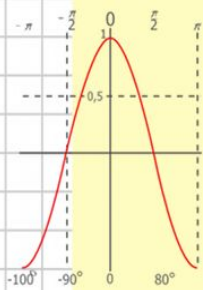
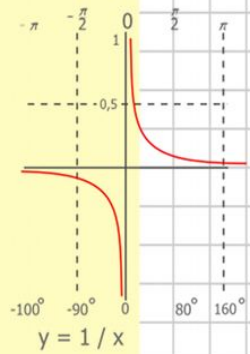
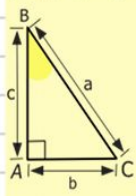
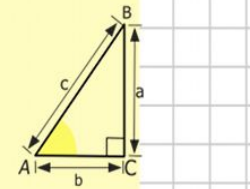
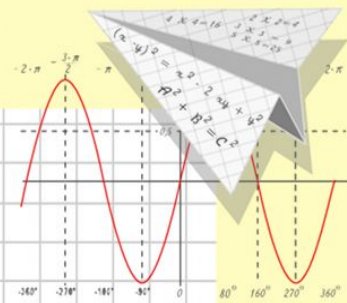
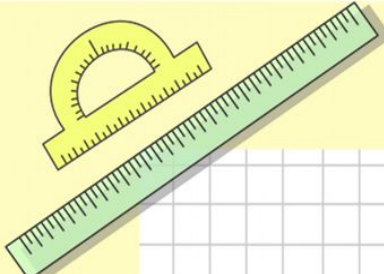


Математик

а

Дополнительные признаки равенства треугольников



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} y = \cos x \\ 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \end{array}$$

Серова Наталья Александровна,
Мурзина Наталья Викторовна,
учителя математики, информатики и ИКТ
г.Омск МОУ «Средняя общеобразовательная школа № 16»

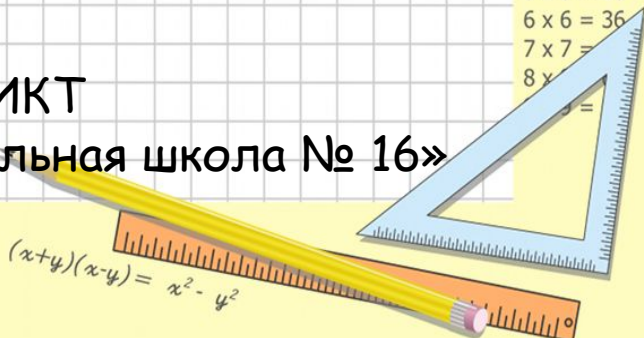
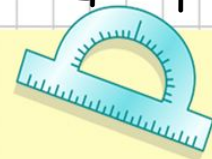
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

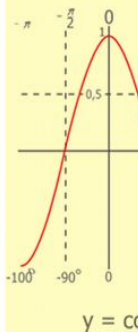
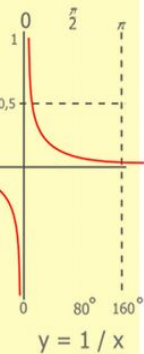
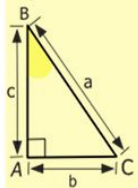
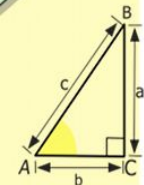
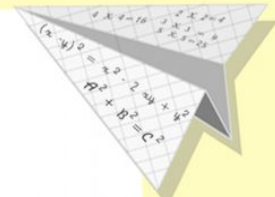
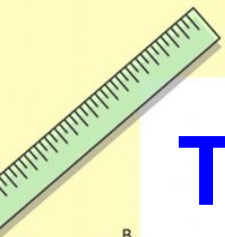
$$(x+y)(x-y) = x^2 - y^2$$



Теорема

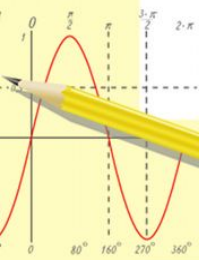
1 Если угол, сторона, противолежащая этому углу, и высота, опущенная на другую сторону, одного треугольника соответственно равны углу, стороне и высоте другого треугольника, то такие треугольники равны.

Для доказательства используются признаки равенства прямоугольных треугольников.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ 105000 \\ \hline \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

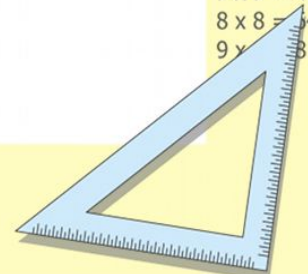
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

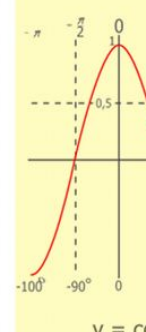
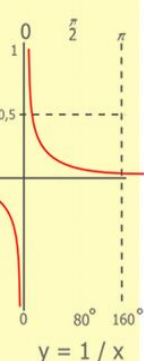
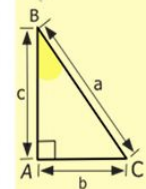
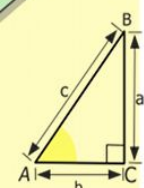
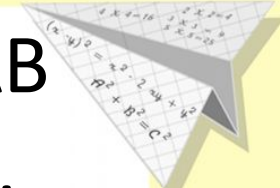
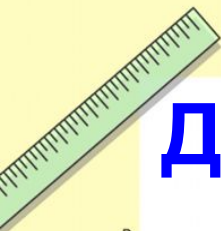
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $\angle C = \angle C_1$, $AB = A_1B_1$, высота AH равна высоте A_1H_1 .

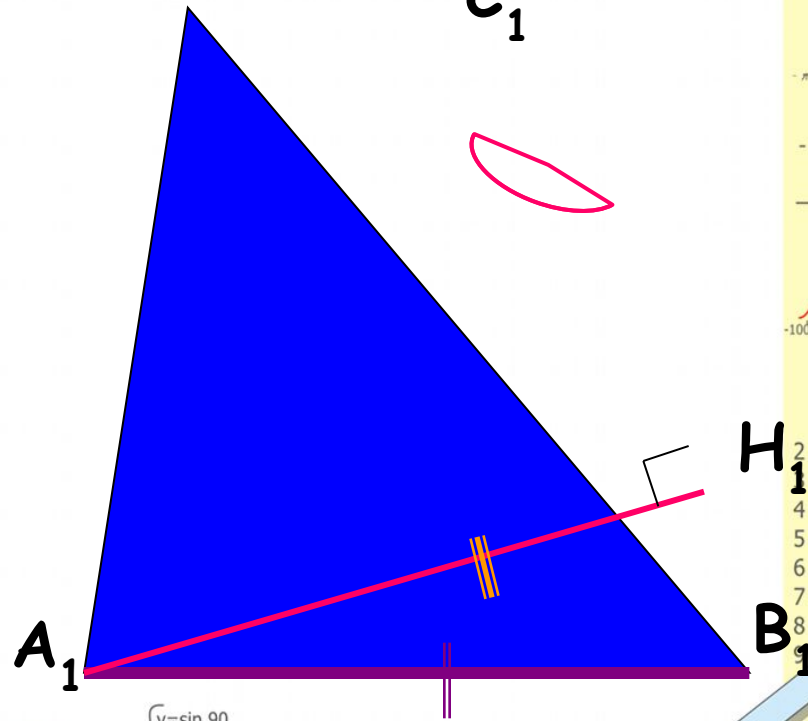
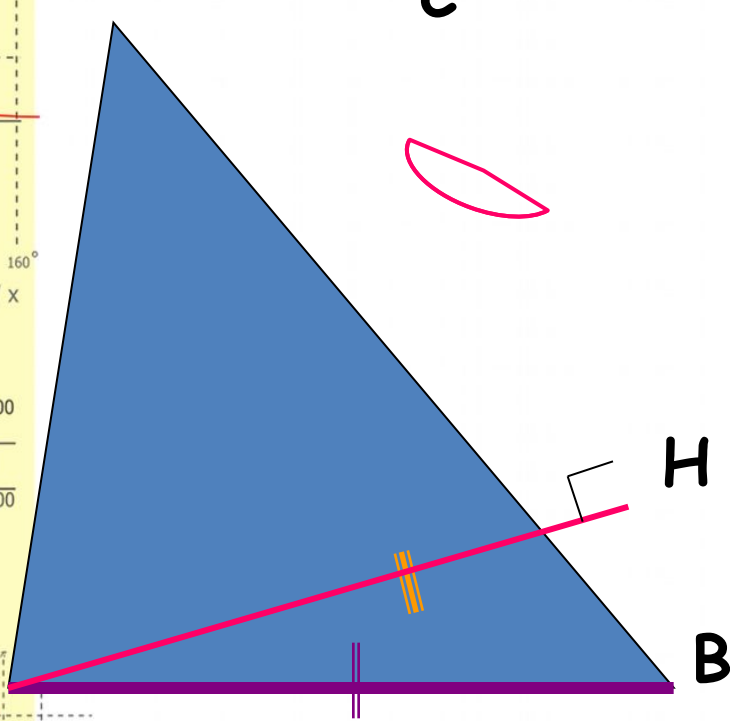
Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$y = 1/x$

$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$2 \times 2 = 4$
 $3 \times 3 = 9$
 $4 \times 4 = 16$
 $5 \times 5 = 25$
 $6 \times 6 = 36$
 $7 \times 7 = 49$
 $8 \times 8 = 64$
 $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

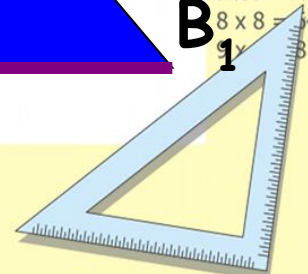
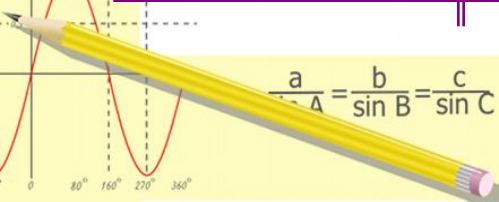
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



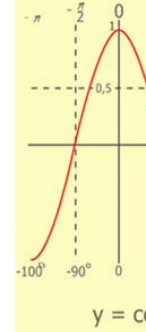
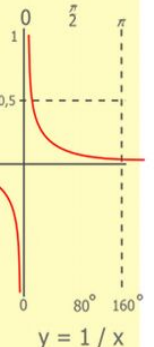
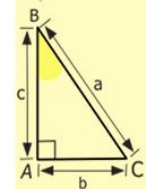
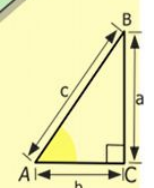
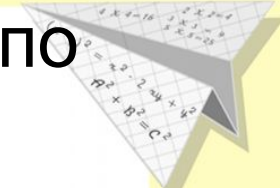
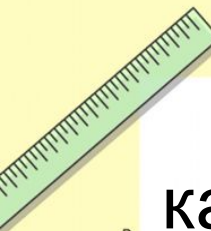
Доказательство:

Прямоугольные $\triangle ABH$ и $\triangle A_1B_1H_1$ равны по катету и гипотенузе. Значит, $\angle B = \angle B_1$.

Учитывая, что $\angle C = \angle C_1$, имеем равенство $\angle A = \angle A_1$. Таким образом, в $\triangle ABC$ и $\triangle A_1B_1C_1$

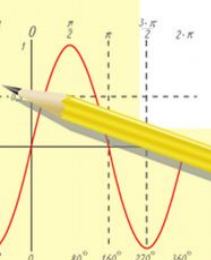
$$AB = A_1B_1, \angle A = \angle A_1, \angle B = \angle B_1.$$

Следовательно, эти треугольники равны по второму признаку равенства треугольников.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



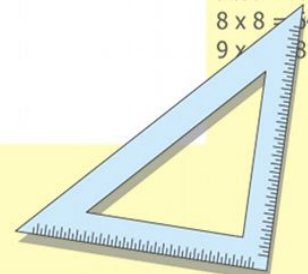
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases} \quad \begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

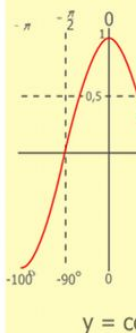
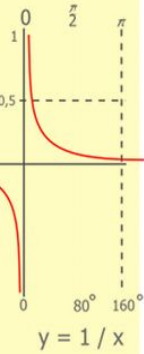
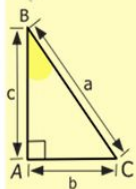
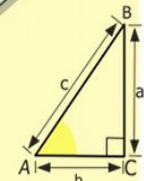
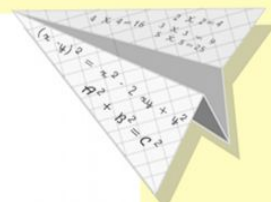
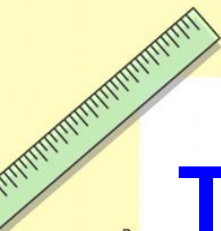
$$(x+y)(x-y) = x^2 - y^2$$



Теорема

2

Если две стороны и медиана, заключенная между ними, одного треугольника соответственно равны двум сторонам и медиане другого треугольника, то такие треугольники равны.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

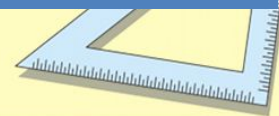


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

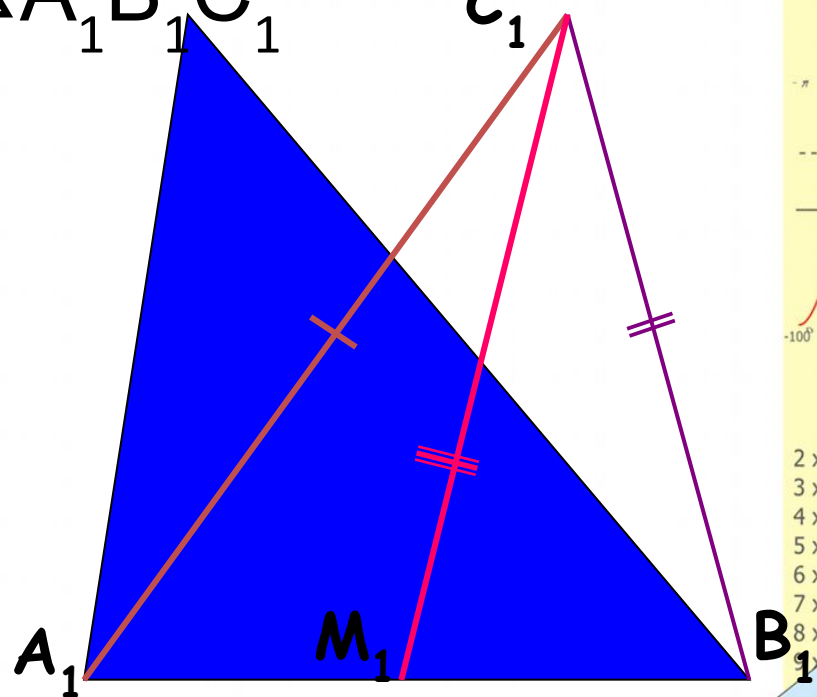
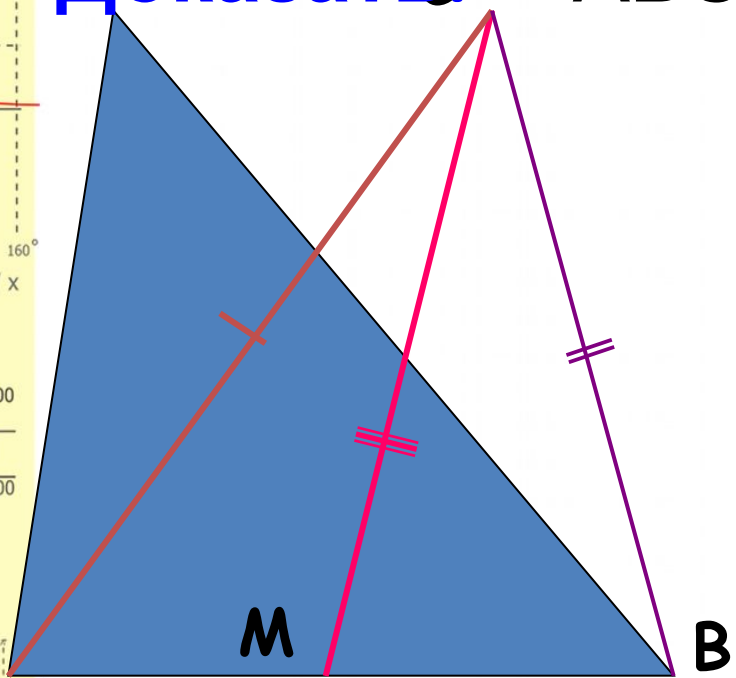
$$(x+y)(x-y) = x^2 - y^2$$

Теорема 8



Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $AC = A_1C_1$, $BC = B_1C_1$, медиана CM равна медиане C_1M_1 .

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

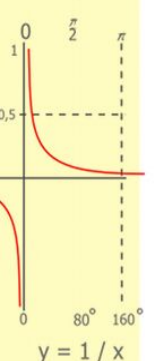
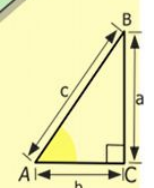
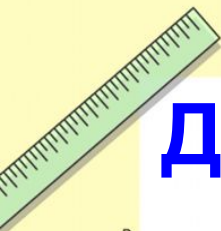
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

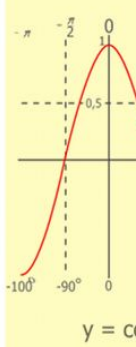
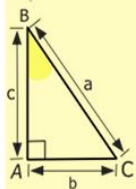
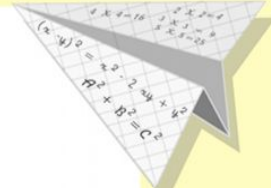
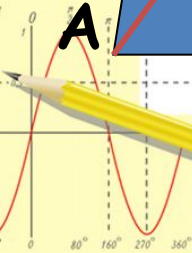
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

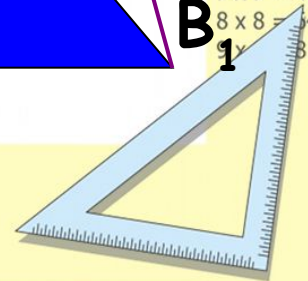
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



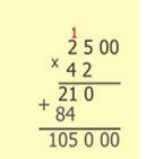
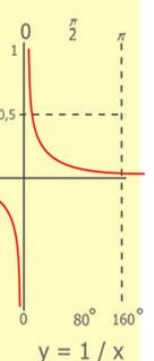
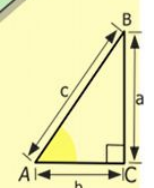
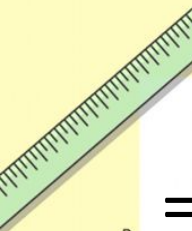
Доказательство

чертеж

Продолжим медианы и отложим отрезки $MD = CM$ и $M_1D_1 = C_1M_1$. Четырехугольники $ACBD$ и $A_1C_1B_1D_1$ — параллелограммы. $\triangle ACD = \triangle A_1C_1D_1$ по трем сторонам. Следовательно, $\angle ACD = \angle A_1C_1D_1$.

Аналогично, $\triangle BCD = \triangle B_1C_1D_1$ по трем сторонам. Следовательно, $\angle BCD = \angle B_1C_1D_1$.

Значит, $\angle C = \angle C_1$ и треугольники ABC и $A_1B_1C_1$ равны по двум сторонам и углу между ними (по первому признаку равенства треугольников).



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

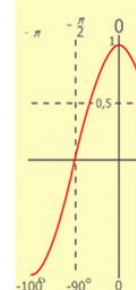
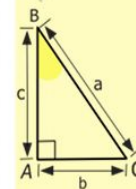
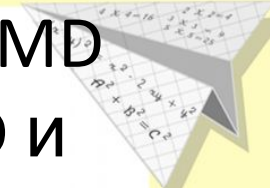
$$\sin 90^\circ = 1$$



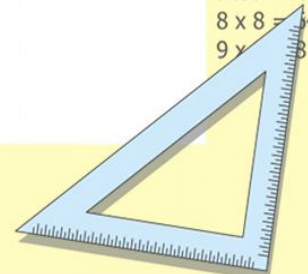
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

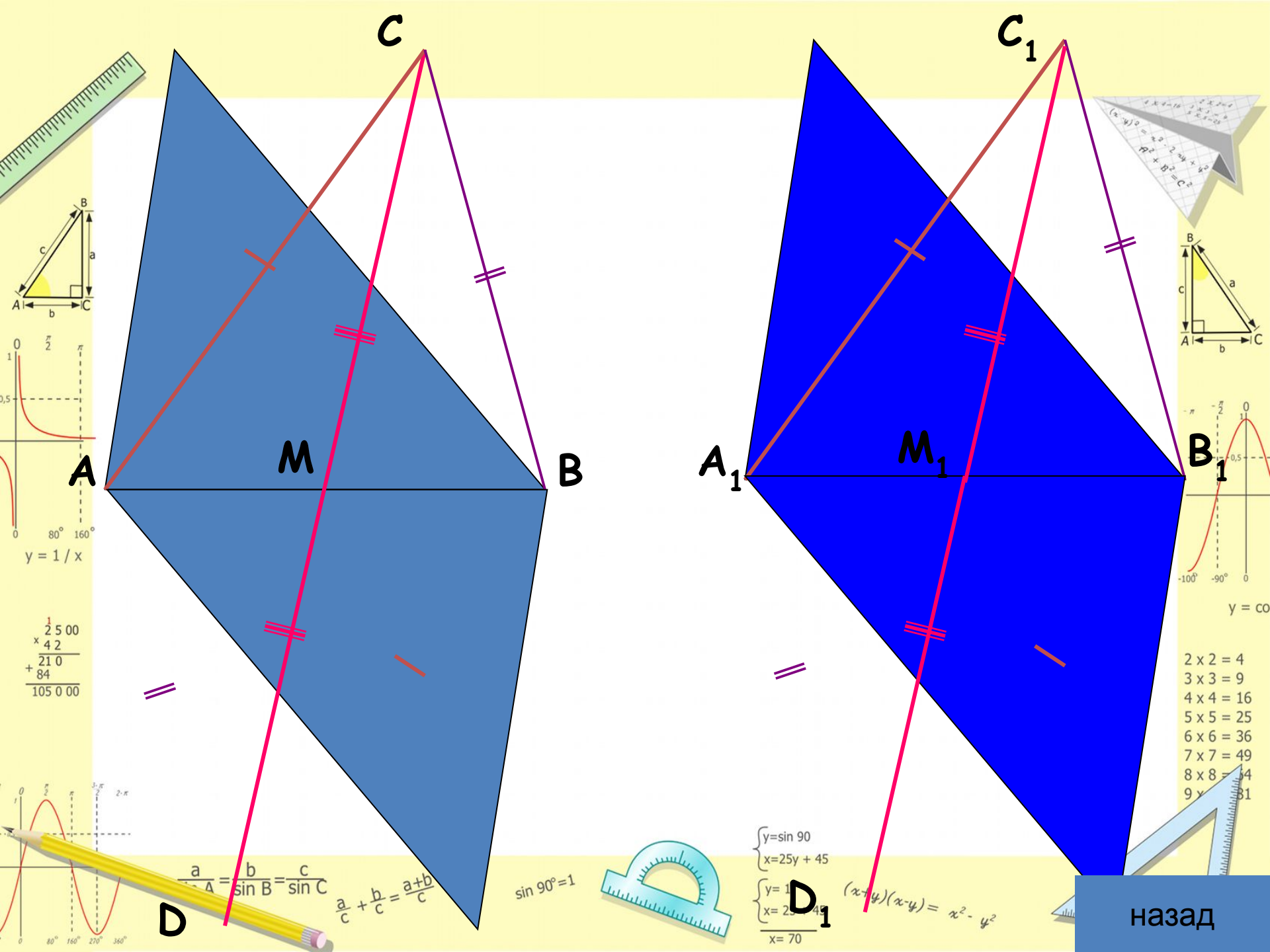
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$





C

C₁

A

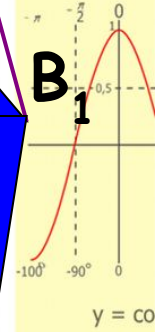
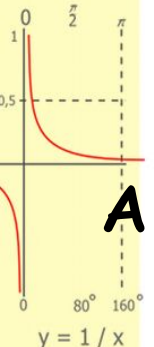
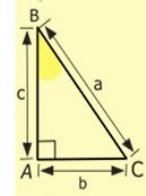
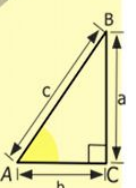
B

M

A₁

B₁

M₁



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 \cdot 1 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

D

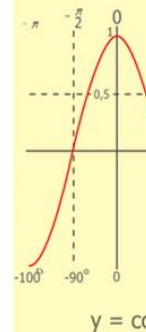
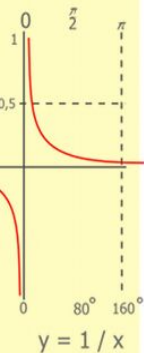
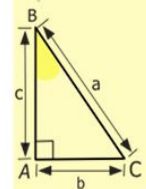
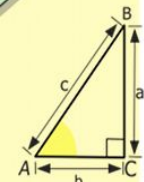
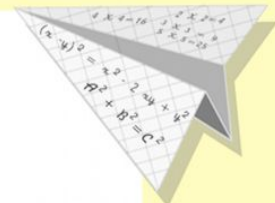
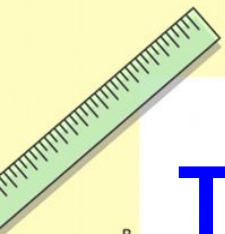
D₁

назад

Теорема

3

Если сторона и две медианы, проведенные к двум другим сторонам, одного треугольника соответственно равны стороне и двум медианам другого треугольника, то такие треугольники равны.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

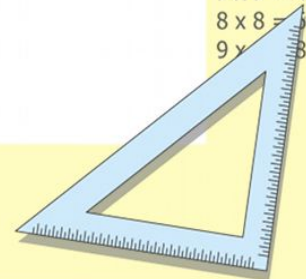
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

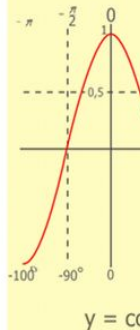
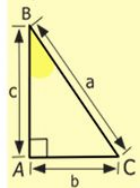
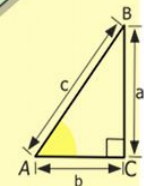
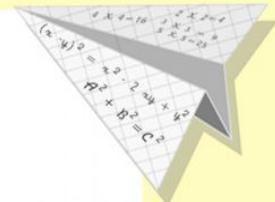
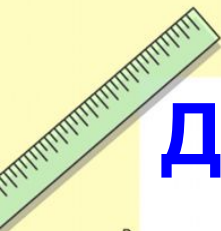
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $AB = A_1B_1$,
 медианы $AM = A_1M_1$, $BK = B_1K_1$.

Доказать: $\triangle ABC = \triangle A_1B_1C_1$

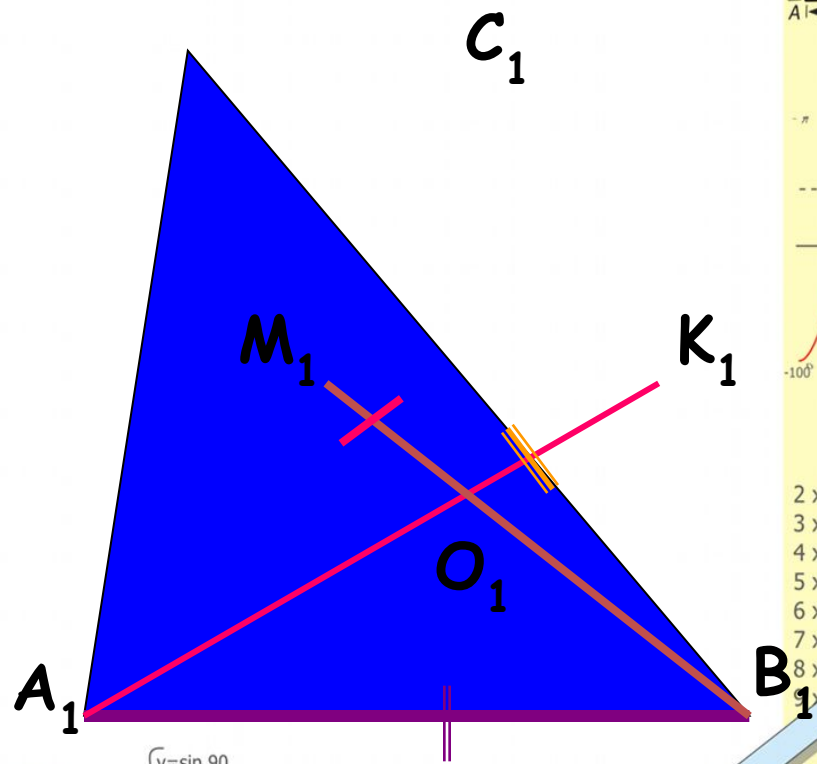
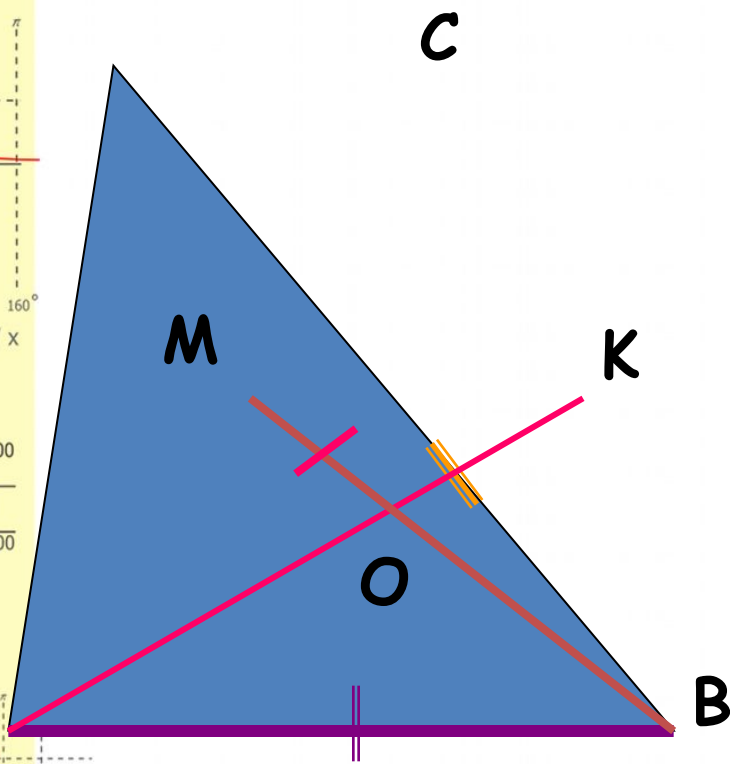


$y = 1/x$

$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$y = \cos$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

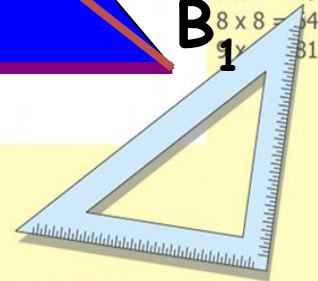


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

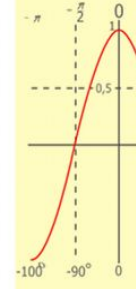
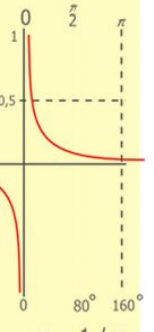
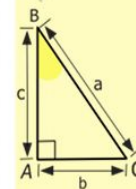
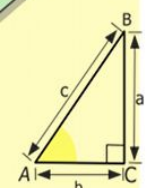
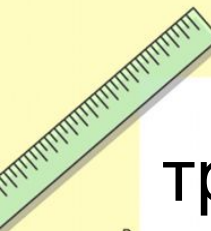


Доказательство:

Точки O и O_1 пересечения медиан данных треугольников делят медианы в отношении $2:1$, считая от вершины. Значит, $\triangle ABO = \triangle A_1B_1O_1$ по трем сторонам. Следовательно, $\angle BAO = \angle B_1A_1O_1$, значит, $\triangle ABM = \triangle A_1B_1M_1$ равны по двум сторонам и углу между ними. Поэтому $\angle ABC = \angle A_1B_1C_1$.

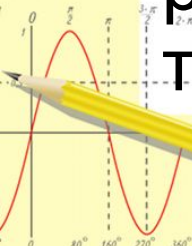
Аналогично доказывается, что $\angle BAC = \angle B_1A_1C_1$.

Таким образом, треугольники $\triangle ABC$ и $\triangle A_1B_1C_1$ равны по стороне и двум прилежащим к ней углам. Следовательно, $\triangle ABC = \triangle A_1B_1C_1$ равны по второму признаку равенства треугольников.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

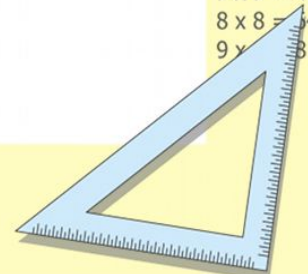
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

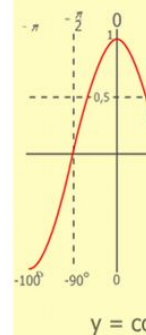
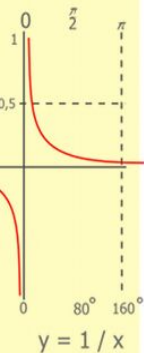
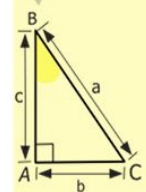
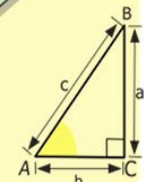
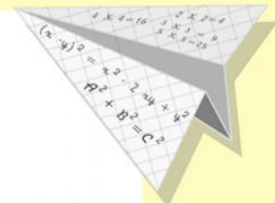
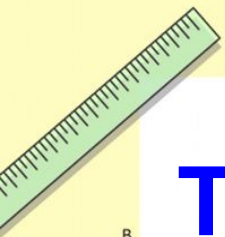
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



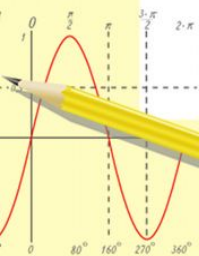
Теорема 4

Если две стороны и биссектриса, заключенная между ними, одного треугольника соответственно равны двум сторонам и биссектрисе, заключенной между ними, другого треугольника, то такие треугольники равны.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

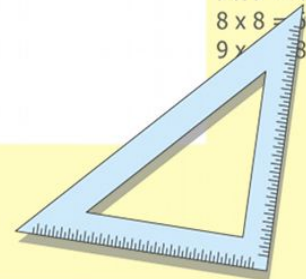
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

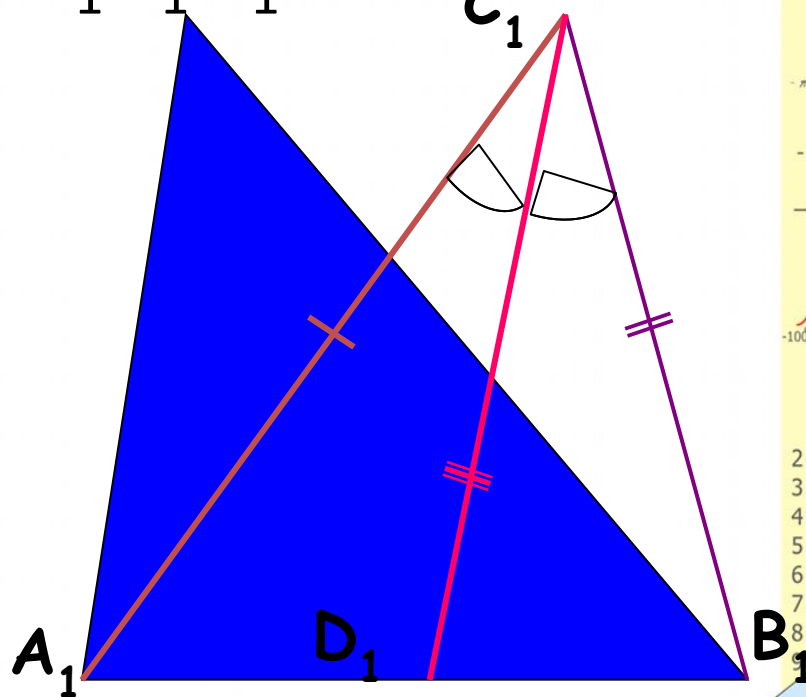
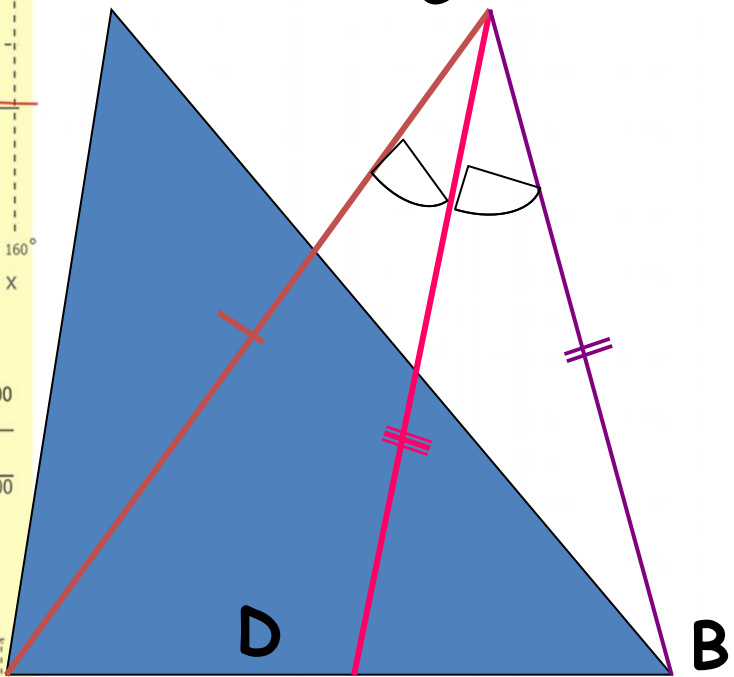
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



• **Дано:** $\triangle ABC$ и $\triangle A_1B_1C_1$, $AC = A_1C_1$, $BC = B_1C_1$, биссектриса CD равна биссектрисе C_1D_1 .

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

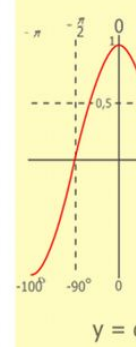
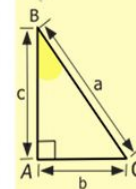
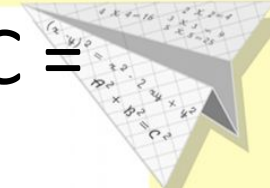
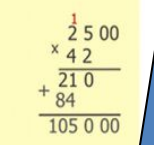
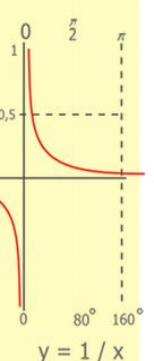
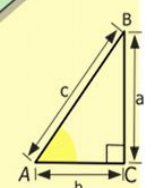
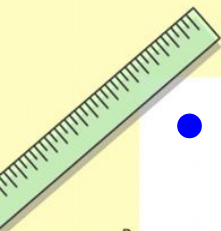
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

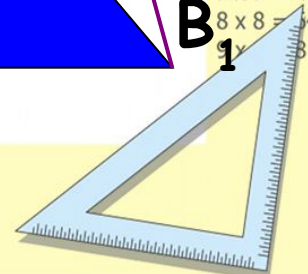
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Доказательство

чертеж

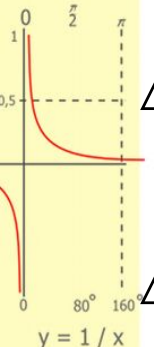
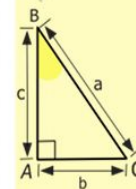
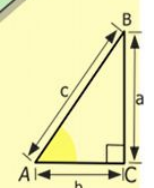
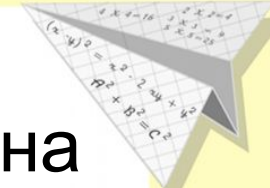
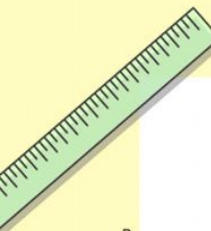
Продолжим стороны AC и A_1C_1 и отложим на их продолжениях отрезки $CE = BC$ и $C_1E_1 = B_1C_1$.

Тогда $BE = CD \frac{AE}{AC}$, $B_1E_1 = C_1D_1 \frac{A_1E_1}{A_1C_1}$

$\triangle BCE = \triangle B_1C_1E_1$ по трем сторонам. Значит, $\angle E = \angle E_1$ и $BE = B_1E_1$.

$\triangle ABE = \triangle A_1B_1E_1$ по двум сторонам и углу между ними. Значит, $AB = A_1B_1$.

Таким образом, $\triangle ABC = \triangle A_1B_1C_1$ по трем сторонам (3 признак равенства треугольников).



$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

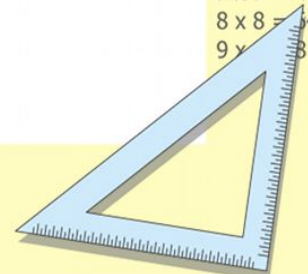
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

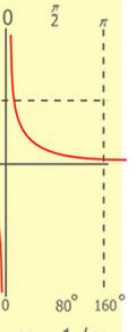
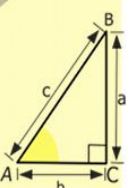
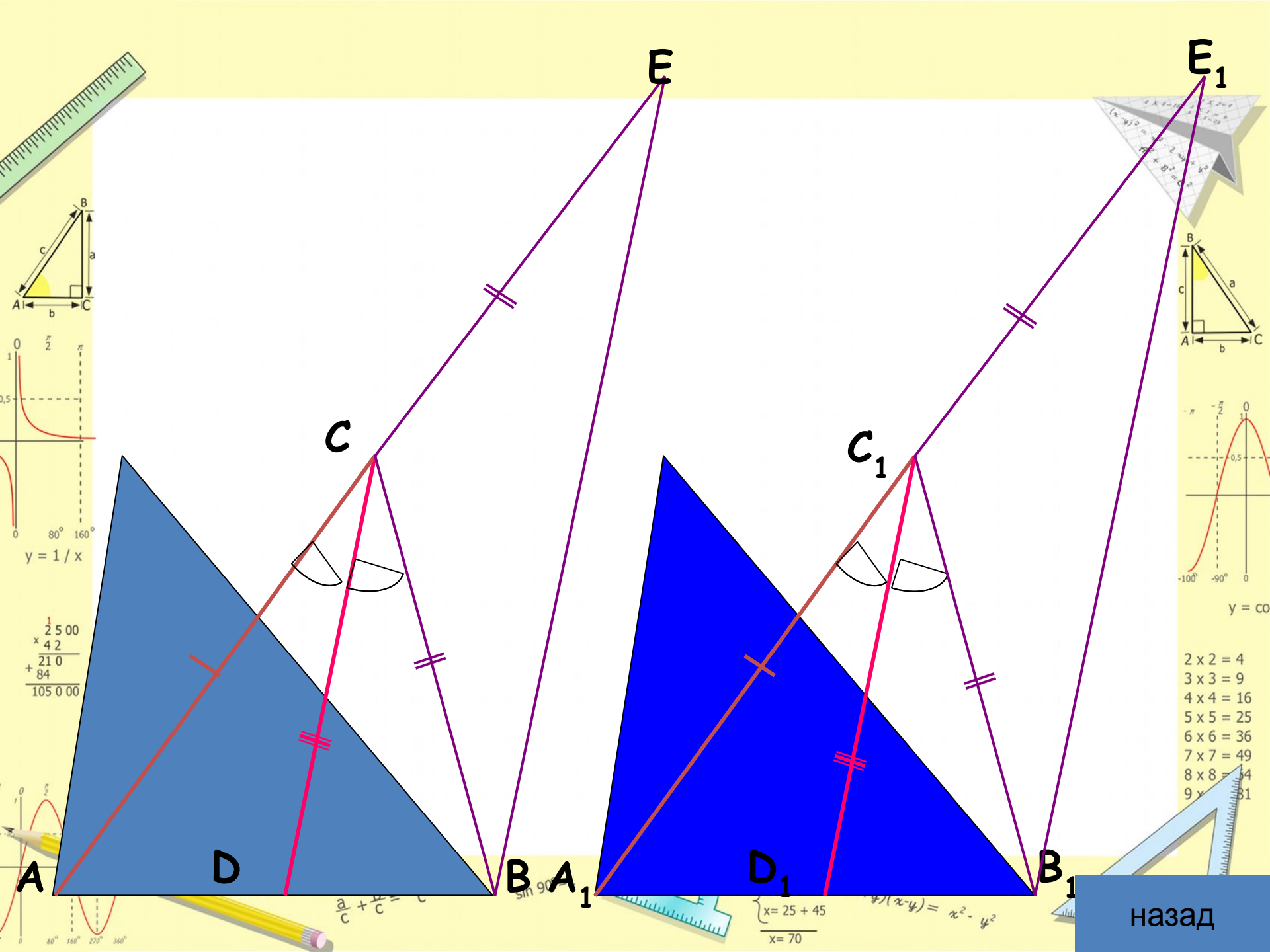
$$\sin 90^\circ = 1$$



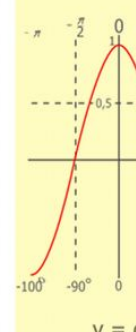
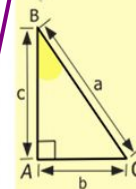
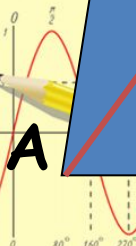
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases} \quad \begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$





1	2 5 00
x	4 2
+	21 0
	84
	105 0 00



2 x 2 =	4
3 x 3 =	9
4 x 4 =	16
5 x 5 =	25
6 x 6 =	36
7 x 7 =	49
8 x 8 =	64
9 x 9 =	81

$$\frac{a}{c} + \frac{b}{c} = 1$$

$$\sin 90^\circ = 1$$

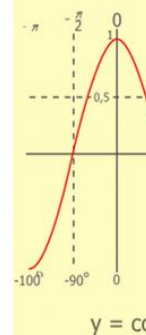
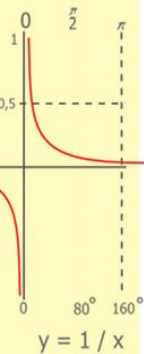
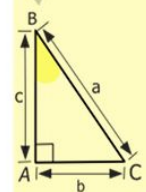
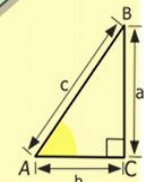
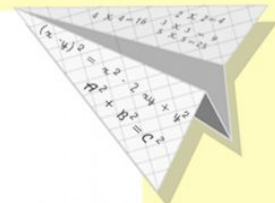
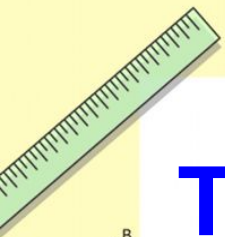
$$\begin{cases} x = 25 + 45 \\ x = 70 \end{cases}$$

$$y(x-y) = x^2 - y^2$$

назад

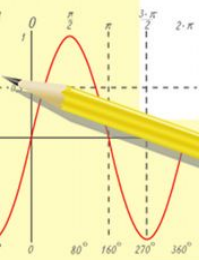
Теорема 5

Два треугольника равны, если сторона, медиана и высота, проведенные к другой стороне, одного треугольника соответственно равны стороне, медиане и высоте другого треугольника.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

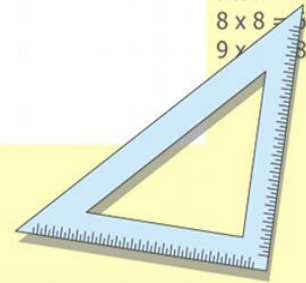
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

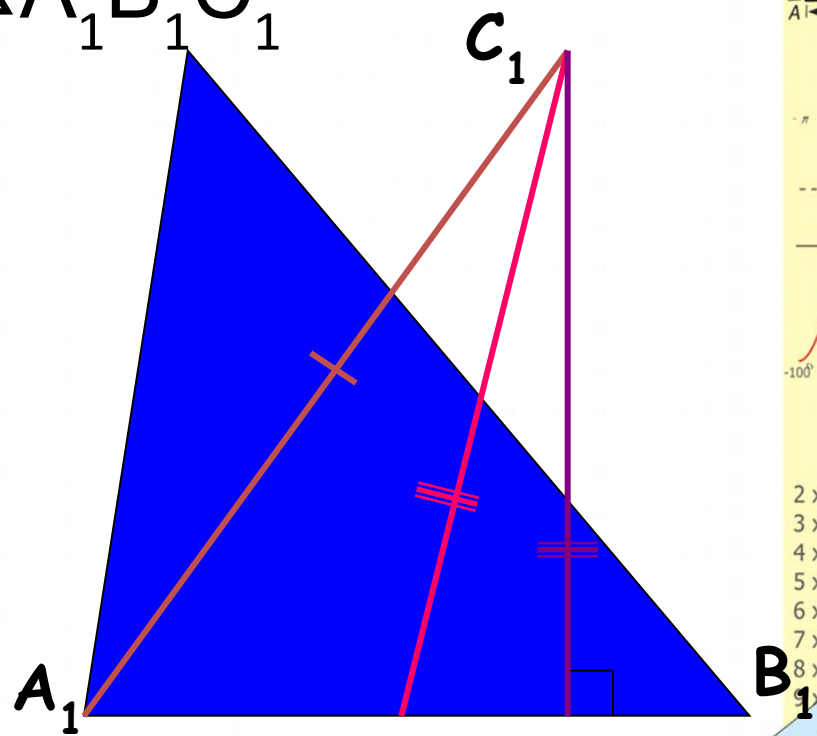
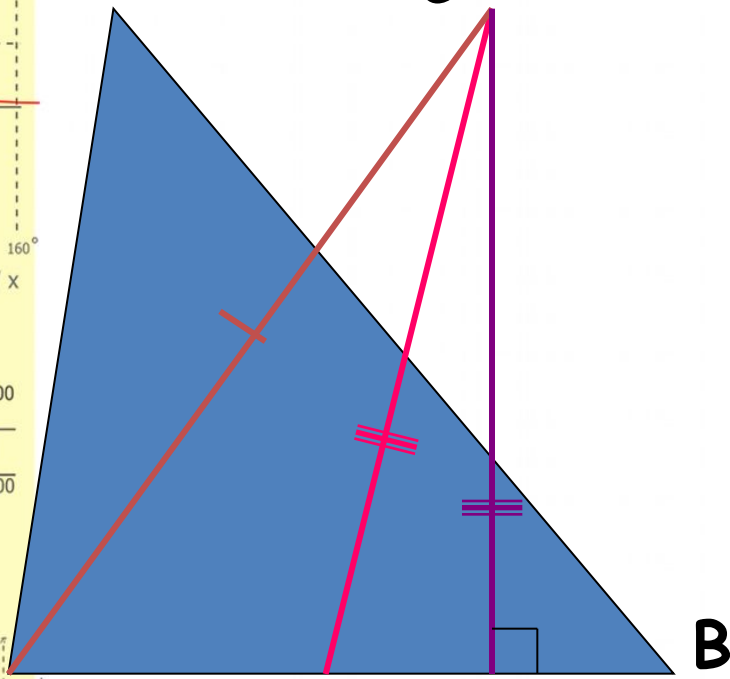
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $AC = A_1C_1$, $AC = A_1C_1$, медианы CM и C_1M_1 равны, высоты CH и C_1H_1 равны.

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

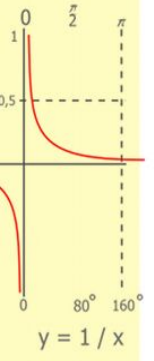
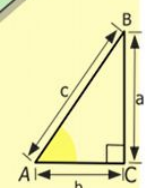
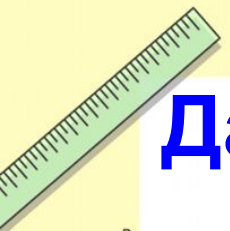
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$\sin 90^\circ = 1$

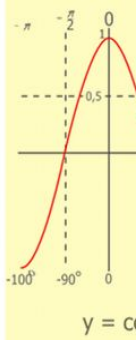
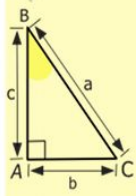
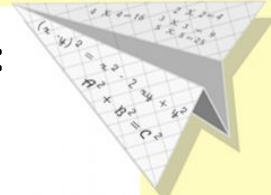
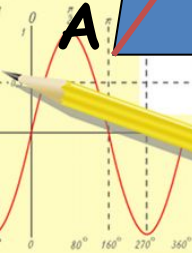
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

M_1 H_1

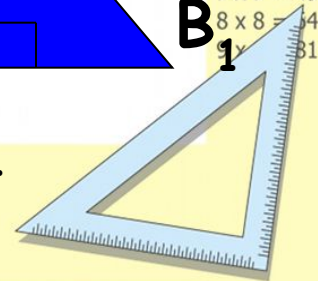
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

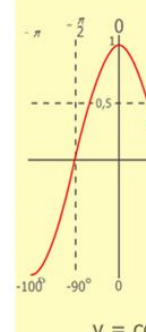
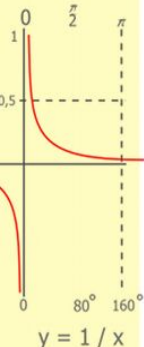
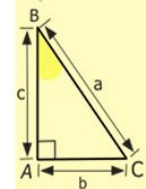
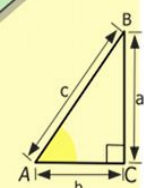


- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Доказательство:

Прямоугольные $\triangle ACH = \triangle A_1C_1H_1$ по гипотенузе и катету. Следовательно, $\angle A = \angle A_1$ и $AH = A_1H_1$. Прямоугольные треугольники $\triangle CMH = \triangle C_1M_1H_1$ по гипотенузе и катету. Следовательно, $MH = M_1H_1$, откуда $AM = A_1M_1$, значит, $AB = A_1B_1$. Таким образом, $\triangle ABC = \triangle A_1B_1C_1$ по двум сторонам и углу между ними (по первому признаку равенства треугольников).



$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

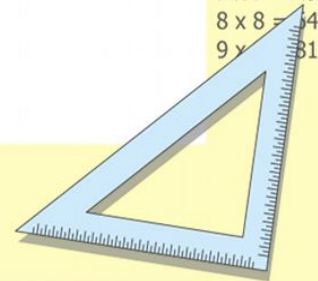
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

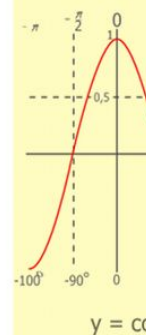
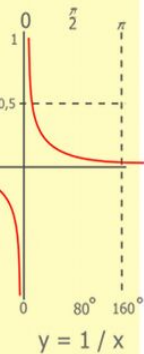
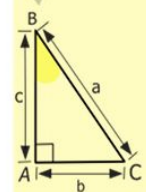
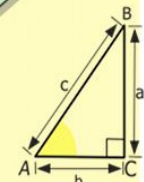
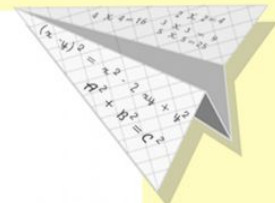
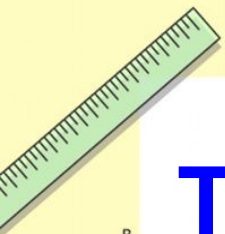
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



Теорема 6

Два треугольника равны, если медиана и два угла на которые делит угол медиана, одного треугольника соответственно равны медиане и двум углам, на которые делит медиана угол другого треугольника.



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ \times 42 \\ \hline 2100 \\ + 84 \\ \hline 105000 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

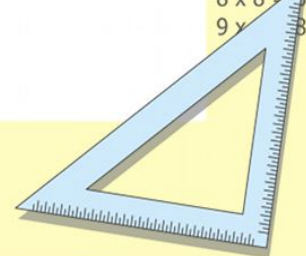
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

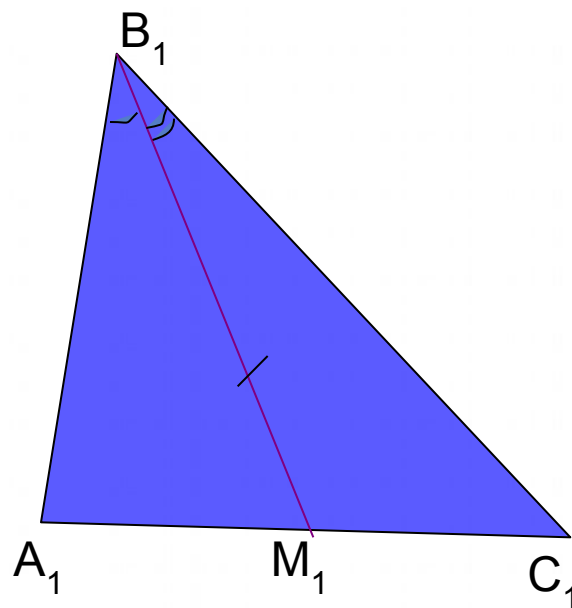
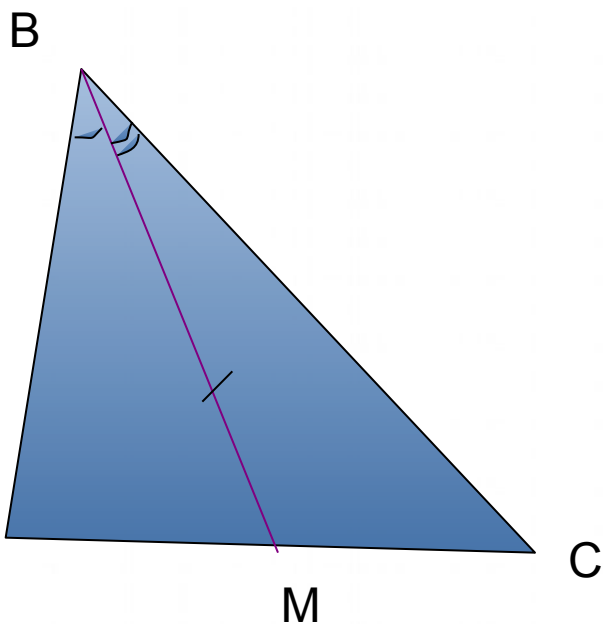
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $BM = B_1M_1$,
 $\angle ABM = \angle A_1B_1M_1$, $\angle CBM = \angle C_1B_1M_1$.

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

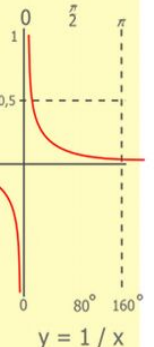
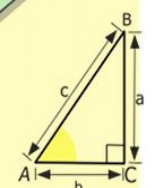
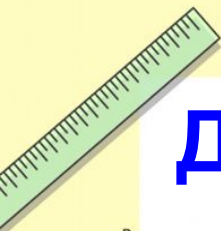
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

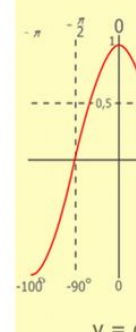
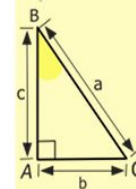
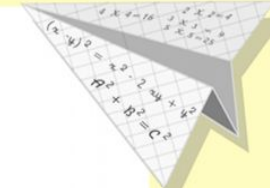
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

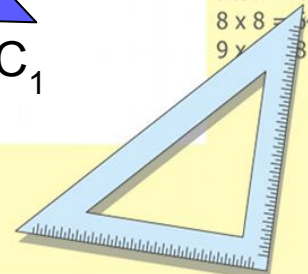
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Доказательство:

В данных треугольниках удвоим медианы $BM=MD$ и $B_1M_1=M_1D_1$.

1. $\triangle AMD = \triangle CMB$, $\triangle A_1M_1D_1 = \triangle C_1M_1B_1$ (по 1 признаку)

Из равенства этих треугольников следуют равенства:

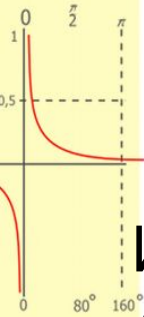
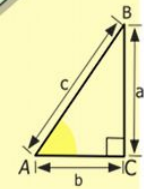
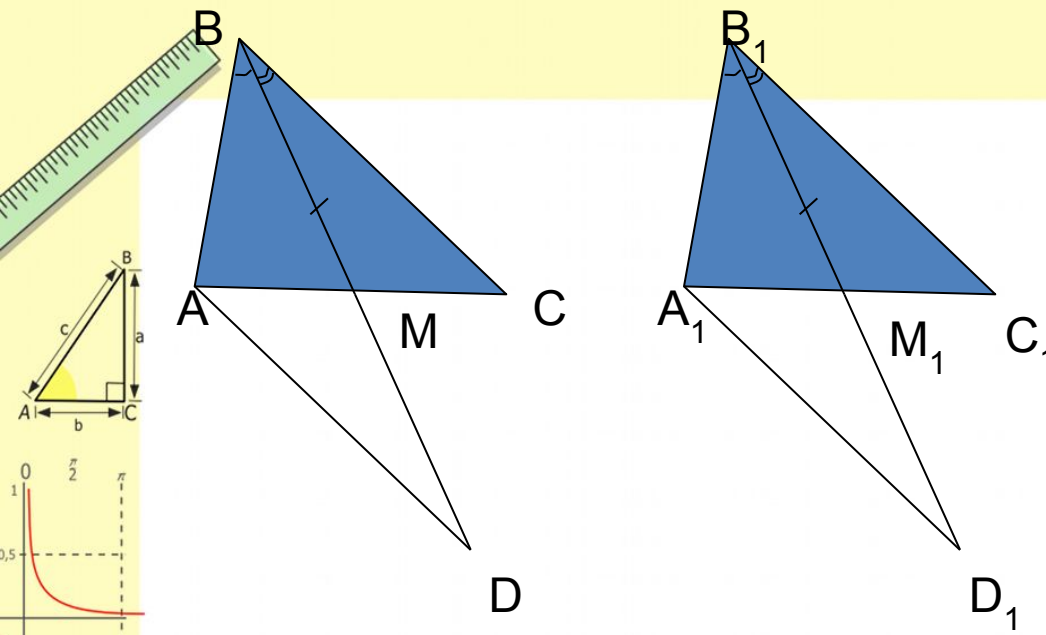
$AD=BC$, $A_1D_1=B_1C_1$ и $\angle ADM = \angle CBM$, $\angle A_1D_1M_1 = \angle C_1B_1M_1$

2. $\triangle ABD = \triangle A_1B_1D_1$ (по 2 признаку)

Из равенства этих треугольников следуют равенства:

$AB=A_1B_1$, а значит, $BC=AD=B_1C_1=A_1D_1$

3. $\triangle ABC = \triangle A_1B_1C_1$ (по первому признаку равенства треугольников)



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 4\ 2 \\ \hline 21\ 0 \\ + 84 \\ \hline 105\ 0\ 00 \end{array}$$



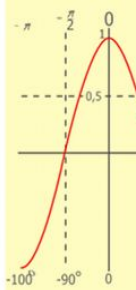
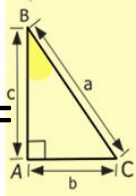
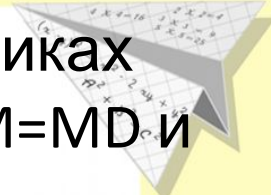
$$\frac{a}{c} = \frac{\sin B}{\sin C} \quad \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



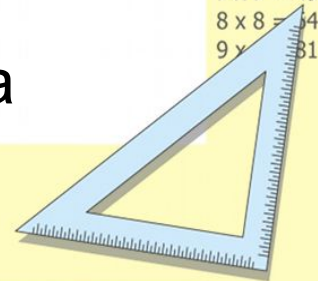
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



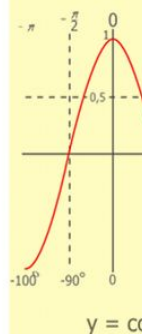
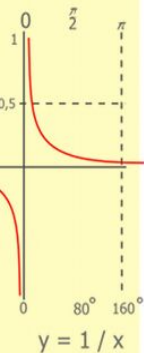
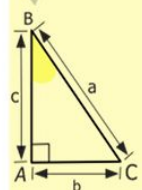
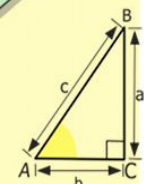
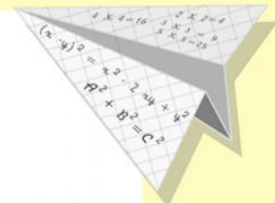
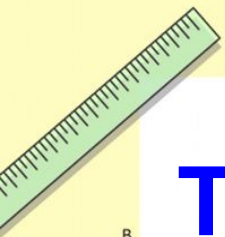
$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



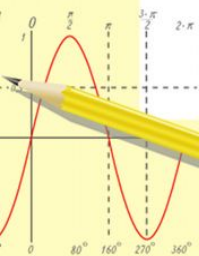
Теорема 7

Два треугольника равны, если сторона, и две высоты, опущенные на две другие стороны, одного треугольника соответственно равны стороне и двум высотам, опущенным на две другие стороны другого треугольника.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

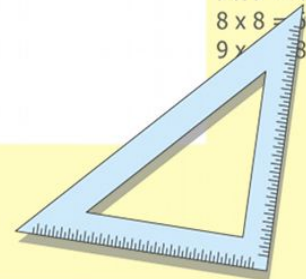
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

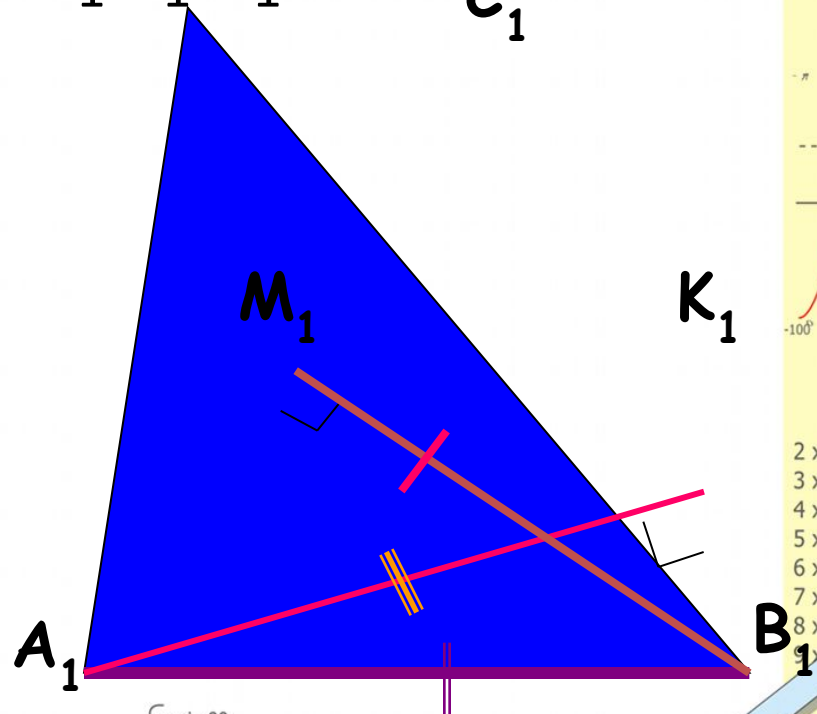
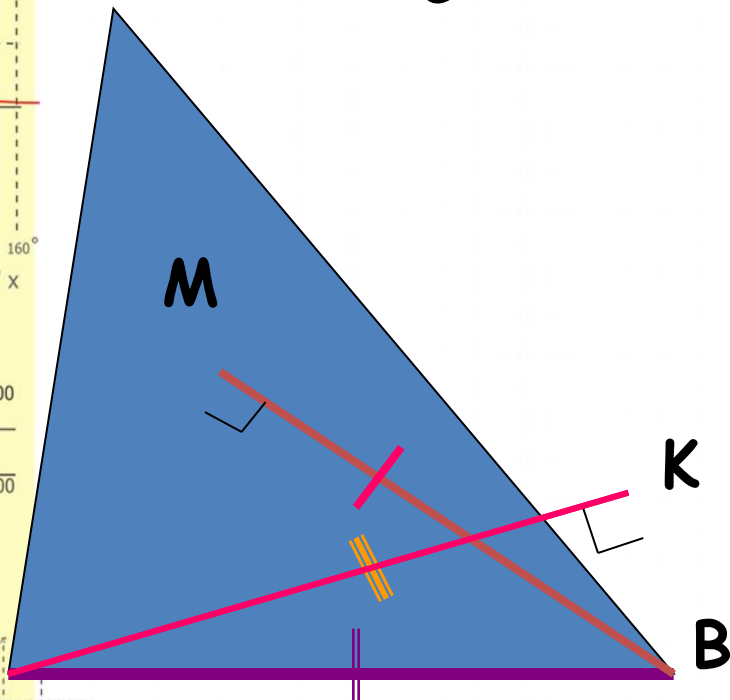
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $AB = A_1B_1$,
 высота AM равна высоте A_1M_1 , высота
 BK равна высоте B_1K_1 .

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

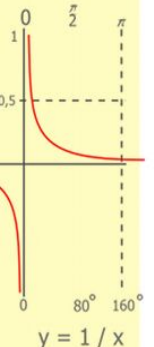
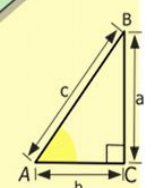
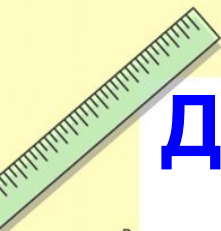
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

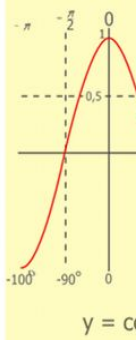
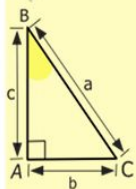
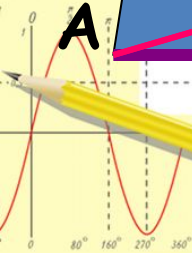
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

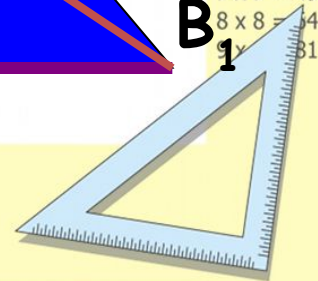
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



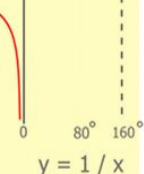
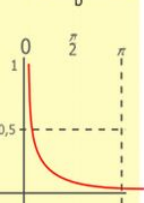
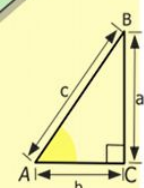
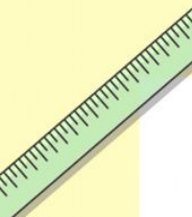
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Доказательство:

Из равенства прямоугольных треугольников $\triangle AMB = \triangle A_1M_1B_1$, $\triangle ВКА = \triangle В_1К_1А_1$ (по катету и гипотенузе) следует равенство углов: $\angle ВАС = \angle В_1А_1С_1$, $\angle АВС = \angle А_1В_1С_1$.

Поэтому $\triangle ABC = \triangle A_1B_1C_1$ по стороне ($AB = A_1B_1$) и двум прилежащим к ней углам (по второму признаку равенства треугольников).



$$\begin{array}{r} 1 \\ \times 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

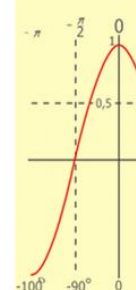
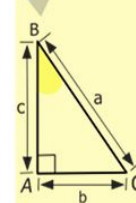
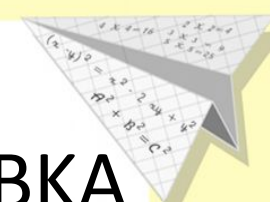


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

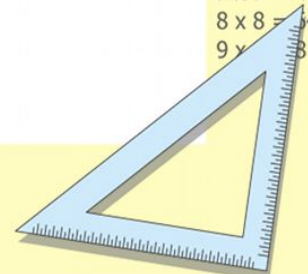
$$\underline{x = 70}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

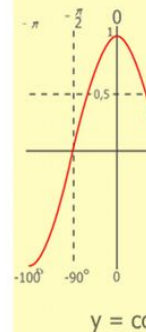
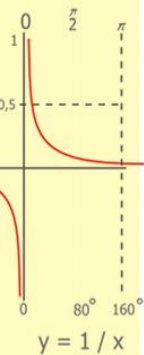
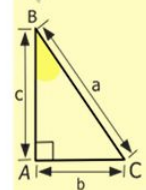
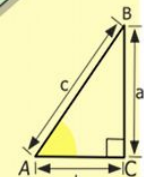
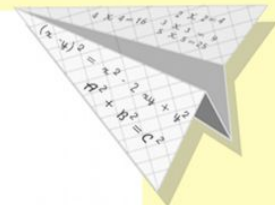
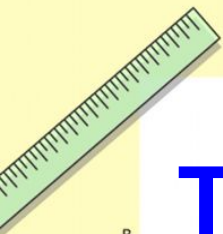
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Теорема

8

Два треугольника равны, если три медианы одного треугольника соответственно равны трем медианам другого.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

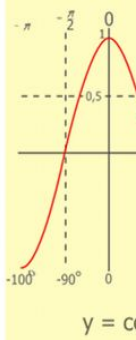
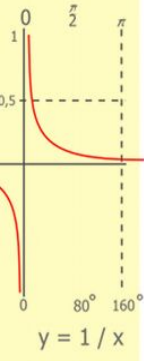
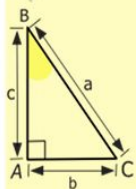
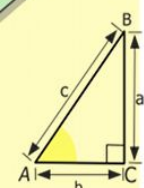
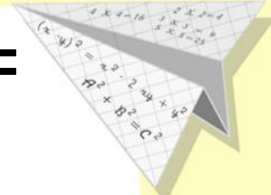
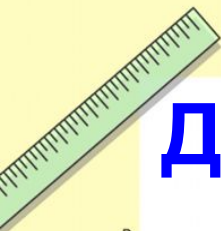
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



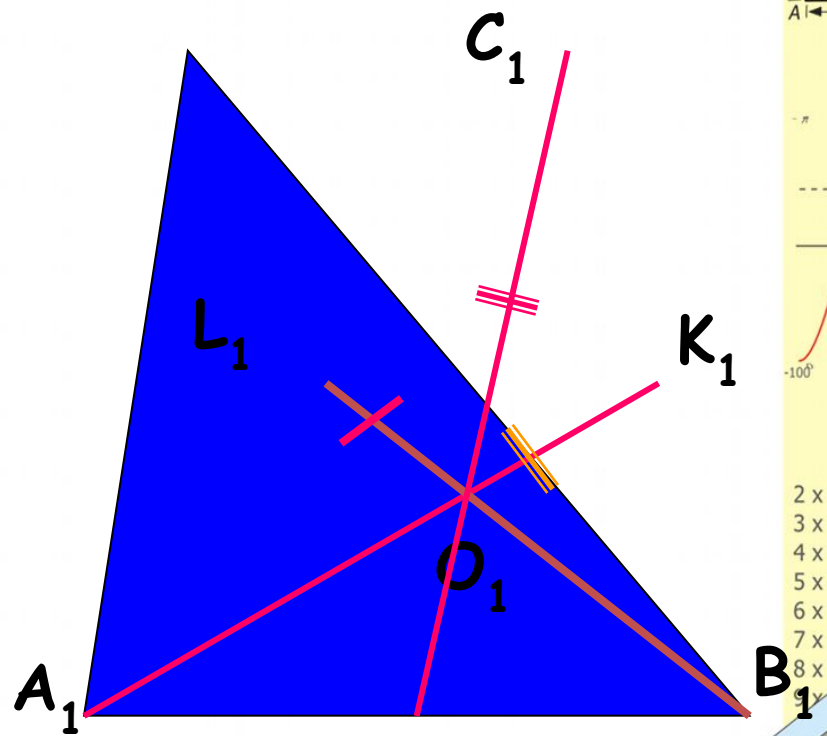
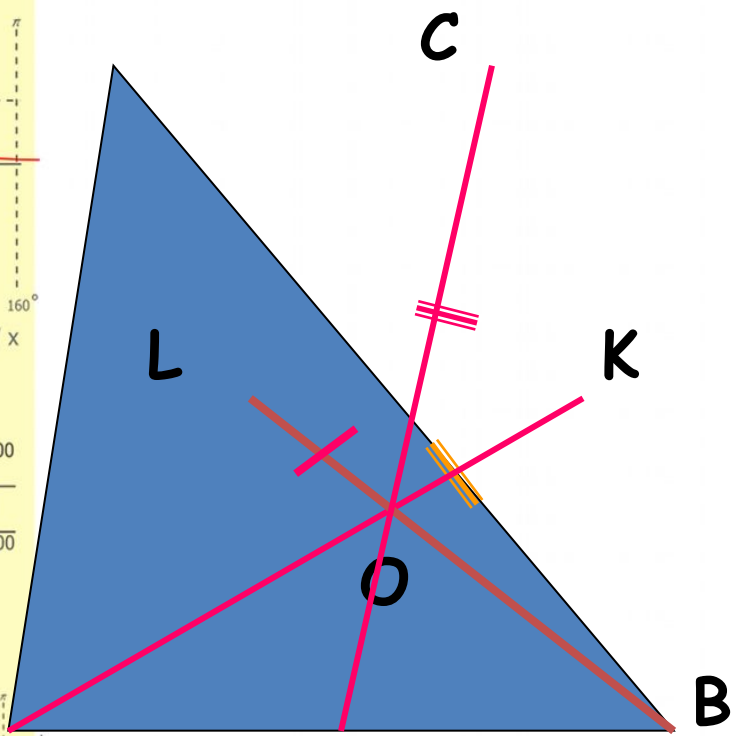
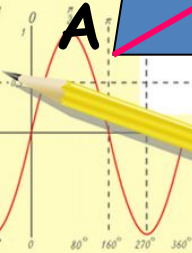
Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, медианы $AK = A_1K_1$, $BL = B_1L_1$, $CM = C_1M_1$.

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

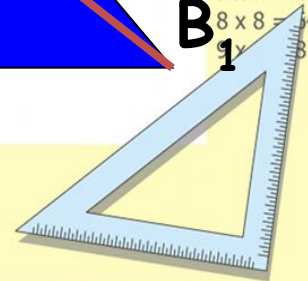
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



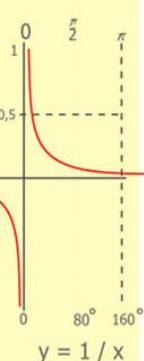
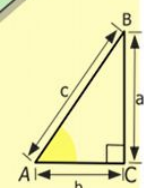
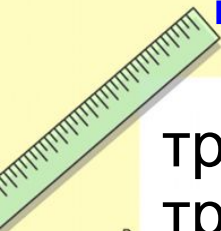
Доказательство:

Пусть O и O_1 — точки пересечения медиан данных треугольников. Заметим, что медианы OM и O_1M_1 треугольников $\triangle ABO$ и $\triangle A_1B_1O_1$ равны, так как они составляют одну треть часть соответствующих медиан данных треугольников. Аналогично равны AO и A_1O_1 , BO и B_1O_1 , так как они составляют две третьих соответствующих медиан данных треугольников.

По признаку равенства треугольников, доказанному нами под номером [2](#), $\triangle ABO = \triangle A_1B_1O_1$, значит, $AB = A_1B_1$.

Аналогично доказывается, что $BC = B_1C_1$ и $AC = A_1C_1$.

Таким образом, $\triangle ABC$ и $\triangle A_1B_1C_1$ равны по трем сторонам (по третьему признаку равенства треугольников) .



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

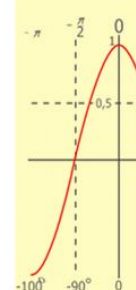
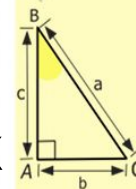
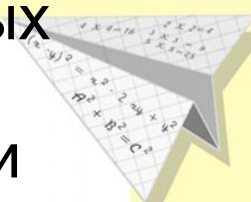


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

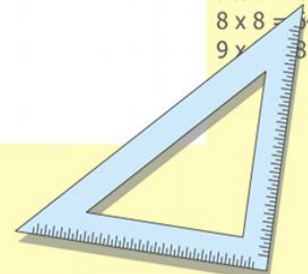
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

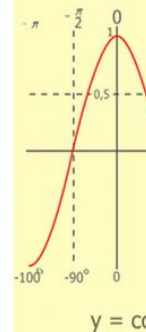
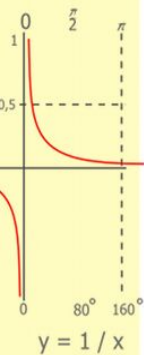
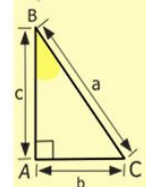
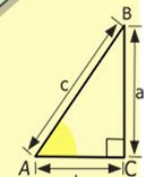
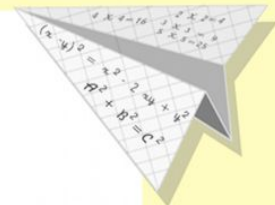
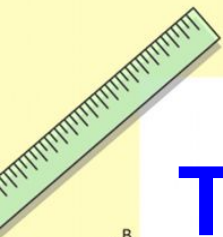
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Теорема

9

Два треугольника равны, если три высоты одного треугольника соответственно равны трем высотам другого треугольника.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

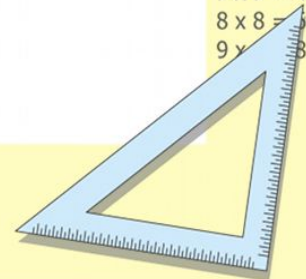
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

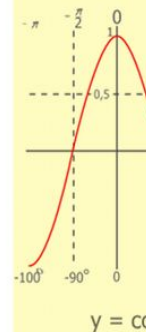
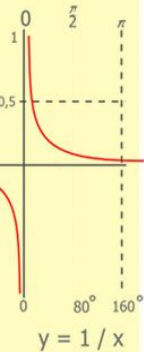
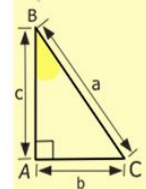
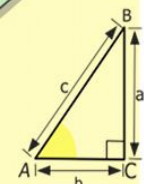
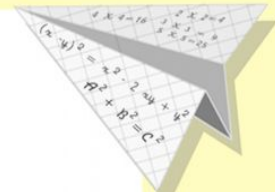
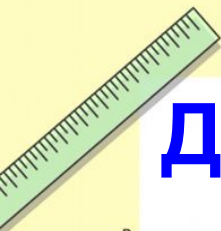
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



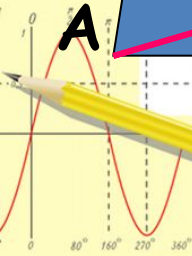
Дано: $\triangle ABC$ и $\triangle A_1B_1C_1$, $AB = A_1B_1$,
 ВЫСОТЫ $AH = A_1H_1$, $BG = B_1G_1$, $CF = C_1F_1$.

Доказать: $\triangle ABC = \triangle A_1B_1C_1$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

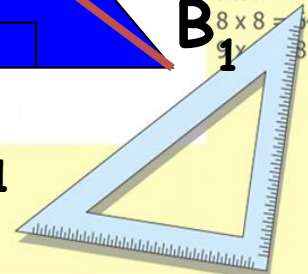
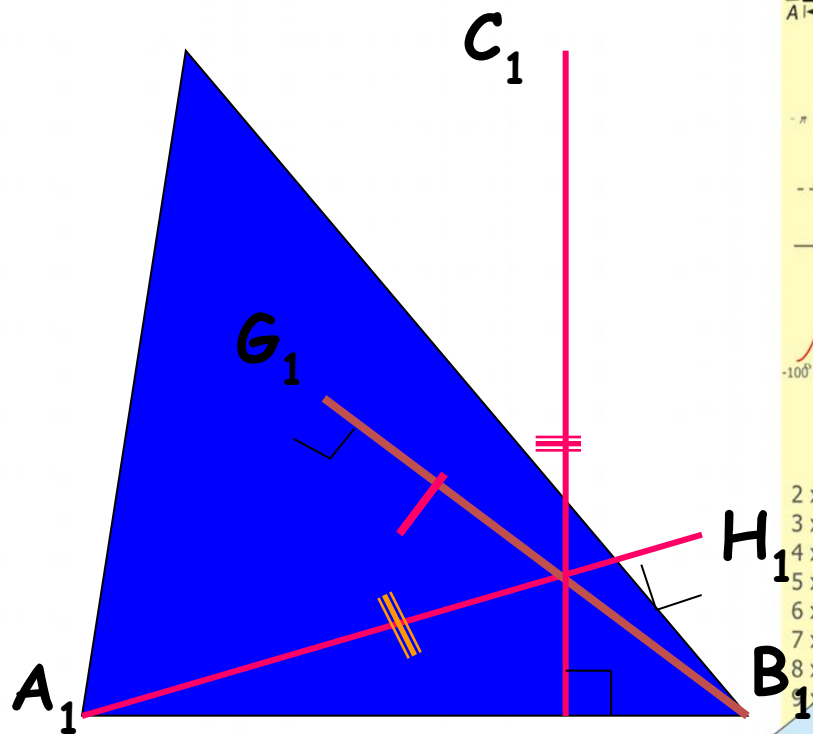
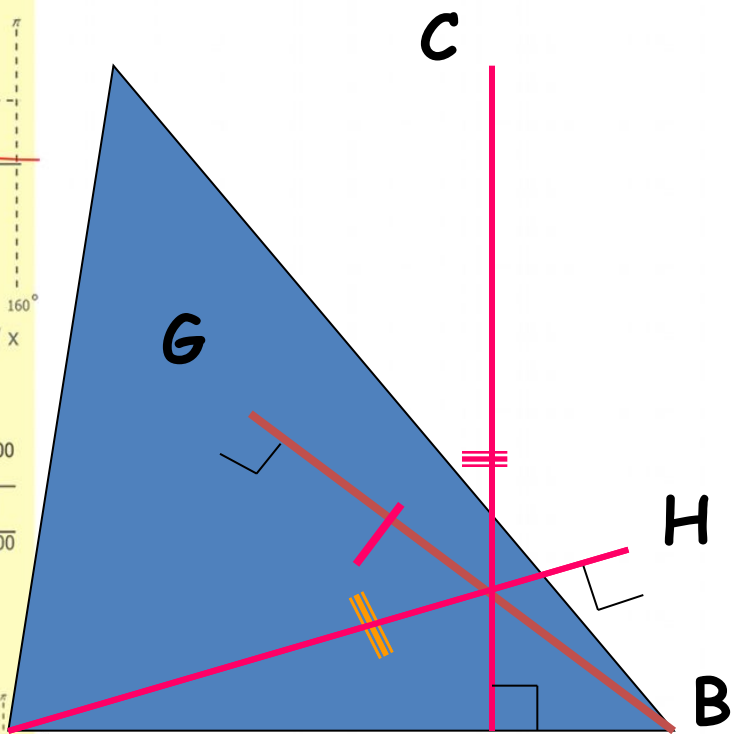
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



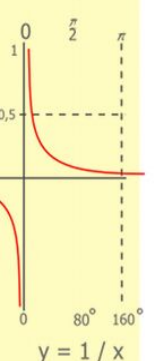
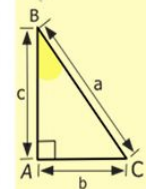
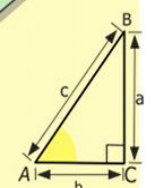
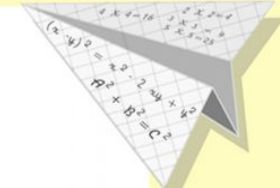
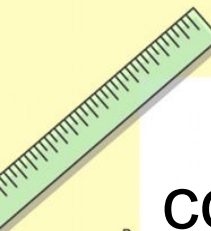
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



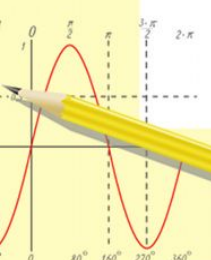
Доказательство:

Обозначим стороны треугольников соответственно a, b, c и a_1, b_1, c_1 , а соответствующие высоты h_a, h_b, h_c и h_{1a}, h_{1b}, h_{1c} .
 Имеют место равенства $ah_a = bh_b = ch_c$ и $a_1h_{1a} = b_1h_{1b} = c_1h_{1c}$.
 Разделив почленно первые равенства на вторые, получим равенства $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$
 из которых следует, что треугольники ABC и $A_1B_1C_1$ подобны. А так как соответствующие высоты этих треугольников равны, то они не только подобны, но и равны.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
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$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

