BBA182 Applied Statistics
Week 9 (1)
Calculating the probability of a continuous random variable – Normal Distribution

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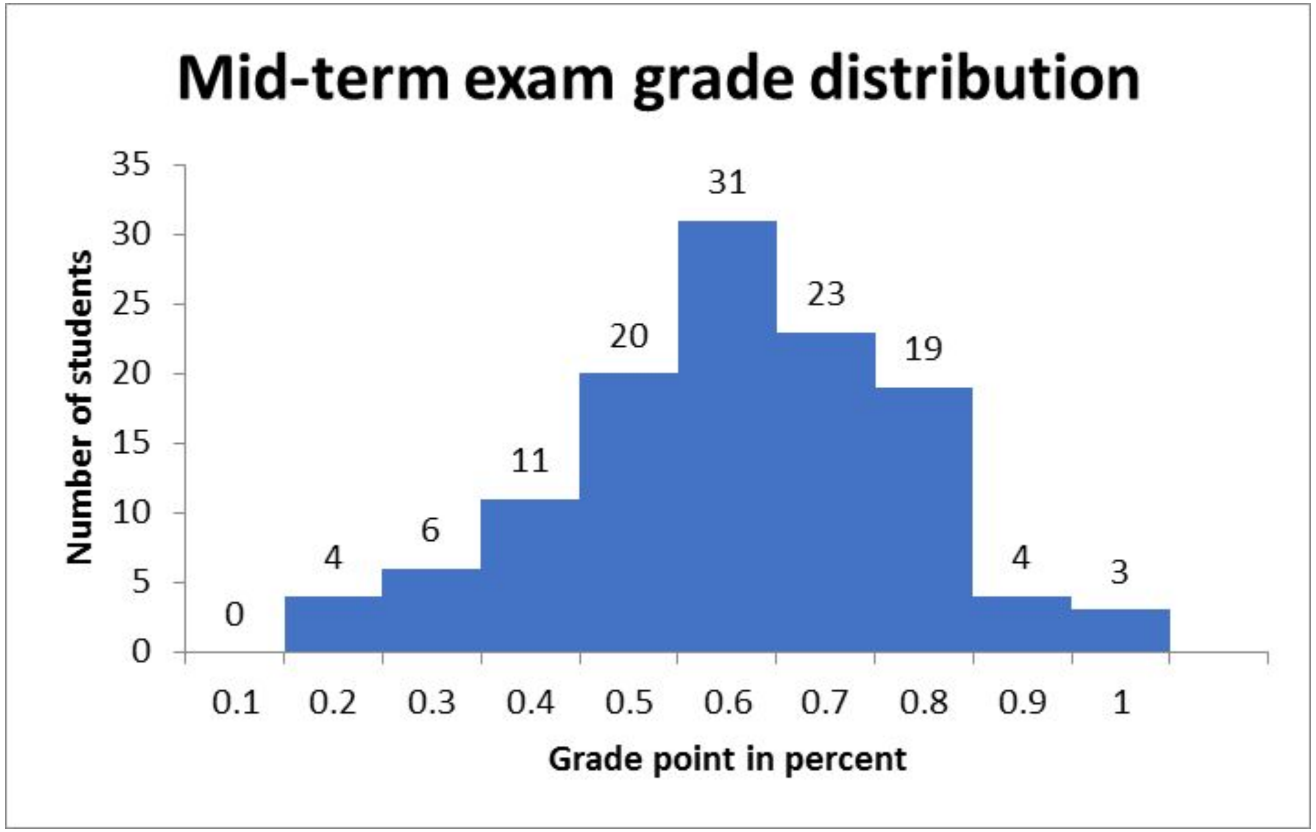
Mid-term exam statistics

Mid-term statistics	
Mean	0.563554
Median	0.57
Mode	0.61
Standard Deviation	0.173872
Sample Variance	0.030231
Kurtosis	0.080928
Skewness	-0.28804
Range	0.885
Minimum	0.115
Maximum	1
Sum	68.19
Count	121

Mid-term exam statistics



Continuous random variable

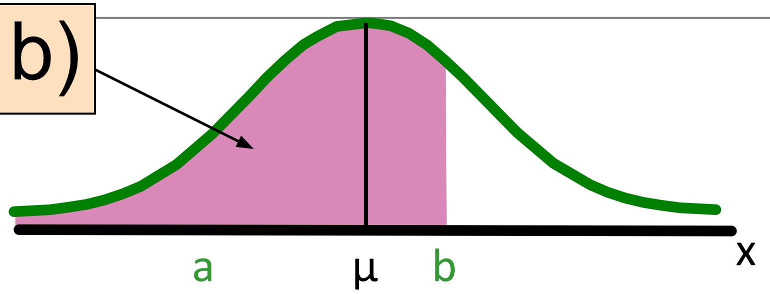
A **continuous random variable** can assume any value in an interval on the real line or in a collection of intervals.

It is not possible to talk about the probability of the random variable assuming a particular value, because the probability will **be close to 0.**

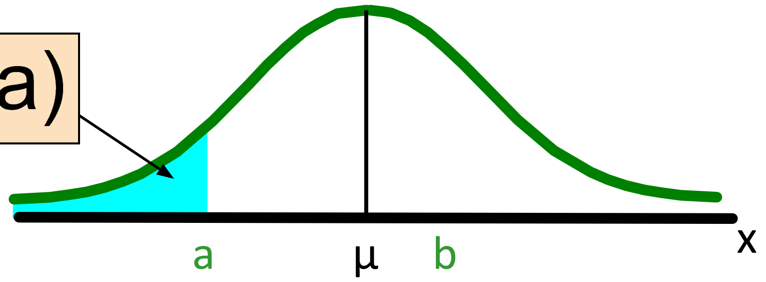
Instead, we talk about the probability of the random variable assuming a value **within a given interval**.

Calculating probabilities of continuous random variables

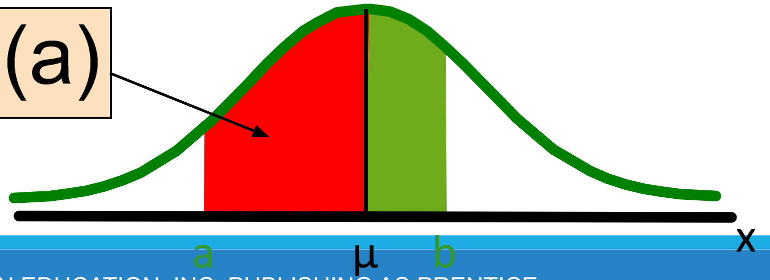
$$F(b) = P(X < b)$$



$$F(a) = P(X < a)$$



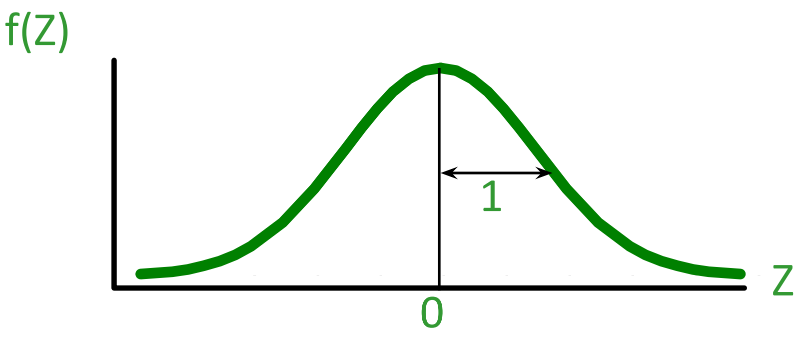
$$P(a < X < b) = F(b) - F(a)$$



The Standard Normal Distribution – -values

Any normal distribution, F(x) (with any mean and standard deviation combination) can be transformed into the **standardized normal distribution F(z), with mean 0 and standard deviation 1**

$$\boxed{Z \sim N(0,1)}$$



We say that Z follows the standard normal distribution.

rocedure for calculating the probability of x using the **Standard Normal Table**

For $\mu = 100$, $\sigma = 15$, find the probability that X is less than 130 = P(x < 130)

Transforming x - random variable into a z - standard random variable:

$$Z = \frac{X - \mu}{\sigma} = \frac{130\text{-}100}{15}$$

$$= \frac{30}{15} = 2 \text{ std dev}$$

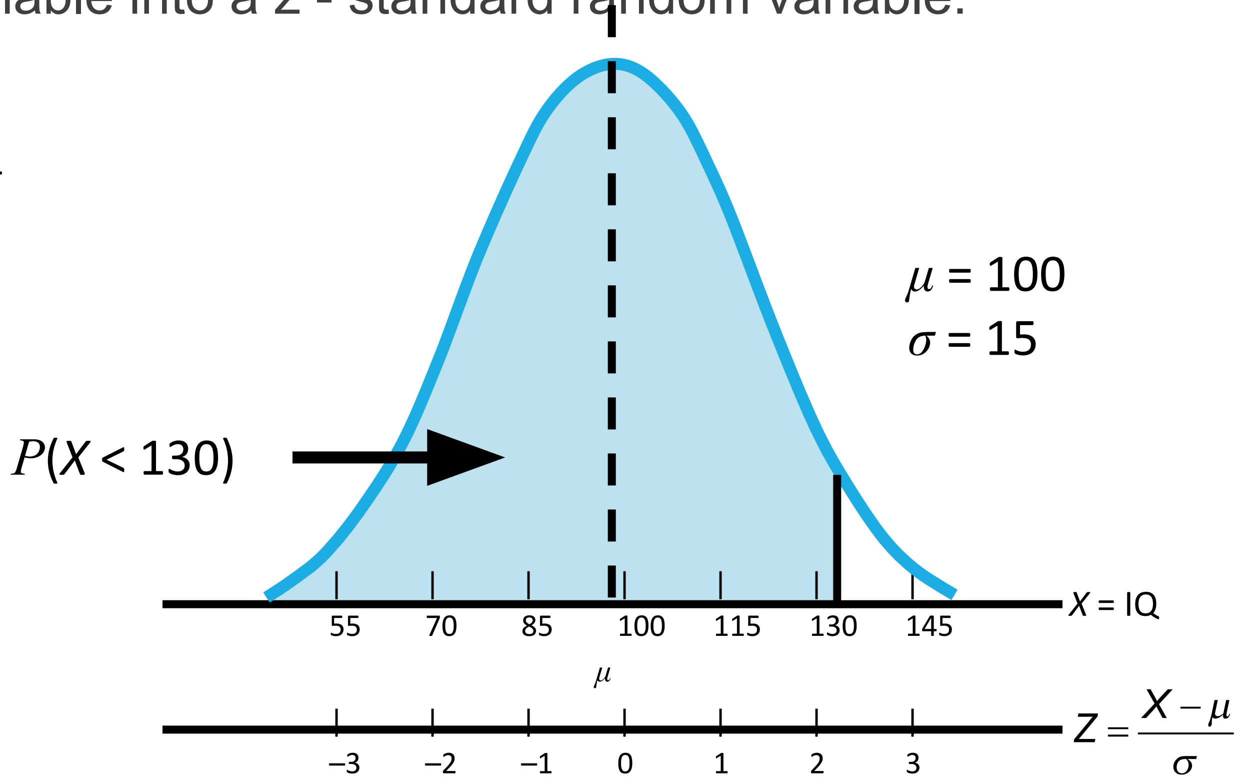


FIGURE 2.9
– Normal Distribution

Procedure for calculating the probability of x using the **Standard Normal Table** (continued)

Step 2

- Look up the probability from the table of normal curve areas
- Column on the left is Z value
- Row at the top has second decimal places for Z values

Using the Standard Normal Table

TABLE 2.10 – Standardized Normal Distribution (partial)

	AREA UNDER THE NORMAL CURVE			
Z	**0.00**	**0.01**	**0.02**	**0.03**
1.8	0.96407	0.96485	0.96562	0.96638
1.9	0.97128	0.97193	0.97257	0.97320
2.0	$\boxed{0.97725}$	0.97784	0.97831	0.97882
2.1	0.98214	0.98257	0.98300	0.98341
2.2	0.98610	0.98645	0.98679	0.98713

For $Z = 2.00$

$$P(X < 130) = P(Z < 2.00) = 0.97725$$

$$P(X > 130) = 1 - P(X \leq 130) = 1 - P(Z \leq 2)$$

$$= 1 - 0.97725 = 0.02275$$

$P(z < +2) = P(z > -2) = .9772$

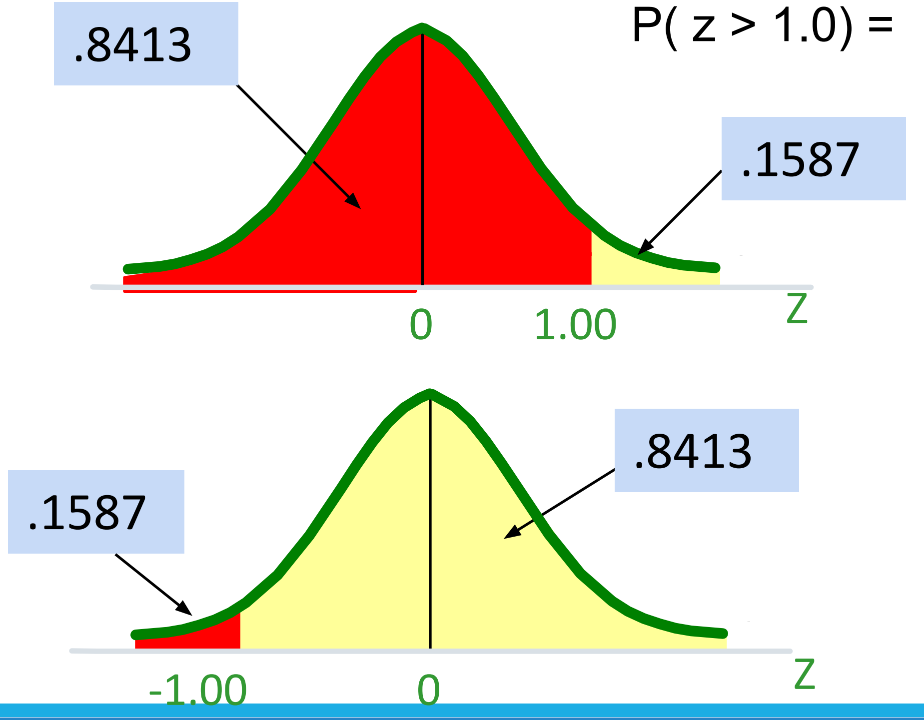
In probability terms, a z-score of -2.0 and +2.0 has the same probability, because they are mirror images of each other.

If we look for the **z-score 2.0 in the table we find a value of 9772.**

The Standard Normal Table

To find the probability of: P (z > 1) and P (z < -1) we will use the **complement rule:**

P(z > 1.0) = 1 - .8413 = 0.1587



P(z < - 1.0) = 1 - .8413 = 0.1587

Finding the probability of z-scores with two decimals and graph the probability

P (z < + 0.55) = **0.7088 or 70.88 %**

P (z > + .55) = **1.0 – 0.7088 = 0.2912 or 29.12%**

P (z > - 0.55) = **0.7088 or 70.88 %**

P (z < - 0.55) = **1.0 - .7088 = 0.2912 or 29.12 %**

P (z < + 1.65) = **0.9505 or 95.05 %**

P (z > + 1.65) = **1.0 – 0.9505 = 0.0495 or 4.96 %**

P(z > - 2.36) = **.9909 or 99.09 %**

P (z < + 2.36) = **.9909 or 99.09 %**

Determine for shampoo filling machine 1 the proportion of bottles that:

μ = 500 ml σ= 10ml

Contain less than 510 ml $P(x < 510)$

Contain more than 515 ml $P(x > 515)$

Contains more than 480 ml $P(x > 480)$

Contain less than 490 ml $P(x < 490)$

Contain more than 505 ml $P(x > 505)$

Solution: Contain more than 515 ml
$P(x > 515ml)$

1. Draw the graph to see which area we are looking for:

2. Z –score = $\frac{515-500}{10}$ = 1.5 = $P(z > 1.5)$

3. We can find $P(z < 1.5) = .9332$ directly from the table

$P(z > 1.5) = 1 - .9332 = .0668$

6.68% of the shampoo bottles contain more than 515 ml.

Solution: Contain more than 505 ml

P(x > 505) ?

1. Draw the curve so you see which probability area we are looking for.

2. Z –score = $\frac{505-500}{10}$ = 0.5 = P(z < .5) = .6915

3. P(z > 0.5) = 1 - . 6915= .3085

30.85 % of the shampoo bottles contain more than 505ml shampoo.

Draw a graph of the below probabilities and find the probability of z in the standard normal table with $\mu = 0$, $\sigma = 1$

P (z < + 1.05) =

P (z > -1.05) =

P (z < - 3.34) =

P (z > - 3.34) =

P (z > - 2.47) =

P (z < + 1.87) =

P (z > + 2.57) =

P (z < - 0.32) =

Exercise:
Find the probability of z-scores and draw a graph of the probability

P (z < + 1.05) = **0.8531 or 85.31 %**

P (z > -1.05) = **0.8531 or 85.31 %**

P (z < - 3.34) = **1.0 – 0.9996 = 0.0004 or 0.04 %**

P (z > - 3.34) = **0.9996 or 99.96 %**

P (z > - 2.47) = **0.9932 or 99.32 %**

P (z < + 1.87) = **0.9693 or 96.93 %**

P (z > + 2.57) = **1.0 – 0.9949 = 0.0054 or 0.054 %**

P(z < - 0.32) = **1.0 – 0.6255 = 0.3745   or 37. 45 %**

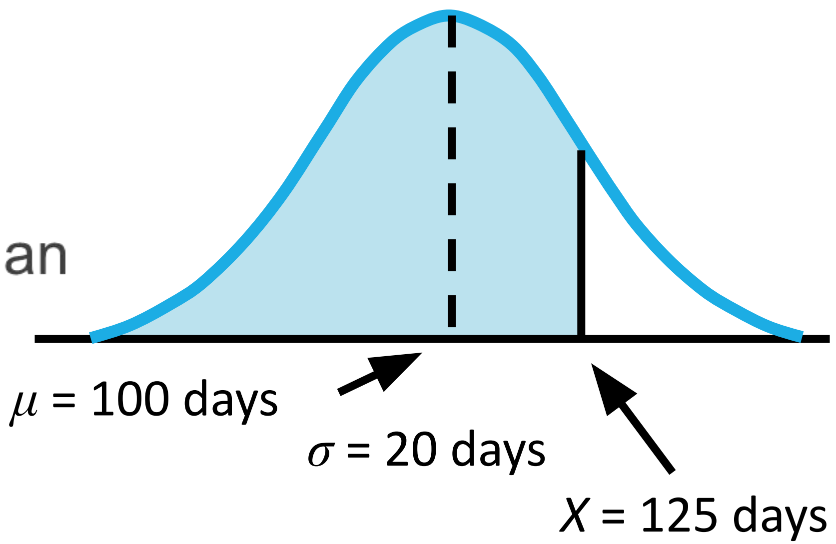
Haynes Construction Company Example

Builds three- and four-unit apartment buildings:

- Total construction time follows a normal distribution
- For triplexes, μ = 100 days and σ = 20 days
- Contract calls for completion in 125 days
- Late completion will incur a severe penalty fee

- Calculate the probability of completing in less than 125 days P(x <125)

FIGURE 2.10



Haynes Construction Company

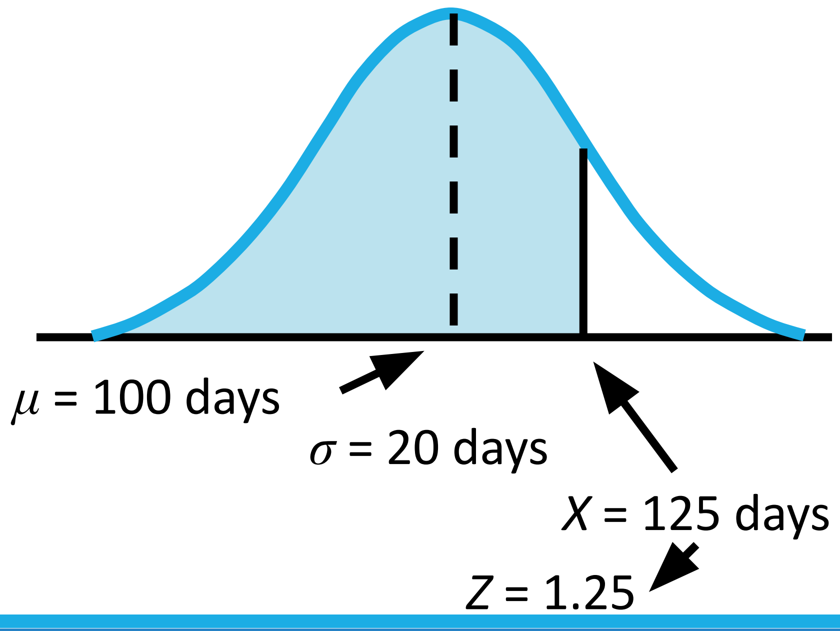
Compute *Z:*

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20}$$

$$= \frac{25}{20} = 1.25 \quad P(z < 1.25)\ ?$$

– From the table for $Z = 1.25$ area $P(z < 1.25) = 0.8944$

FIGURE 2.10



Haynes Construction Company

- Con

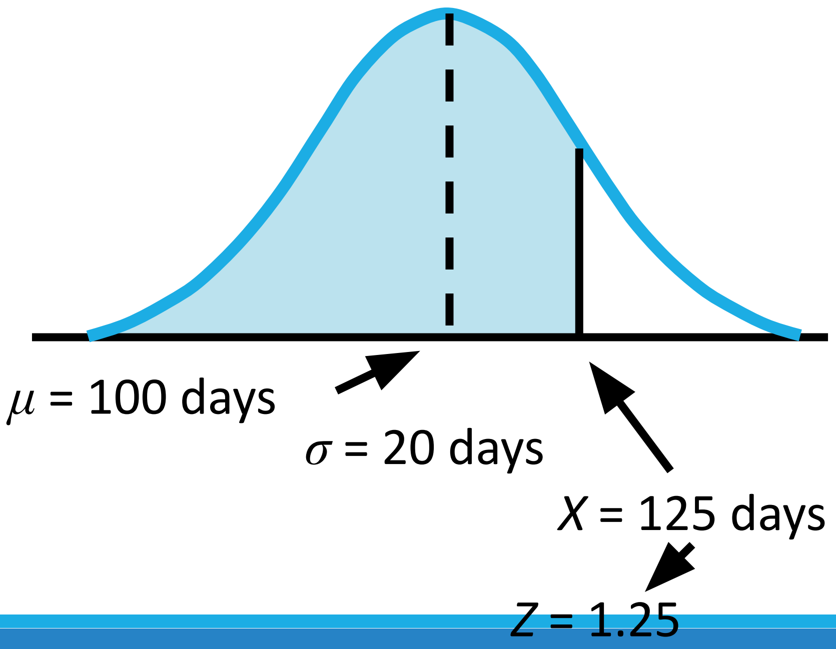
The probability is about 0.89 or 89 % that Haynes will not violate the contract

$Z =$

$$= \frac{25}{20} = 1.25$$

- From the table for $Z = 1.25$ area = 0.89435

FIGURE 2.10

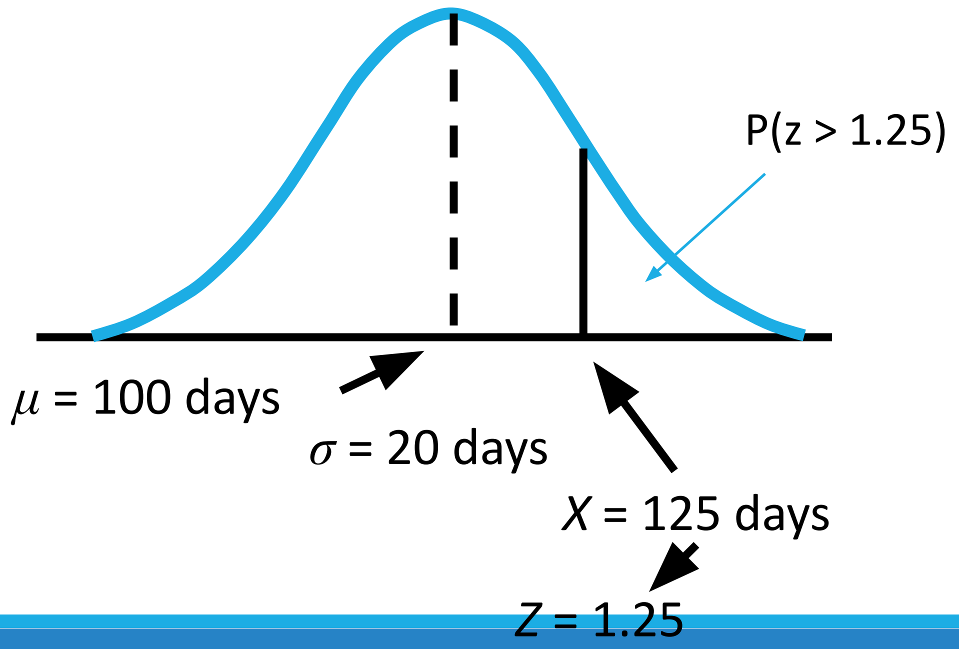


Haynes Construction Company

What is the probability that the company will **not** finish in 125 days and therefore will have to pay a penalty?

$$Z = \frac{X - \mu}{\sigma} = 125 - 100 / 20 = 1.25$$

FIGURE 2.10



Haynes Construction Company

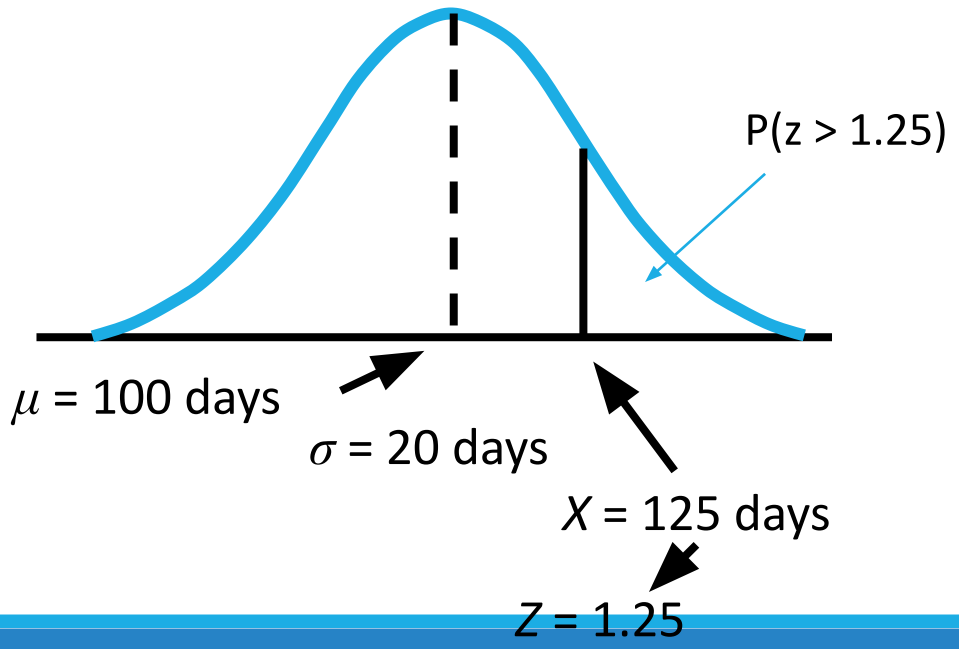
What is the probability that the company will **not** finish in 125 days and therefore will have to pay a penalty?

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20}$$

$$= \frac{25}{20} = 1.25 \quad P(z > 1.25)\,?$$

- From the table for Z = 1.25 area P(z > 1.25) = 1 – P(z < 1.25) = 1 - 0.8944 = 0.1056 or 10.56 %

FIGURE 2.10



Haynes Construction Company

If finished in 75 days or less, Haynes will get a bonus of $5,000

- What is the probability of a bonus? P (x < 75)

μ = 100 days and σ = 20 days

$$Z = \frac{X - \mu}{\sigma}$$

Haynes Construction Company

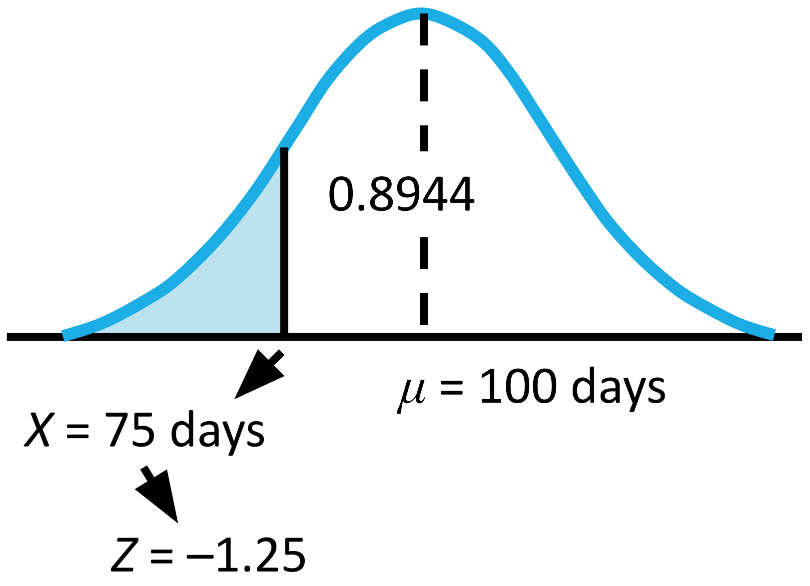
If finished in 75 days or less, bonus = \$5,000

- Probability of bonus? P (x < 75)

$$Z = \frac{X - \mu}{\sigma} = \frac{75 - 100}{20}$$

$$= \frac{-25}{20} = -1.25 \quad P(z < -1.25)\,?$$

FIGURE 2.11



– Because the distribution is symmetrical, equivalent to Z = 1.25 P(z < 1.25) so area = 0.8944

aynes Construction Company

- If fir
  - P

$Z =$

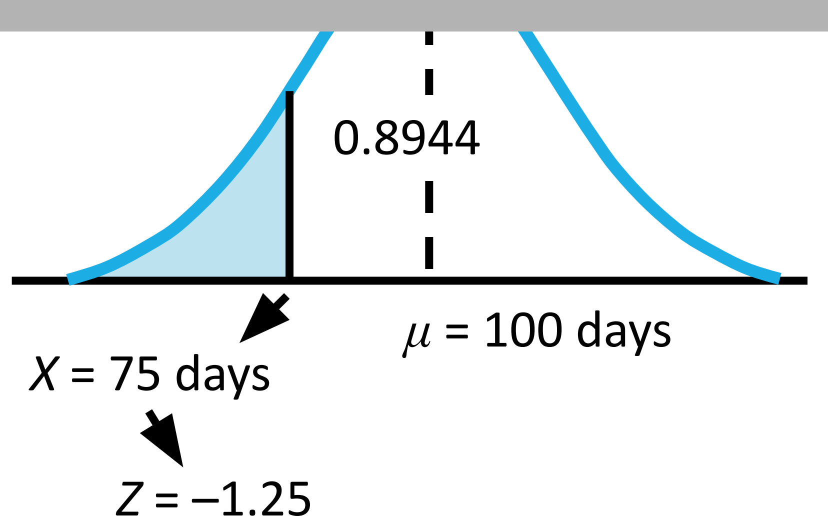
$= \frac{-25}{20} = -1.25$

$P(z < -1.25) \quad = 1.0 - P(z < 1.25)$

$= 1.0 - 0.8944 = 0.1056$

The probability of completing the contract in 75 days or less is about 11%

- Because the distribution is symmetrical, equivalent to $Z = 1.25$ so area = 0.89435



Haynes Construction Company

Probability of completing between 110 and 125 days?

$P(110 < X < 125)$?

$$\boxed{P(a < X < b) = F(b) - F(a)}$$

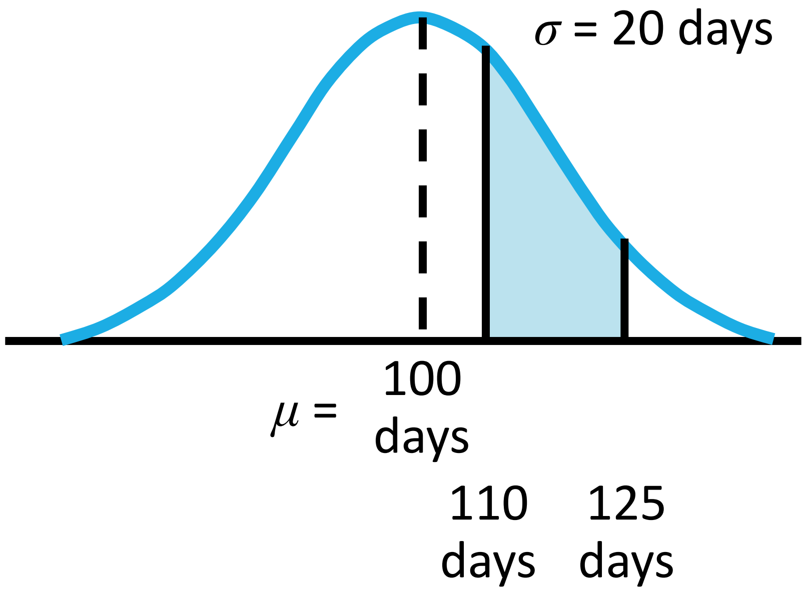


FIGURE 2.12

Haynes Construction Company

Probability of completing between 110 and 125 days?

$P(110 < X < 125)$?

$$\boxed{P(a < X < b) = F(b) - F(a)}$$

$$P\left(\frac{a-\mu}{\sigma} < z < \frac{b-\mu}{\sigma}\right) = P\left(\frac{110-100}{20} < z < \frac{125-100}{20}\right) =$$

$F(b) - F(a)$ = F(1.25) – F(0.5) = .8944 - .6915 = .2029

P (.05 < z < 1.25) = .2029 or 20.29 %

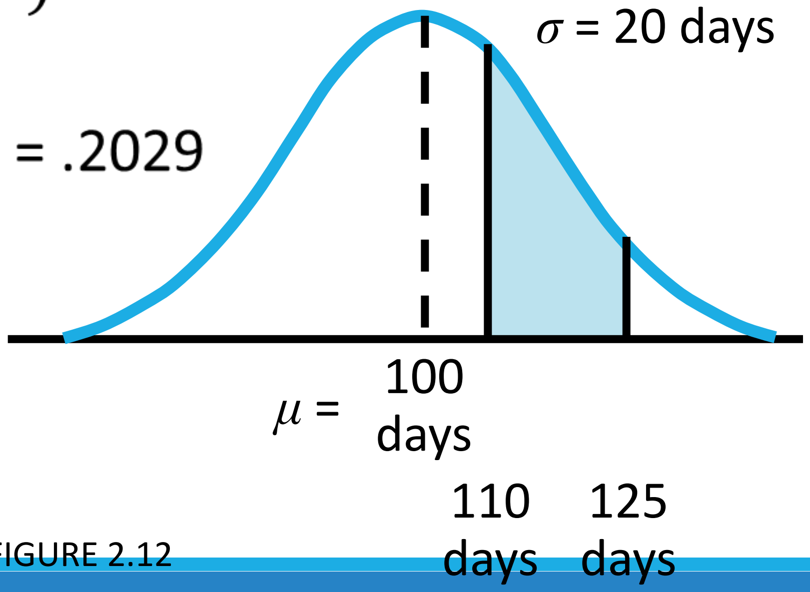


FIGURE 2.12

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Hayne

Probability of co

$P(110 < X < 125)$

$P(110 \le X < 125) = 0.8944 - 0.6915$

$= 0.2029$

The probability of completing between 110 and 125 days is about 20%

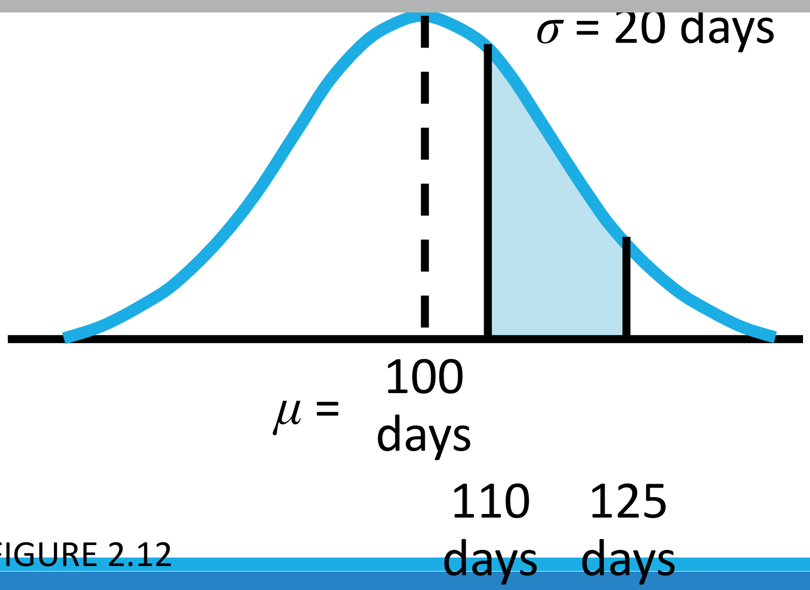

FIGURE 2.12

Calculation procedure to find the probability of the area under the normal curve:

1. First draw the normal curve for the problem, to understand what area under the curve we are looking for.

2. Transform x-values to the standardized random variable, z

$$Z = \frac{X - \mu_X}{\sigma_X}$$

3. Use the standardized normal distribution table to find the probability of the calculated z-value