z < + 1.05) =$$

$$P (z > -1.05) =$$

$$P (z < - 3.34) =$$

$$P (z > - 3.34) =$$

$$P (z > - 2.47) =$$

$$P (z < + 1.87) =$$

$$P (z > + 2.57) =$$

$$P (z < - 0.32) =$$

Exercise:

Find the probability of z-scores and draw a graph of the probability

$$P(z < + 1.05) = \mathbf{0.8531 \text{ or } 85.31 \%}$$

$$P(z > -1.05) = \mathbf{0.8531 \text{ or } 85.31 \%}$$

$$P(z < - 3.34) = \mathbf{1.0 - 0.9996 = 0.0004 \text{ or } 0.04 \%}$$

$$P(z > - 3.34) = \mathbf{0.9996 \text{ or } 99.96 \%}$$

$$P(z > - 2.47) = \mathbf{0.9932 \text{ or } 99.32 \%}$$

$$P(z < + 1.87) = \mathbf{0.9693 \text{ or } 96.93 \%}$$

$$P(z > + 2.57) = \mathbf{1.0 - 0.9949 = 0.0054 \text{ or } 0.054 \%}$$

$$P(z < - 0.32) = \mathbf{1.0 - 0.6255 = 0.3745 \text{ or } 37.45 \%}$$

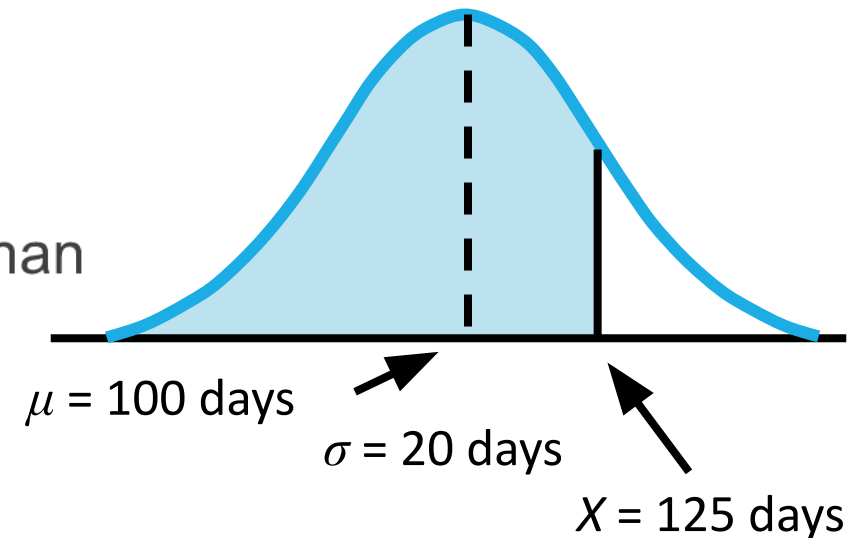


Haynes Construction Company Example

Builds three- and four-unit apartment buildings:

- Total construction time follows a normal distribution
- For triplexes, $\mu = 100$ days and $\sigma = 20$ days
- Contract calls for completion in 125 days
- Late completion will incur a severe penalty fee
- Calculate the probability of completing in less than 125 days $P(x < 125)$

FIGURE 2.10



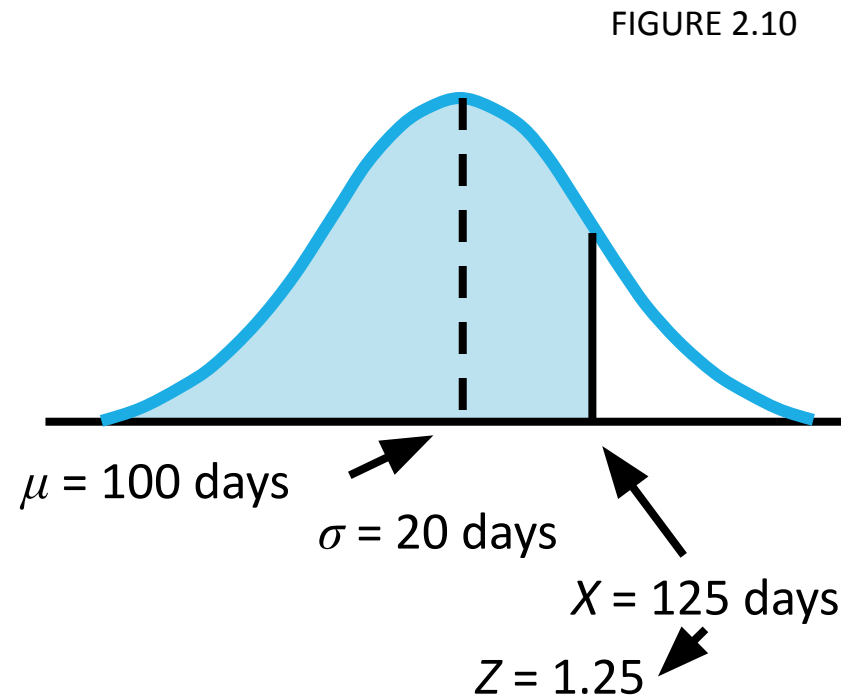


Haynes Construction Company

Compute Z:

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20}$$
$$= \frac{25}{20} = 1.25 \quad P(z < 1.25) ?$$

- From the table for $Z = 1.25$
area $P(z < 1.25) = 0.8944$





Haynes Construction Company

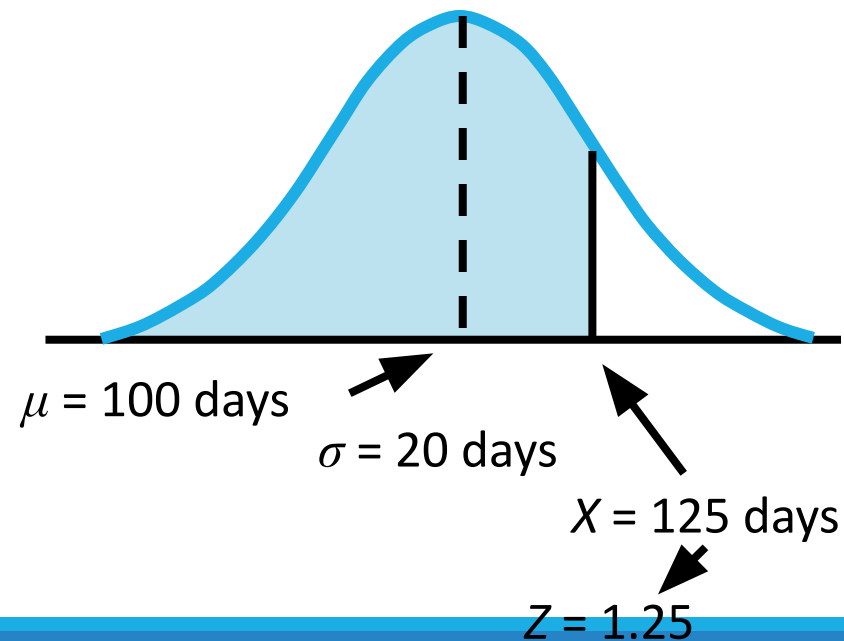
• Con

The probability is about 0.89 or 89 %
that Haynes will not violate the contract

Z =

$$= \frac{25}{20} = 1.25$$

— From the table for $Z = 1.25$
area = 0.89435



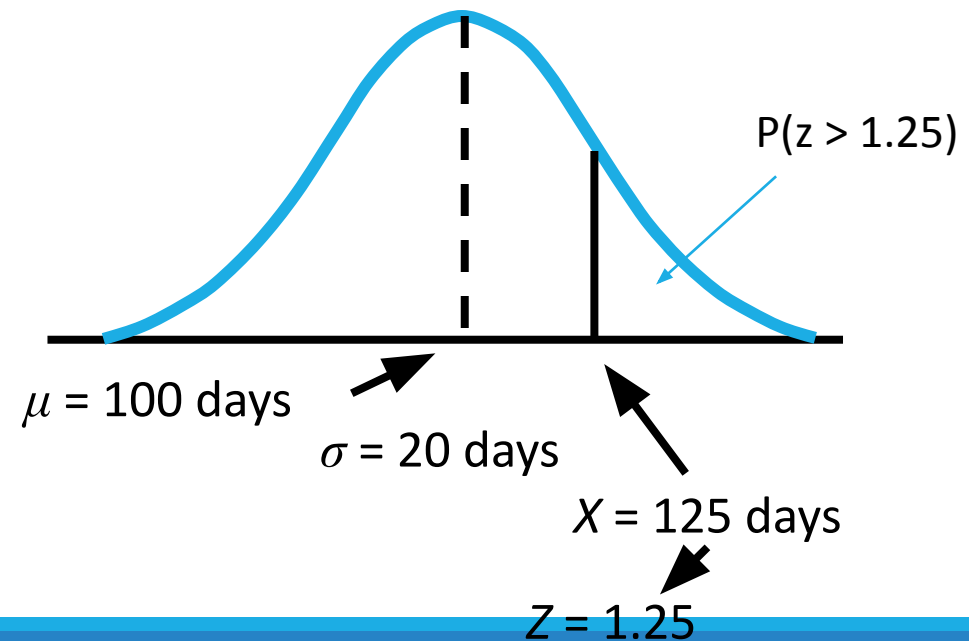


Haynes Construction Company

What is the probability that the company will **not** finish in 125 days and therefore will have to pay a penalty?

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20} = 1.25$$

FIGURE 2.10



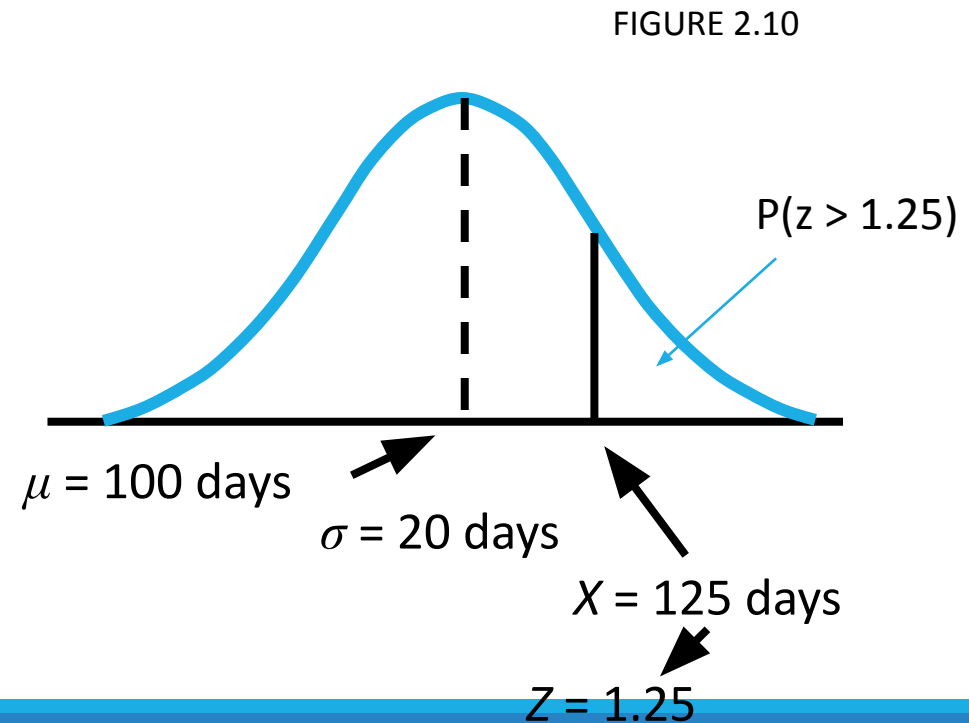


Haynes Construction Company

What is the probability that the company will **not** finish in 125 days and therefore will have to pay a penalty?

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20}$$
$$= \frac{25}{20} = 1.25 \quad P(z > 1.25) ?$$

- From the table for $Z = 1.25$
area $P(z > 1.25) =$
 $1 - P(z < 1.25) = 1 - 0.8944 =$
 0.1056 or 10.56%





Haynes Construction Company

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If finished in 75 days or less, Haynes will get a bonus of \$5,000

- What is the probability of a bonus? $P(x < 75)$

$\mu = 100$ days and $\sigma = 20$ days

$$Z = \frac{X - \mu}{\sigma}$$



Haynes Construction Company

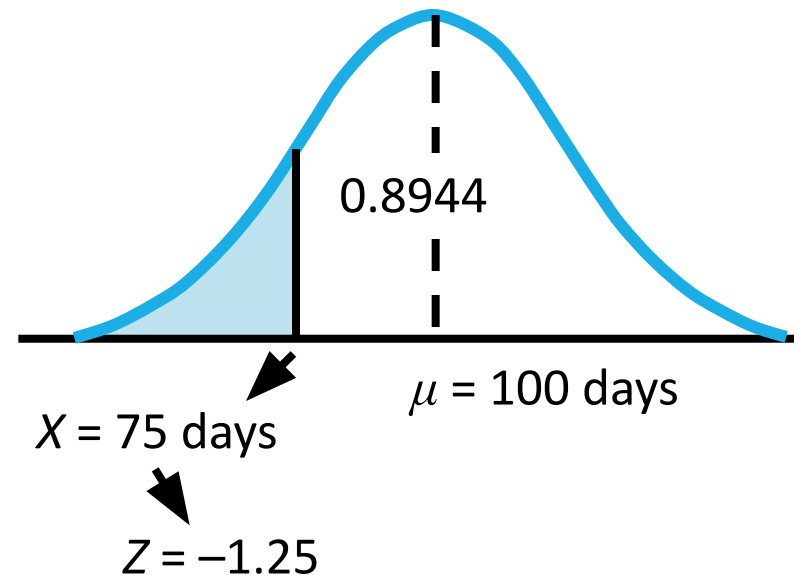
If finished in 75 days or less, bonus = \$5,000

- Probability of bonus? $P(x < 75)$

$$Z = \frac{X - \mu}{\sigma} = \frac{75 - 100}{20}$$
$$= \frac{-25}{20} = -1.25 \quad P(z < -1.25) ?$$

- Because the distribution is symmetrical, equivalent to $Z = 1.25$
 $P(z < 1.25)$ so area = 0.8944

FIGURE 2.11





aynes Construction Company

• If fir

– P

Z =

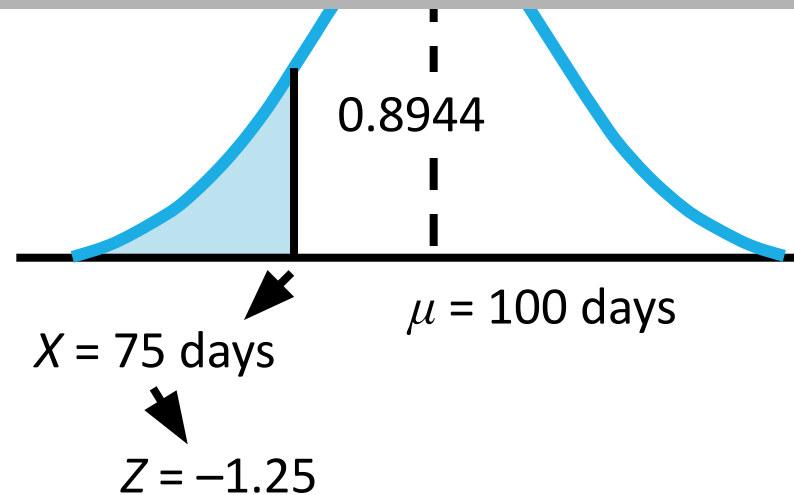
$$= \frac{75 - 100}{20} = -1.25$$

- Because the distribution is symmetrical, equivalent to $Z = 1.25$ so area = 0.89435

$$P(z < -1.25) = 1.0 - P(z < 1.25)$$

$$= 1.0 - 0.8944 = 0.1056$$

The probability of completing the contract in 75 days or less is about 11%





Haynes Construction Company

Probability of completing between 110 and 125 days?

$$P(110 < X < 125) ?$$

$$P(a < X < b) = F(b) - F(a)$$

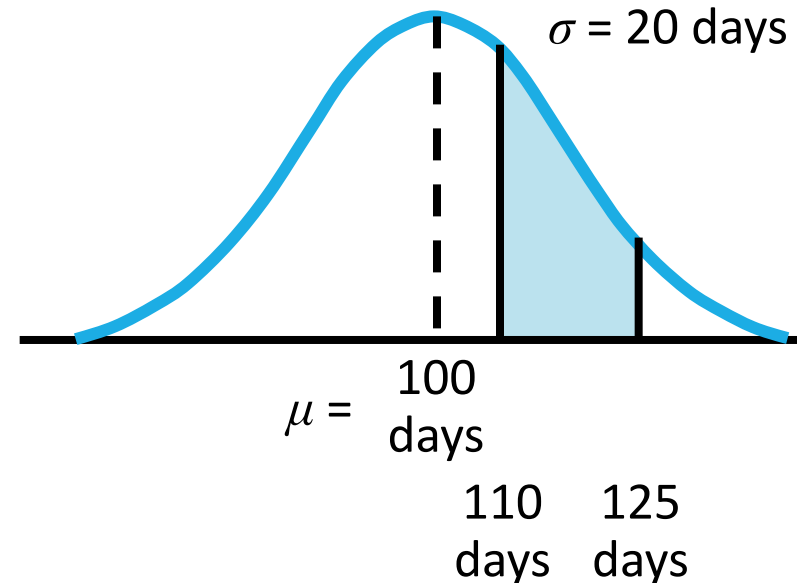


FIGURE 2.12



Haynes Construction Company

Probability of completing between 110 and 125 days?

$$P(110 < X < 125) ?$$

$$P(a < X < b) = F(b) - F(a)$$

$$P\left(\frac{a-\mu}{\sigma} < z < \frac{b-\mu}{\sigma}\right) = P\left(\frac{110-100}{20} < z < \frac{125-100}{20}\right) =$$

$$F(b) - F(a) = F(1.25) - F(0.5) = .8944 - .6915 = .2029$$

$$P(.5 < z < 1.25) = .2029 \text{ or } 20.29 \%$$

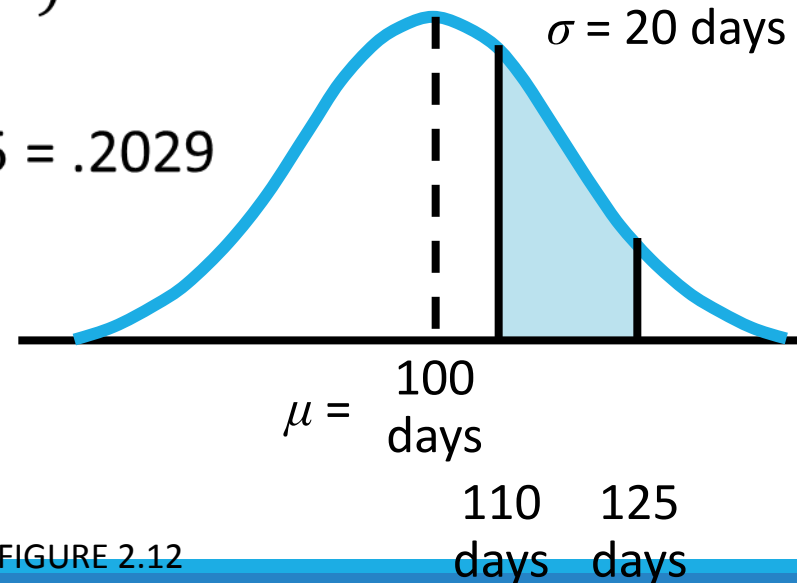


FIGURE 2.12



Probability of co

$$P(110 < X < 125)$$

$$P(110 \leq X < 125) = 0.8944 - 0.6915 = 0.2029$$

The probability of completing between 110 and 125 days is about 20%

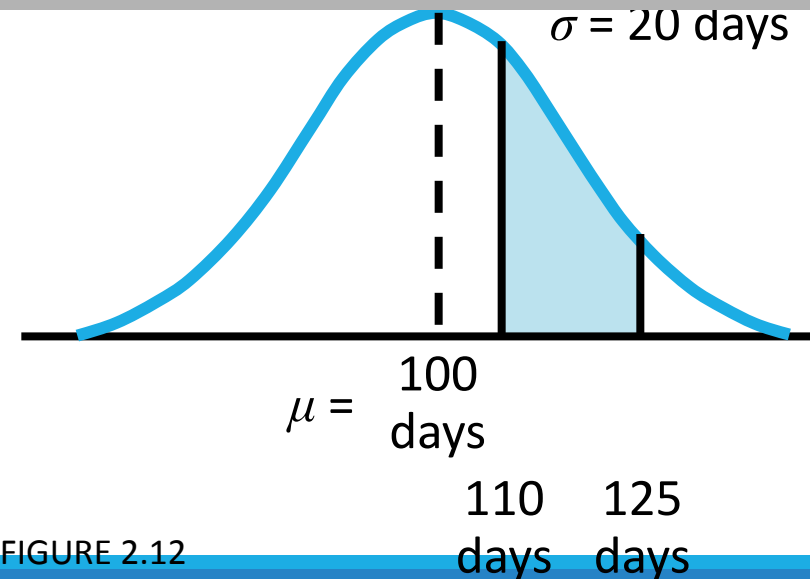


FIGURE 2.12

Calculation procedure to find the probability of the area under the normal curve:

1. First draw the normal curve for the problem, to understand what area under the curve we are looking for.
2. Transform x-values to the standardized random variable, z

$$z = \frac{X - \mu_x}{\sigma_x}$$

3. Use the standardized normal distribution table to find the probability of the calculated z-value