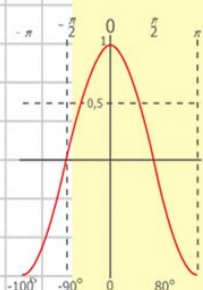
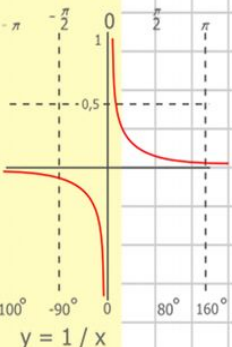
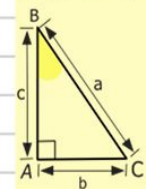
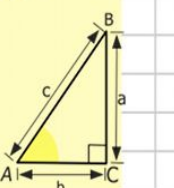
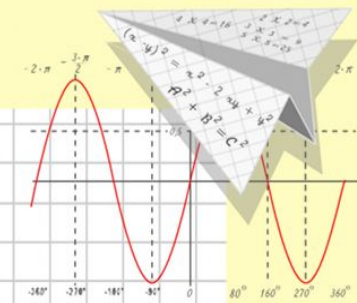
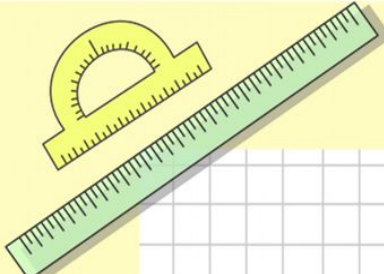


# Математик

а

## Дополнительные признаки равенства треугольников



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \end{array}$$

Серова Наталья Александровна,  
Мурзина Наталья Викторовна,  
учителя математики, информатики и ИКТ  
г.Омск МОУ «Средняя общеобразовательная школа № 16»

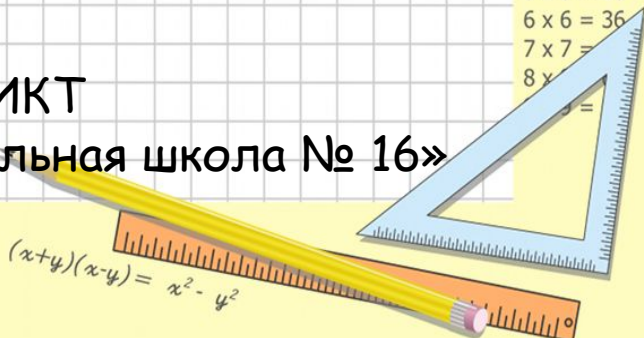
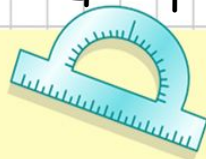
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

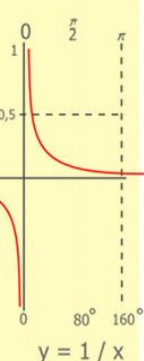
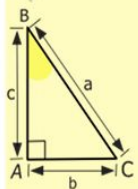
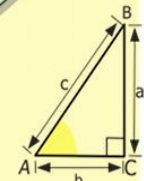
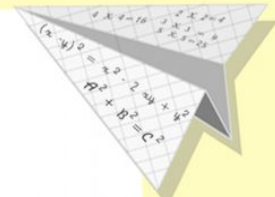
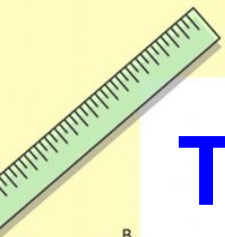
$$(x+y)(x-y) = x^2 - y^2$$



# Теорема

**1** Если угол, сторона, противолежащая этому углу, и высота, опущенная на другую сторону, одного треугольника соответственно равны углу, стороне и высоте другого треугольника, то такие треугольники равны.

Для доказательства используются признаки равенства прямоугольных треугольников.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ 105000 \\ \hline \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

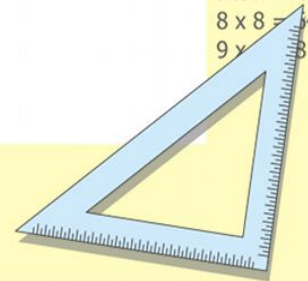
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

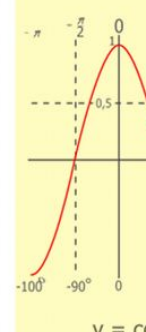
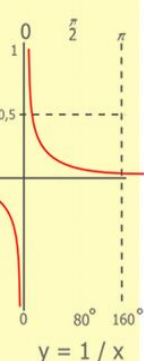
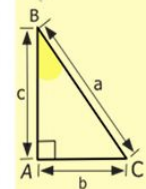
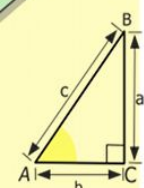
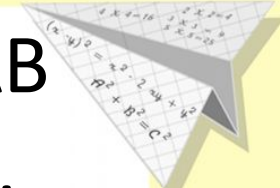
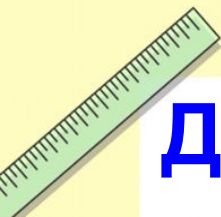
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



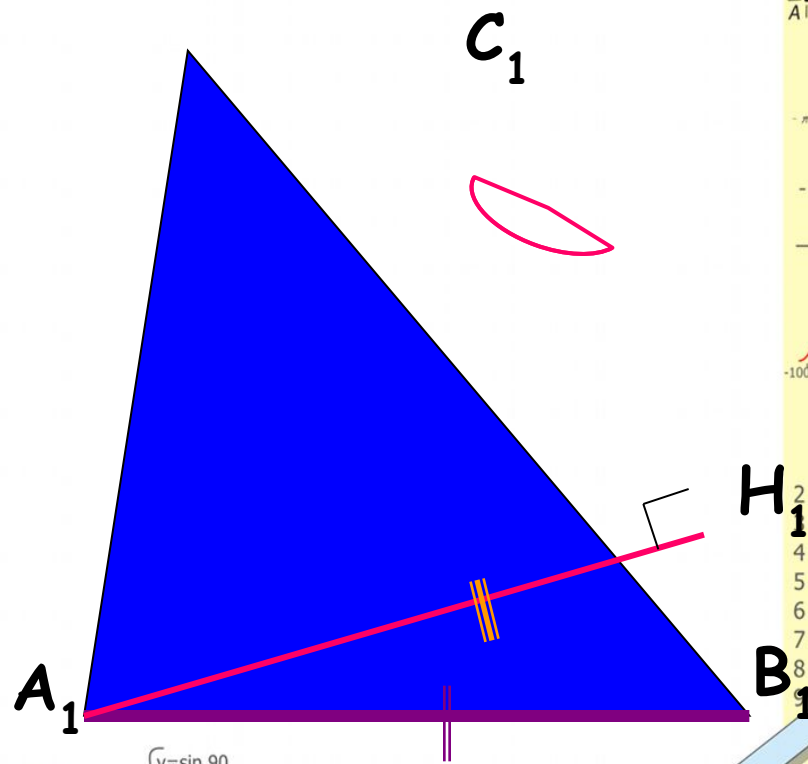
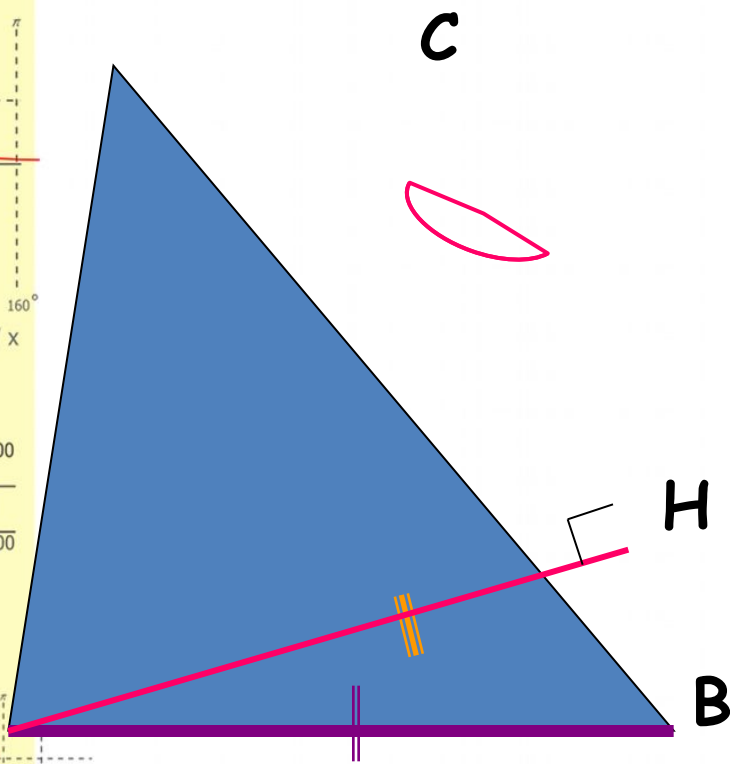
**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $\angle C = \angle C_1$ ,  $AB = A_1B_1$ , высота  $AH$  равна высоте  $A_1H_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$\frac{1}{2} \times 500$   
 $\times 42$   
 $\hline 210$   
 $+ 84$   
 $\hline 105000$

$2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

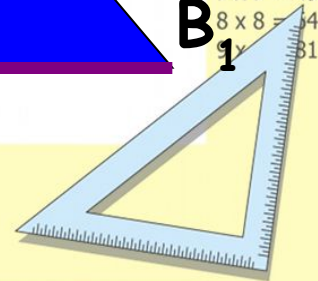
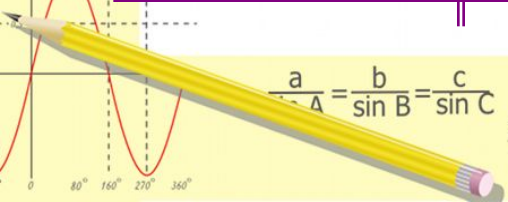
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



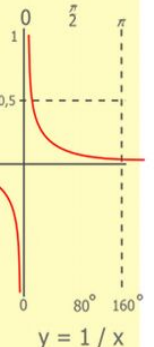
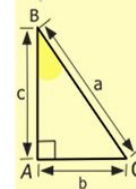
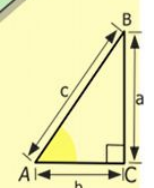
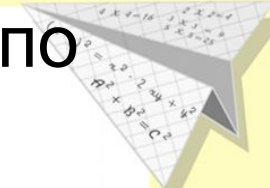
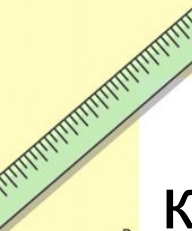
# Доказательство:

Прямоугольные  $\triangle ABH$  и  $\triangle A_1B_1H_1$  равны по катету и гипотенузе. Значит,  $\angle B = \angle B_1$ .

Учитывая, что  $\angle C = \angle C_1$ , имеем равенство  $\angle A = \angle A_1$ . Таким образом, в  $\triangle ABC$  и  $\triangle A_1B_1C_1$

$$AB = A_1B_1, \angle A = \angle A_1, \angle B = \angle B_1.$$

Следовательно, эти треугольники равны по второму признаку равенства треугольников.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



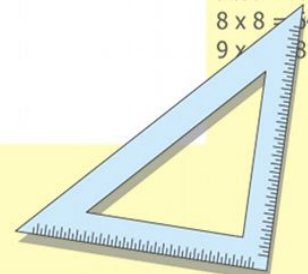
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases} \quad \begin{cases} y = 1 \\ x = 25 + 45 \end{cases} \quad \frac{x}{70}$$

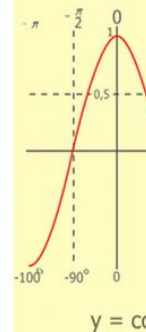
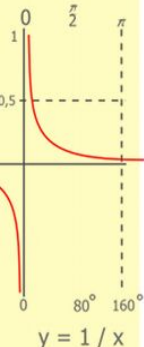
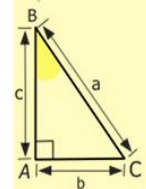
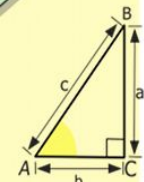
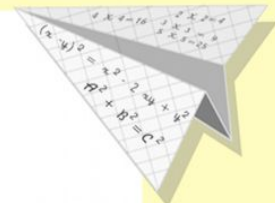
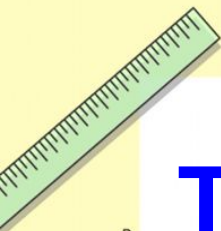
$$(x+y)(x-y) = x^2 - y^2$$



# Теорема

## 2

Если две стороны и медиана, заключенная между ними, одного треугольника соответственно равны двум сторонам и медиане другого треугольника, то такие треугольники равны.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

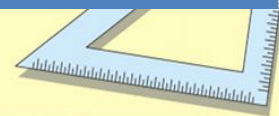


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

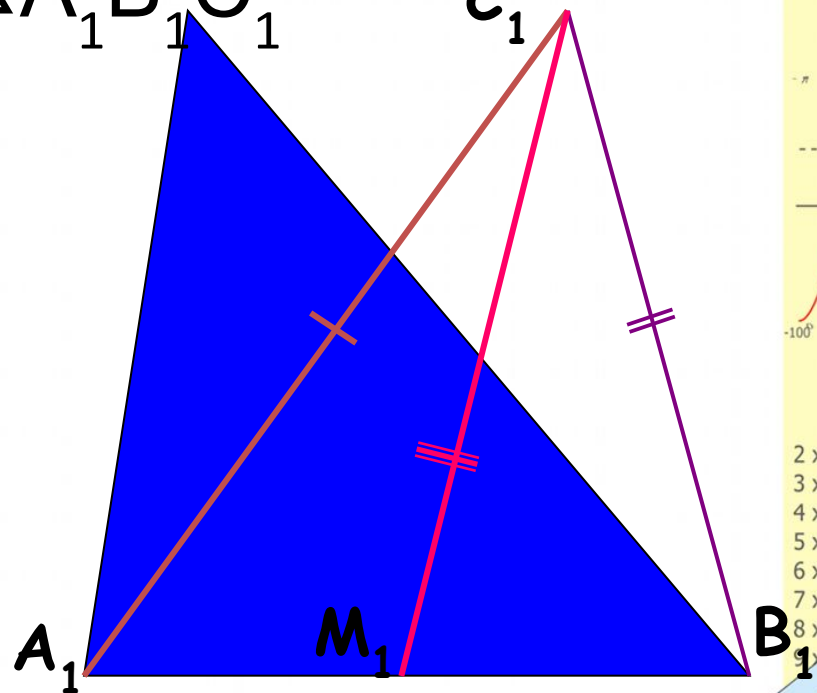
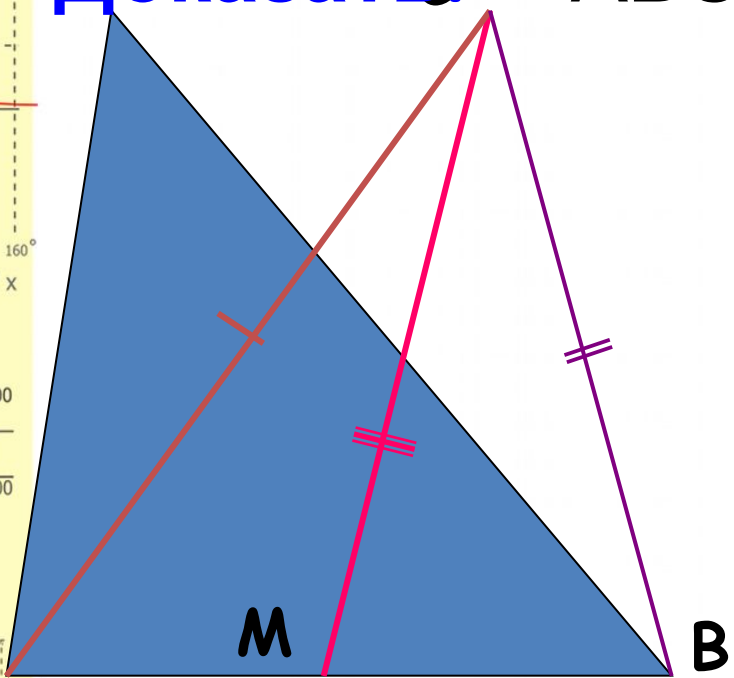
$$(x+y)(x-y) = x^2 - y^2$$

Теорема 8



**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $AC = A_1C_1$ ,  $BC = B_1C_1$ , медиана  $CM$  равна медиане  $C_1M_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

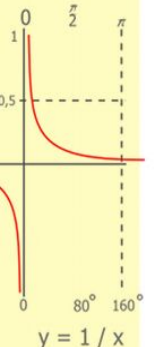
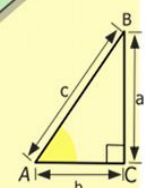
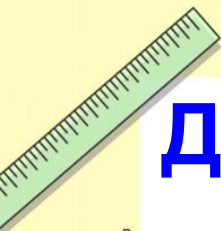
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

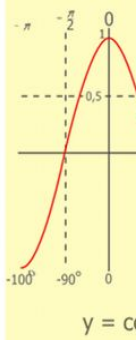
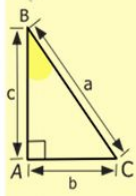
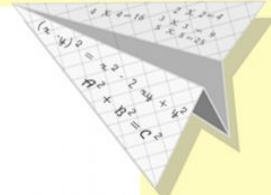
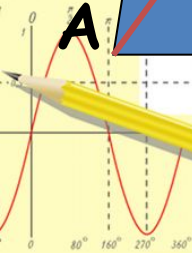
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

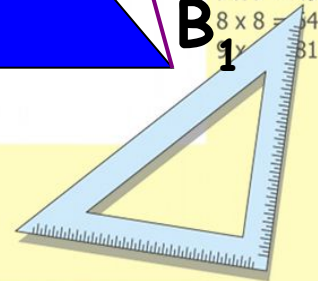
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



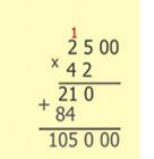
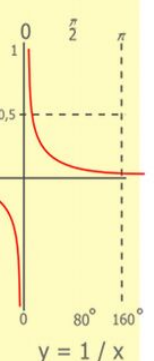
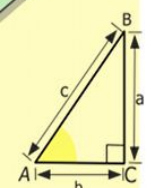
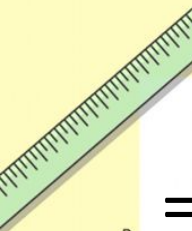
# Доказательство

чертеж

Продолжим медианы и отложим отрезки  $MD = CM$  и  $M_1D_1 = C_1M_1$ . Четырехугольники  $ACBD$  и  $A_1C_1B_1D_1$  — параллелограммы.  $\triangle ACD = \triangle A_1C_1D_1$  по трем сторонам. Следовательно,  $\angle ACD = \angle A_1C_1D_1$ .

Аналогично,  $\triangle BCD = \triangle B_1C_1D_1$  по трем сторонам. Следовательно,  $\angle BCD = \angle B_1C_1D_1$ .

Значит,  $\angle C = \angle C_1$  и треугольники  $ABC$  и  $A_1B_1C_1$  равны по двум сторонам и углу между ними (по первому признаку равенства треугольников).



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

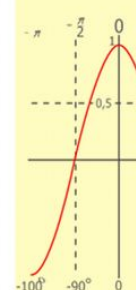
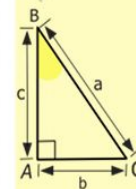
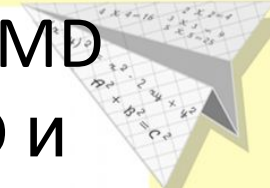
$$\sin 90^\circ = 1$$



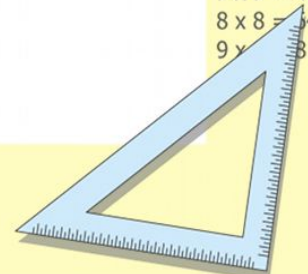
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

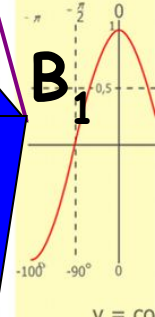
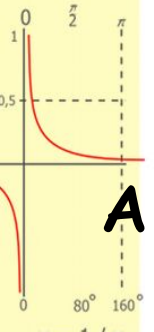
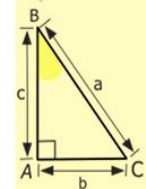
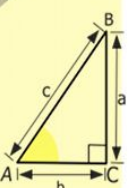
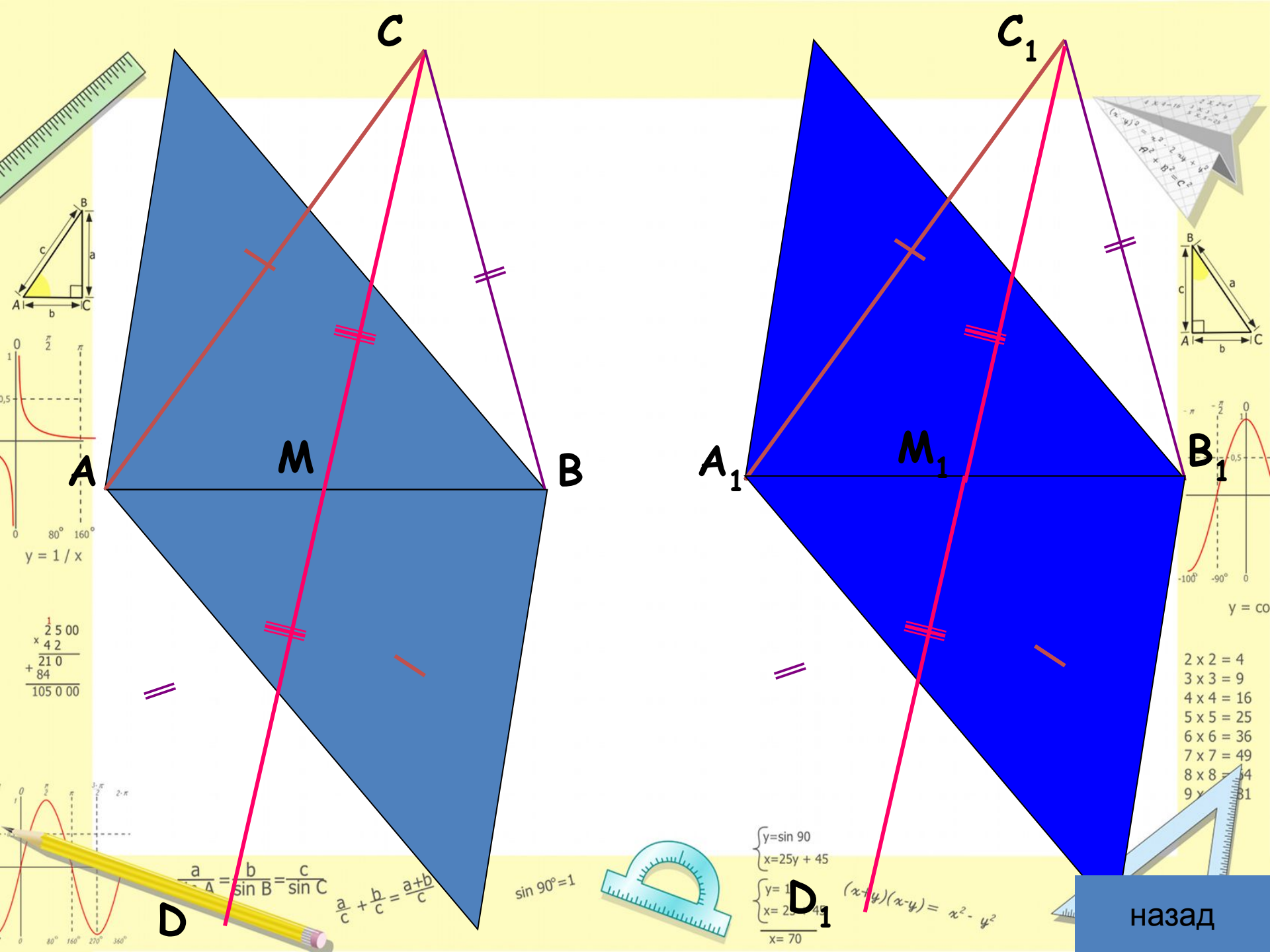
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



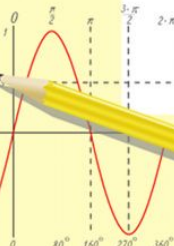
- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$





$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
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$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 \cdot 1 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

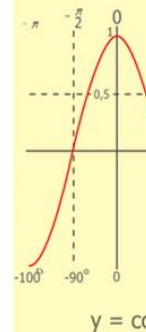
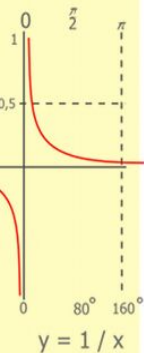
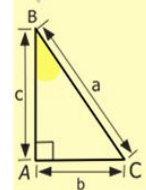
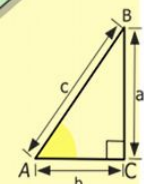
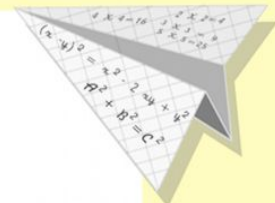
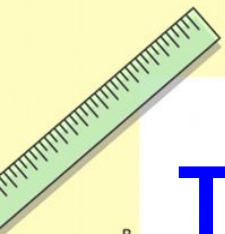
назад



# Теорема

## 3

Если сторона и две медианы, проведенные к двум другим сторонам, одного треугольника соответственно равны стороне и двум медианам другого треугольника, то такие треугольники равны.



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

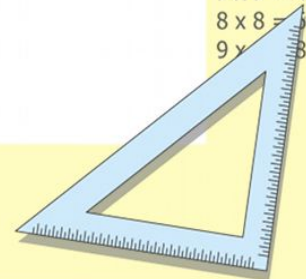
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

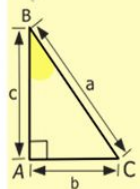
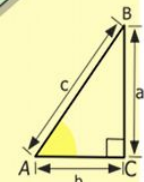
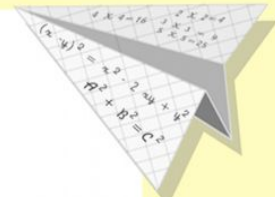
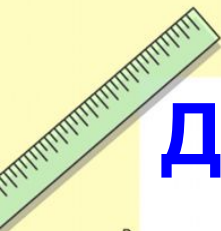
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $AB = A_1B_1$ ,  
 медианы  $AM = A_1M_1$ ,  $BK = B_1K_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

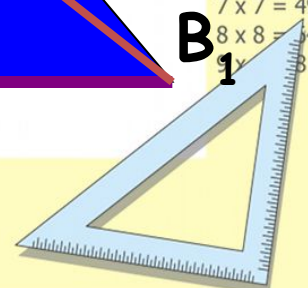
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

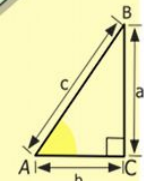
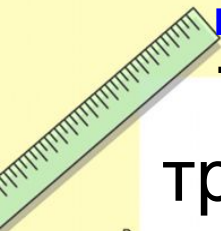


# Доказательство:

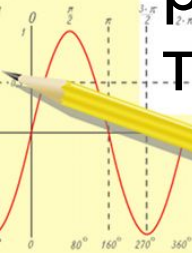
Точки  $O$  и  $O_1$  пересечения медиан данных треугольников делят медианы в отношении  $2:1$ , считая от вершины. Значит,  $\triangle ABO = \triangle A_1B_1O_1$  по трем сторонам. Следовательно,  $\angle BAO = \angle B_1A_1O_1$ , значит,  $\triangle ABM = \triangle A_1B_1M_1$  равны по двум сторонам и углу между ними. Поэтому  $\angle ABC = \angle A_1B_1C_1$ .

Аналогично доказывается, что  $\angle BAC = \angle B_1A_1C_1$ .

Таким образом, треугольники  $\triangle ABC$  и  $\triangle A_1B_1C_1$  равны по стороне и двум прилежащим к ней углам. Следовательно,  $\triangle ABC = \triangle A_1B_1C_1$  равны по второму признаку равенства треугольников.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

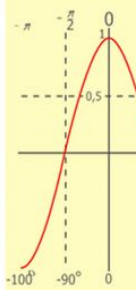
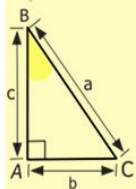
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

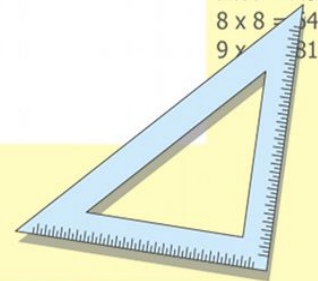
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



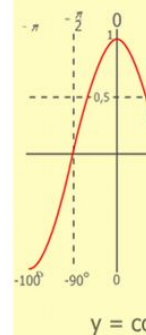
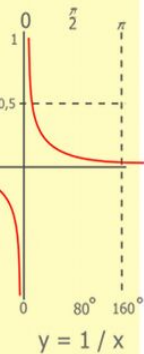
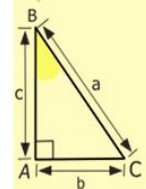
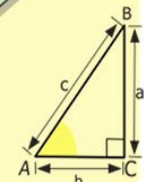
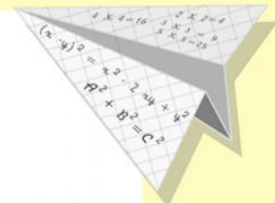
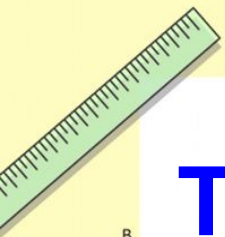
$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



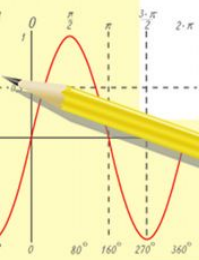
# Теорема 4

Если две стороны и биссектриса, заключенная между ними, одного треугольника соответственно равны двум сторонам и биссектрисе, заключенной между ними, другого треугольника, то такие треугольники равны.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

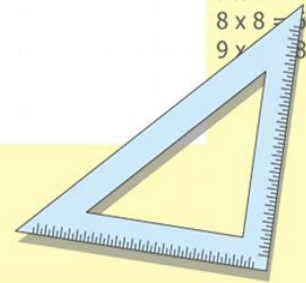
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

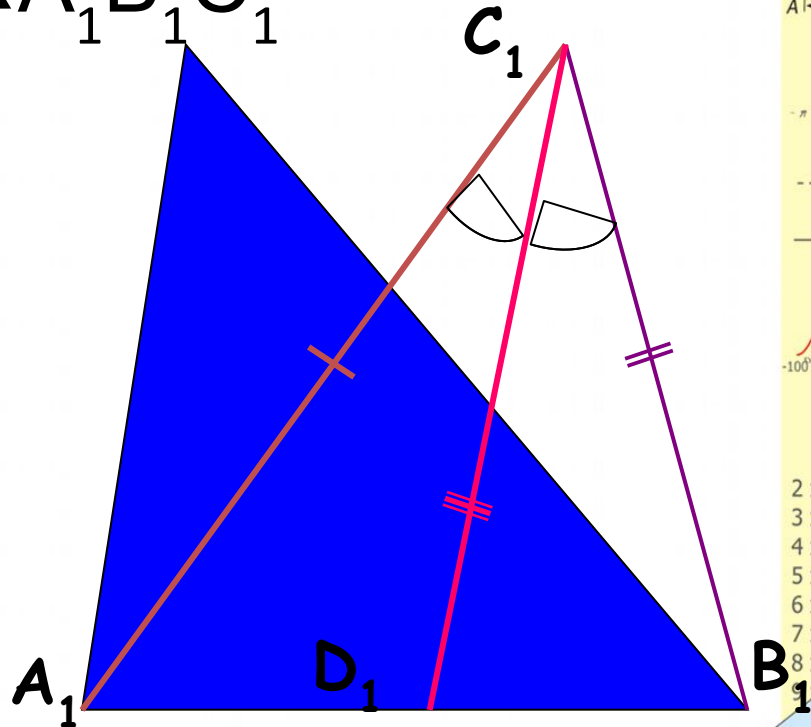
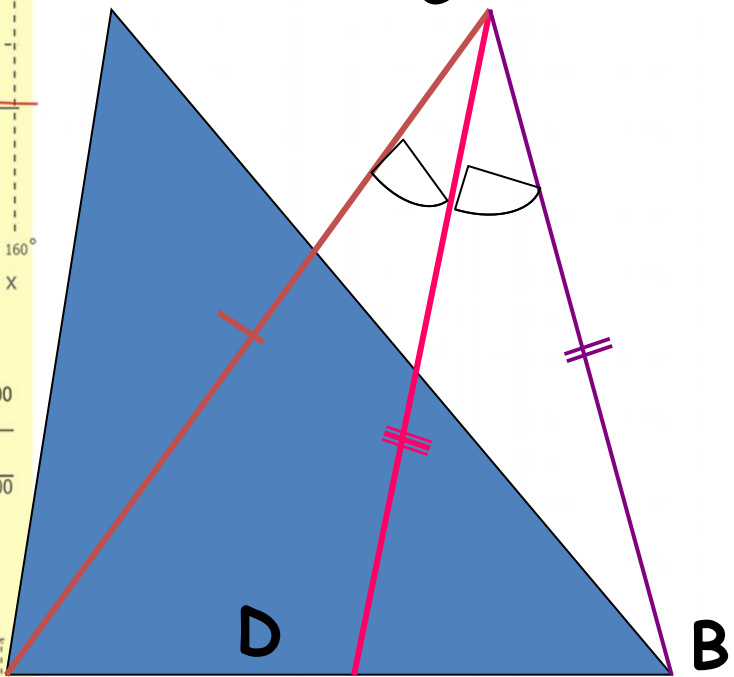
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



• **Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $AC = A_1C_1$ ,  $BC = B_1C_1$ , биссектриса  $CD$  равна биссектрисе  $C_1D_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

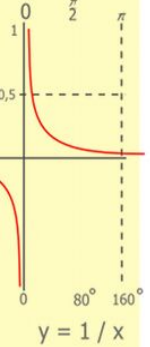
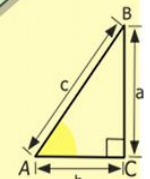
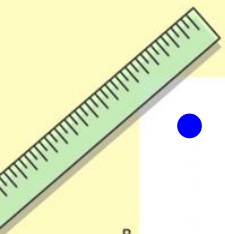
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

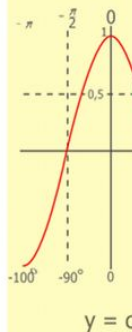
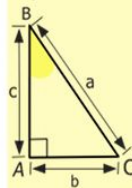
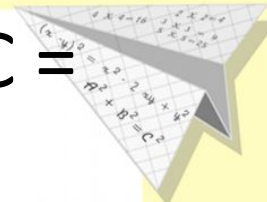
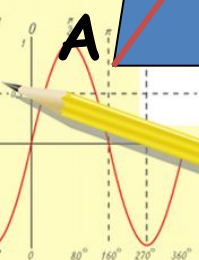
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

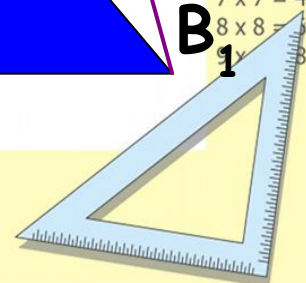
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Доказательство

чертеж

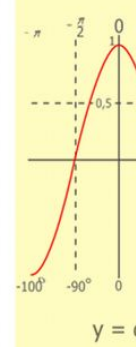
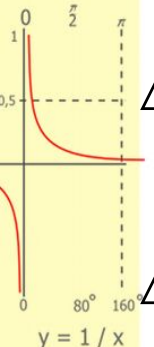
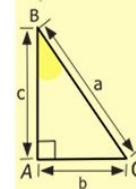
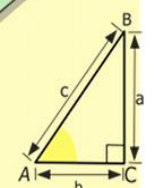
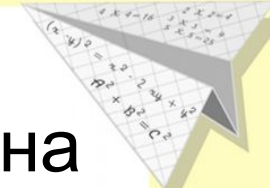
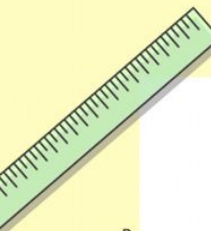
Продолжим стороны  $AC$  и  $A_1C_1$  и отложим на их продолжениях отрезки  $CE = BC$  и  $C_1E_1 = B_1C_1$ .

Тогда  $BE = CD \frac{AE}{AC}$ ,  $B_1E_1 = C_1D_1 \frac{A_1E_1}{A_1C_1}$

$\triangle BCE = \triangle B_1C_1E_1$  по трем сторонам. Значит,  $\angle E = \angle E_1$  и  $BE = B_1E_1$ .

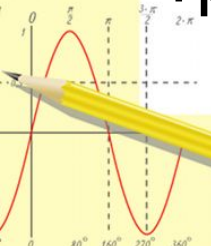
$\triangle ABE = \triangle A_1B_1E_1$  по двум сторонам и углу между ними. Значит,  $AB = A_1B_1$ .

Таким образом,  $\triangle ABC = \triangle A_1B_1C_1$  по трем сторонам (3 признак равенства треугольников).



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

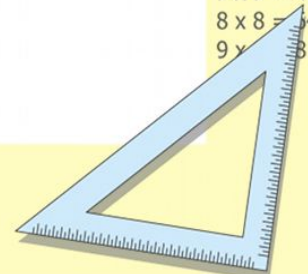
$$\sin 90^\circ = 1$$

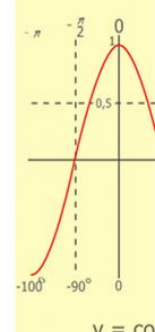
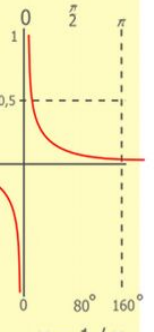
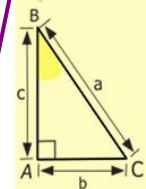
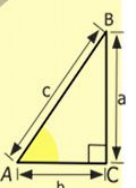
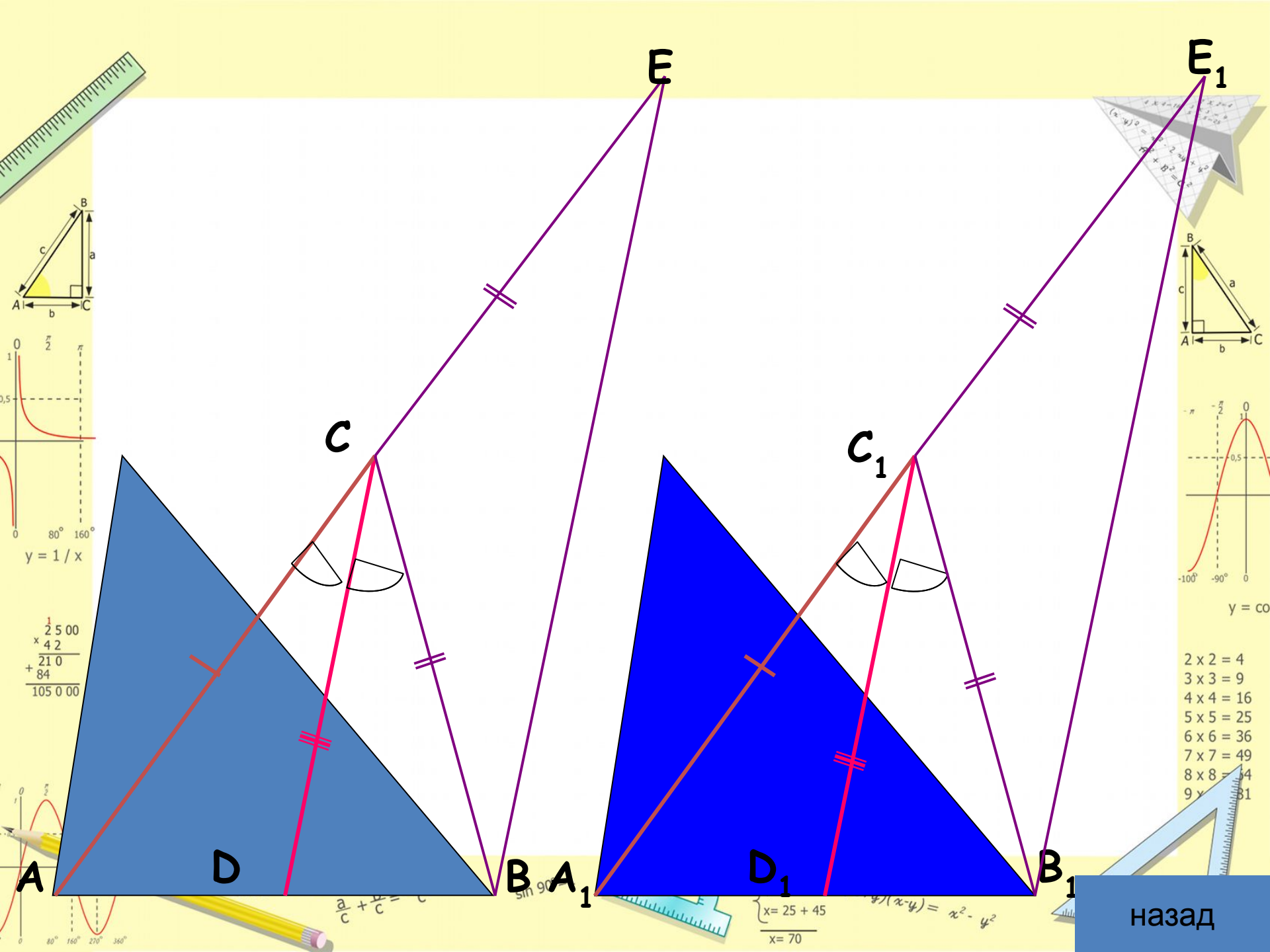


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

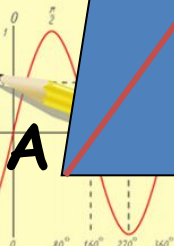
$$(x+y)(x-y) = x^2 - y^2$$





$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$

$2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$



$\frac{a}{c} + \frac{b}{c} = c$

$\sin 90^\circ$

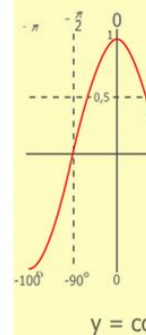
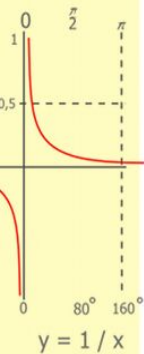
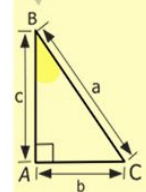
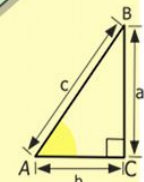
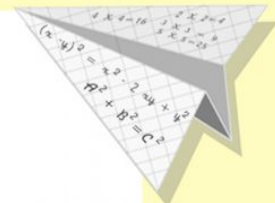
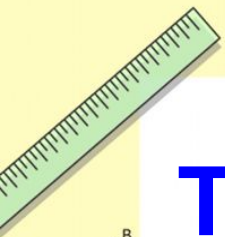
$\begin{array}{l} x = 25 + 45 \\ \hline x = 70 \end{array}$

$y(x-y) = x^2 - y^2$

назад

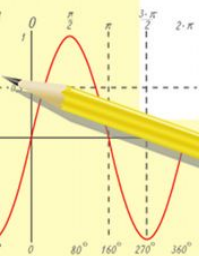
# Теорема 5

Два треугольника равны, если сторона, медиана и высота, проведенные к другой стороне, одного треугольника соответственно равны стороне, медиане и высоте другого треугольника.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

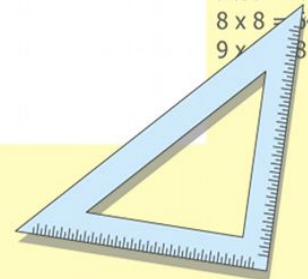
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

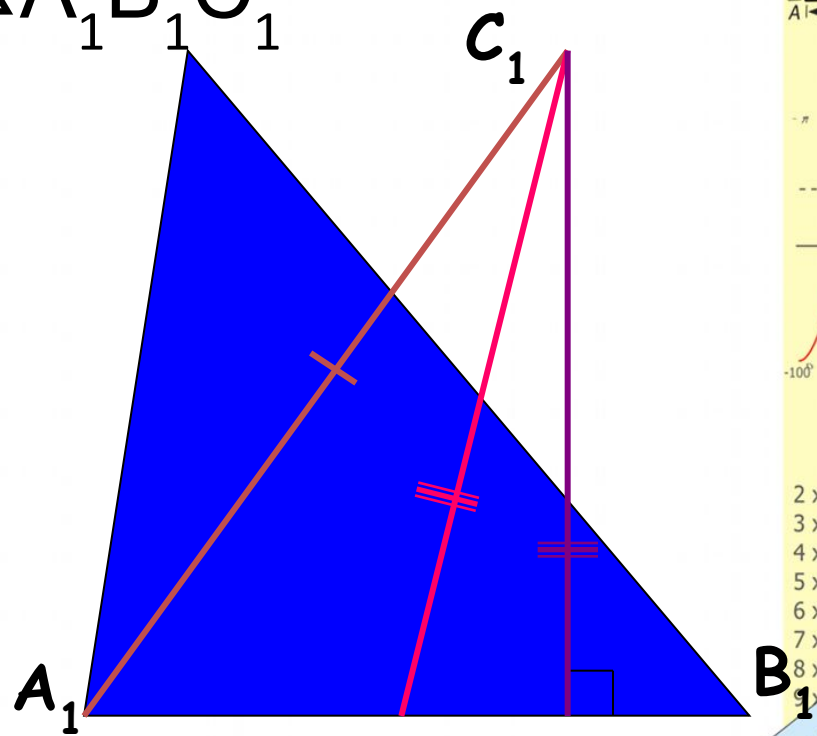
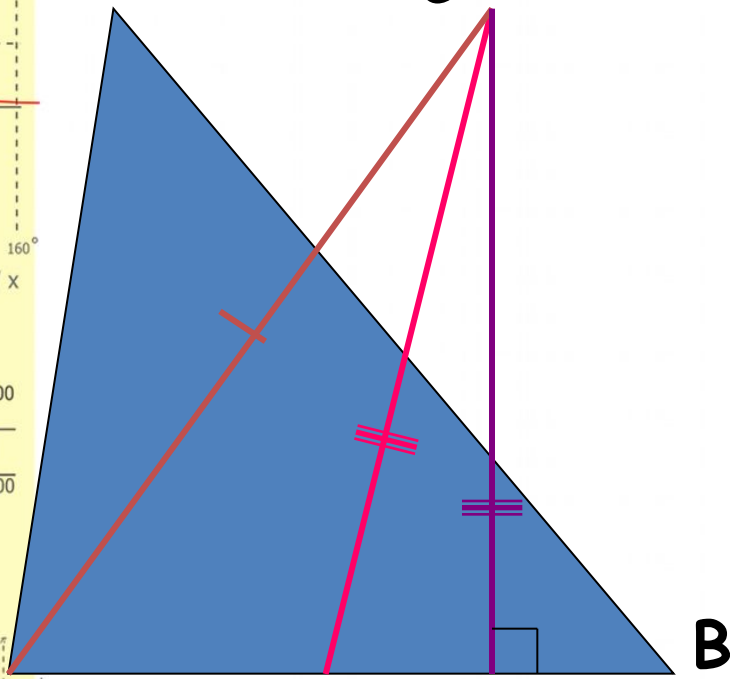
$$(x+y)(x-y) = x^2 - y^2$$





**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $AC = A_1C_1$ ,  $AC = A_1C_1$ , медианы  $CM$  и  $C_1M_1$  равны, высоты  $CH$  и  $C_1H_1$  равны.

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



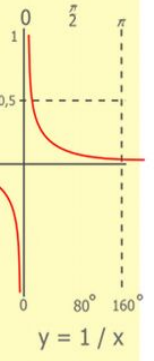
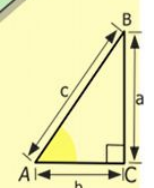
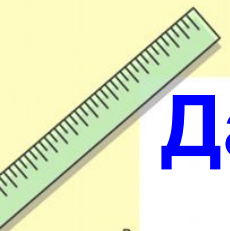
$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

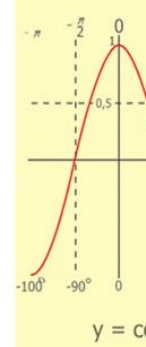
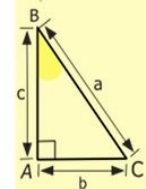
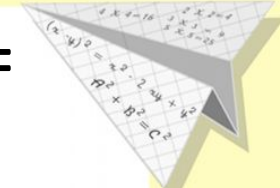
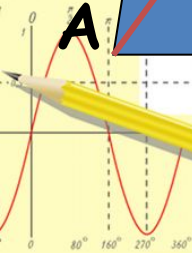
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

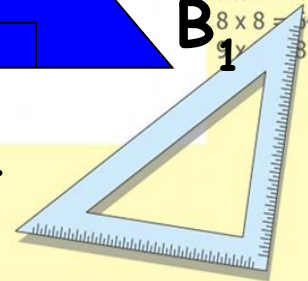
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

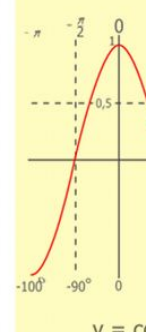
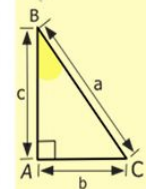
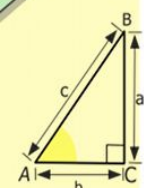
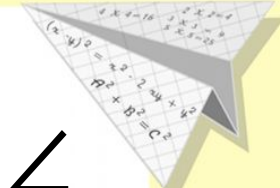
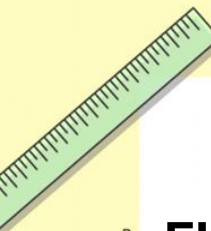


$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Доказательство:

Прямоугольные  $\triangle ACH = \triangle A_1C_1H_1$  по гипотенузе и катету. Следовательно,  $\angle A = \angle A_1$  и  $AH = A_1H_1$ . Прямоугольные треугольники  $\triangle CMH = \triangle C_1M_1H_1$  по гипотенузе и катету. Следовательно,  $MH = M_1H_1$ , откуда  $AM = A_1M_1$ , значит,  $AB = A_1B_1$ . Таким образом,  $\triangle ABC = \triangle A_1B_1C_1$  по двум сторонам и углу между ними (по первому признаку равенства треугольников).



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

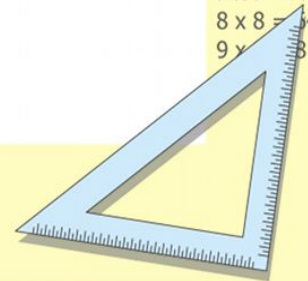
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

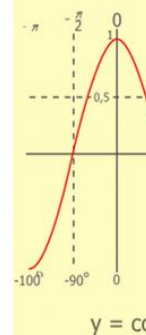
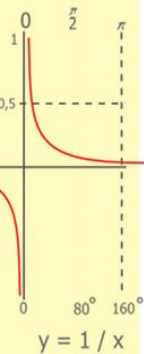
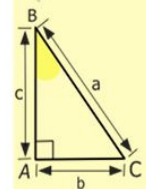
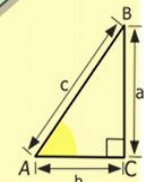
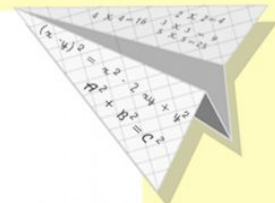
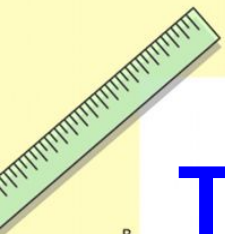
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



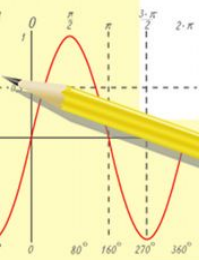
# Теорема 6

Два треугольника равны, если медиана и два угла на которые делит угол медиана, одного треугольника соответственно равны медиане и двум углам, на которые делит медиана угол другого треугольника.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

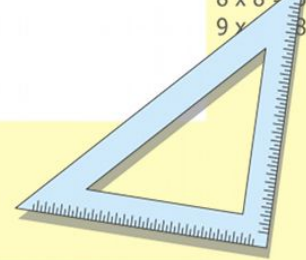
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

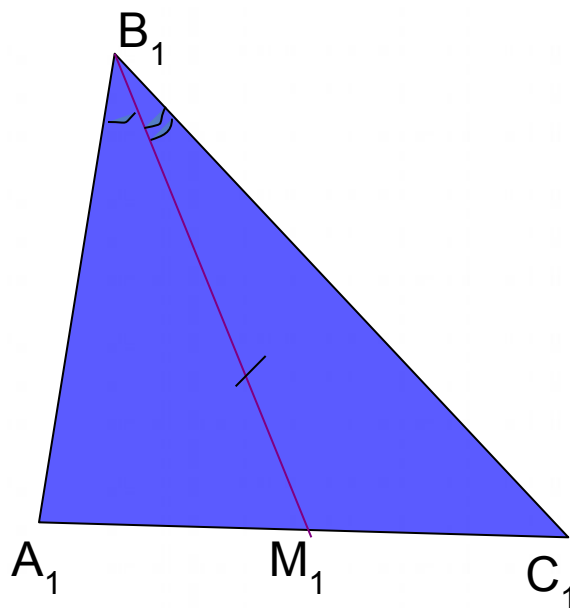
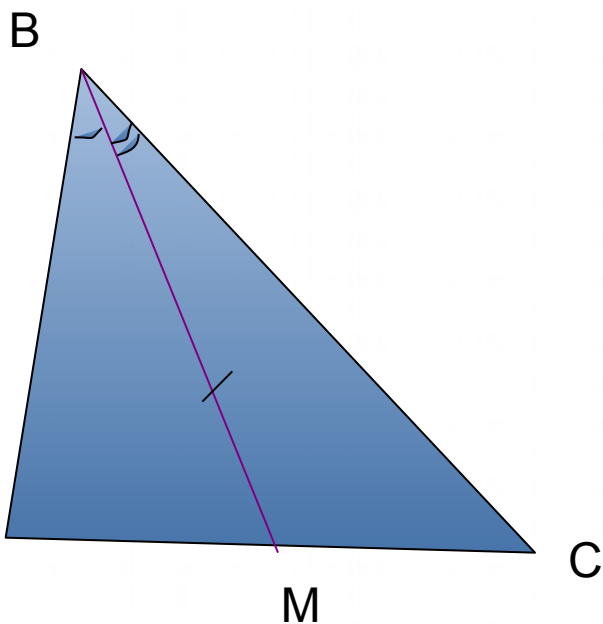
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $BM = B_1M_1$ ,  
 $\angle ABM = \angle A_1B_1M_1$ ,  $\angle CBM = \angle C_1B_1M_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

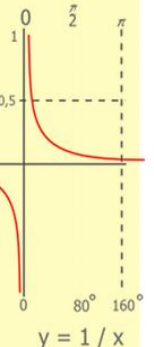
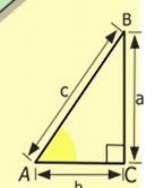
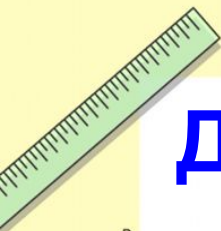
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

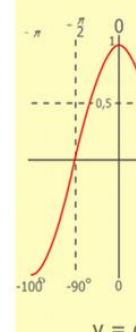
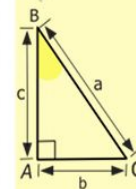
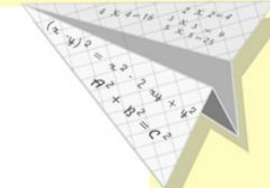
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

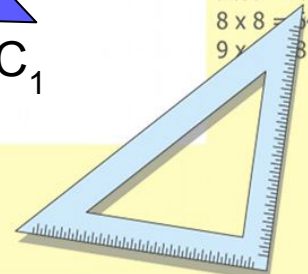
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 4\ 2 \\ \hline 2\ 1\ 0 \\ + 8\ 4 \\ \hline 10\ 5\ 0\ 00 \end{array}$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



## Доказательство:

В данных треугольниках удвоим медианы  $BM=MD$  и  $B_1M_1=M_1D_1$ .

1.  $\triangle AMD = \triangle CMB$ ,  $\triangle A_1M_1D_1 = \triangle C_1M_1B_1$  ( по 1 признаку)

Из равенства этих треугольников следуют равенства:

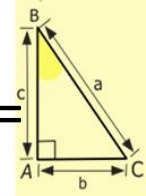
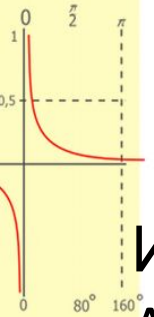
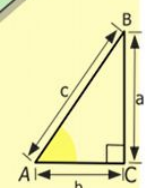
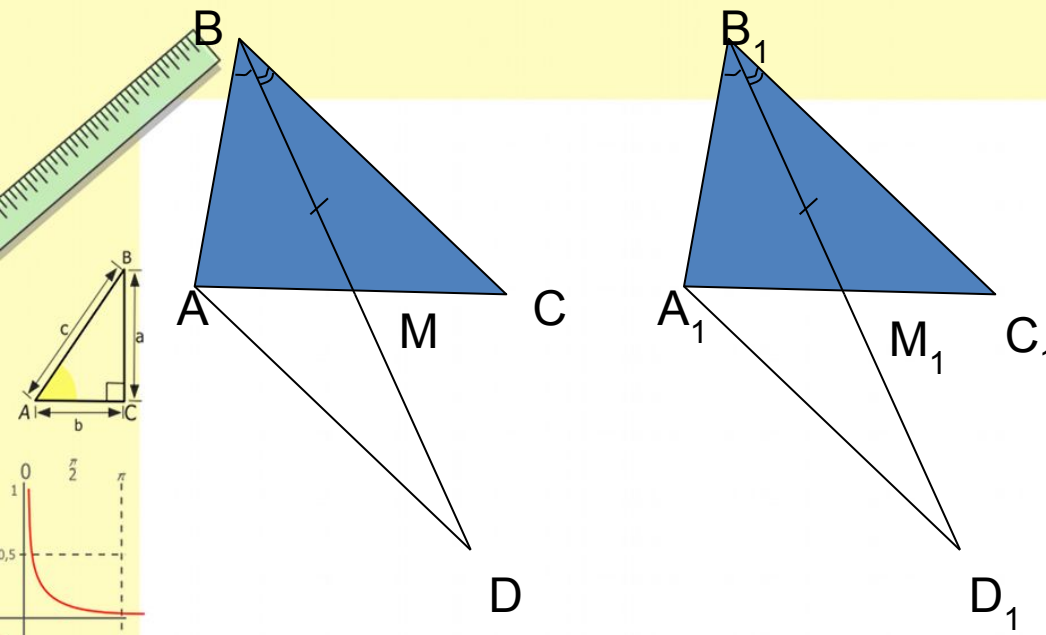
$$AD=BC, A_1D_1=B_1C_1 \text{ и } \angle ADM = \angle CBM, \angle A_1D_1M_1 = \angle C_1B_1M_1$$

2.  $\triangle ABD = \triangle A_1B_1D_1$  ( по 2 признаку)

Из равенства этих треугольников следуют равенства:

$$AB=A_1B_1, \text{ а значит, } BC=AD=B_1C_1=A_1D_1$$

3.  $\triangle ABC = \triangle A_1B_1C_1$  ( по первому признаку равенства треугольников)



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



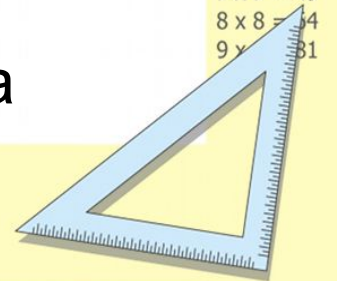
$$\frac{a}{c} = \sin B = \sin C, \quad \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



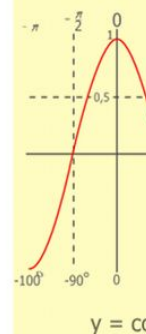
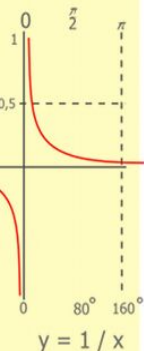
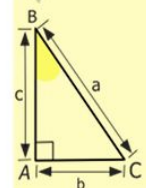
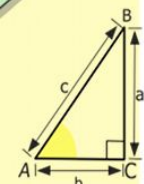
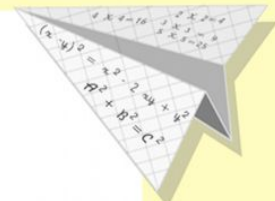
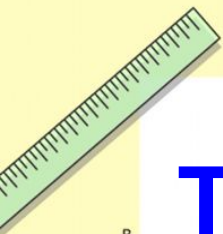
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



# Теорема 7

Два треугольника равны, если сторона, и две высоты, опущенные на две другие стороны, одного треугольника соответственно равны стороне и двум высотам, опущенным на две другие стороны другого треугольника.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

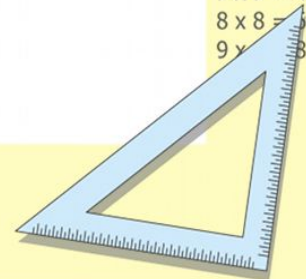
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

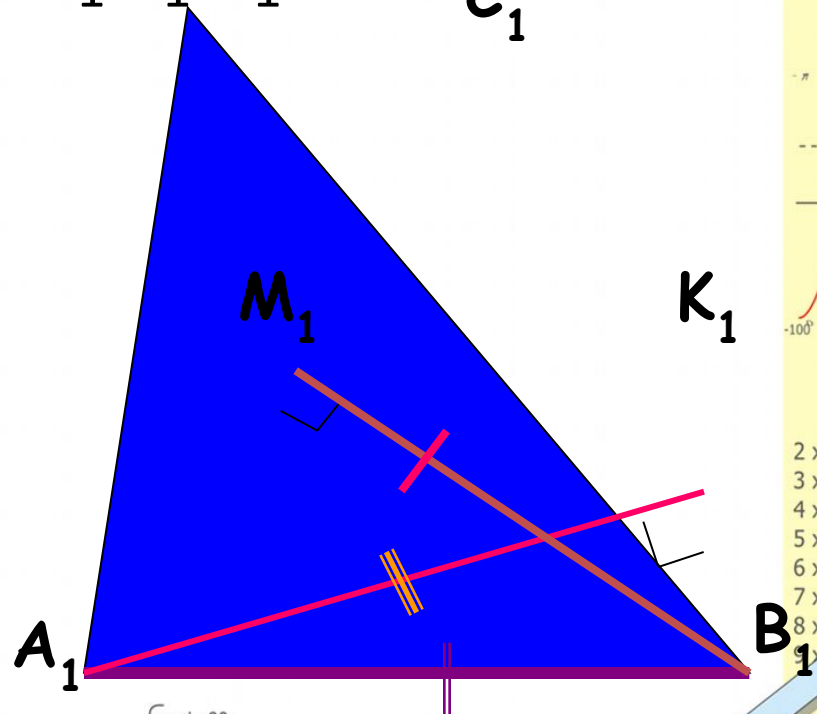
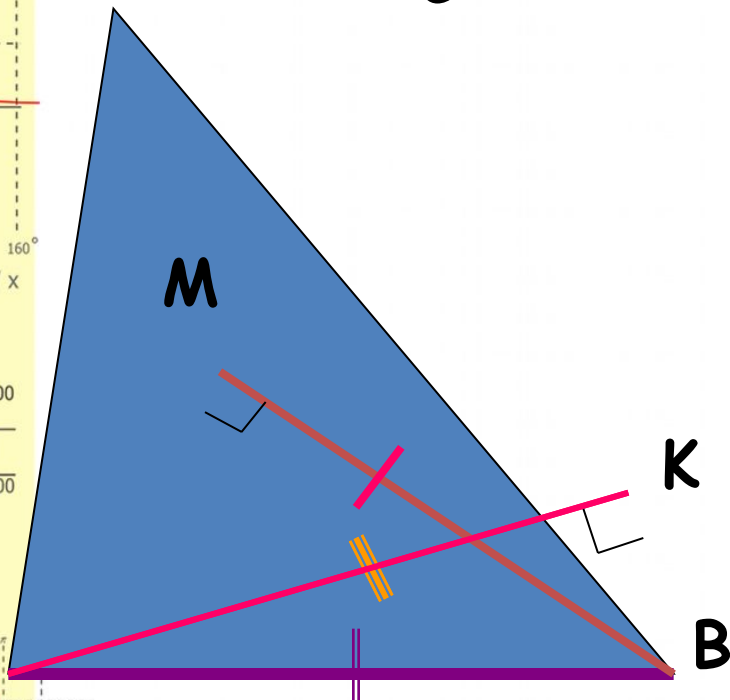
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $AB = A_1B_1$ ,  
 высота  $AM$  равна высоте  $A_1M_1$ , высота  
 $BK$  равна высоте  $B_1K_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

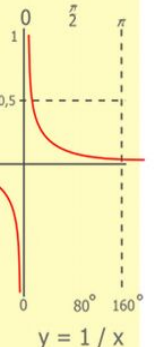
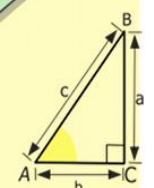
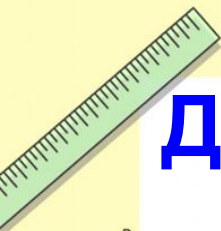
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

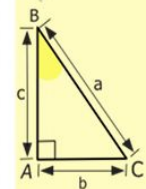
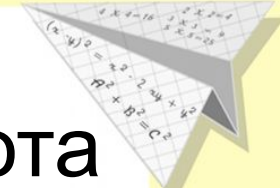
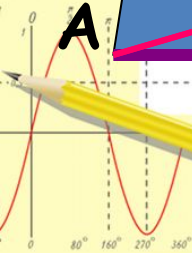
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

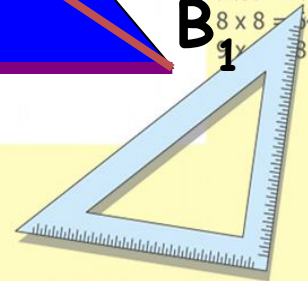
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



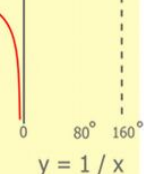
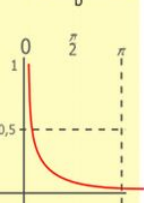
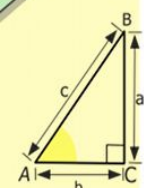
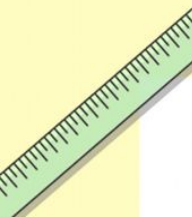
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Доказательство:

Из равенства прямоугольных треугольников  $\triangle AMB = \triangle A_1M_1B_1$ ,  $\triangle ВКА = \triangle В_1К_1А_1$  (по катету и гипотенузе) следует равенство углов:  $\angle ВАС = \angle В_1А_1С_1$ ,  $\angle АВС = \angle А_1В_1С_1$ .

Поэтому  $\triangle ABC = \triangle A_1B_1C_1$  по стороне ( $AB = A_1B_1$ ) и двум прилежащим к ней углам (по второму признаку равенства треугольников).



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 2100 \\ + 840 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

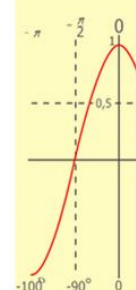
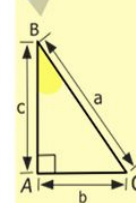
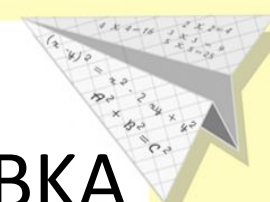


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

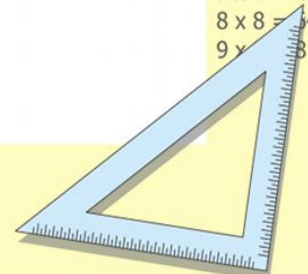
$$\underline{x = 70}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81

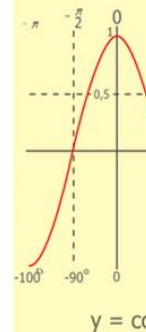
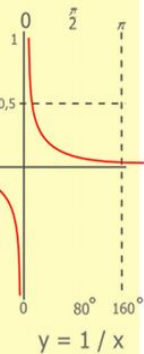
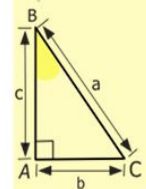
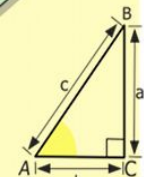
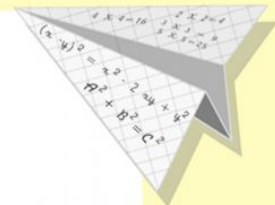
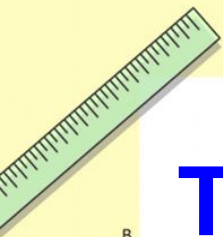




# Теорема

## 8

Два треугольника равны, если три медианы одного треугольника соответственно равны трем медианам другого.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

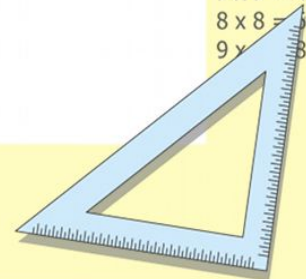
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

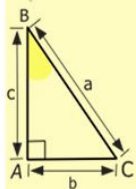
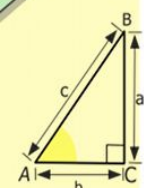
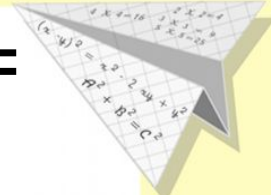
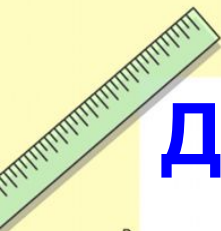
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



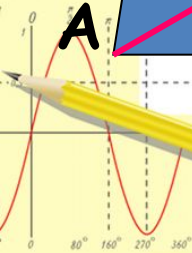
**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ , медианы  $AK = A_1K_1$ ,  $BL = B_1L_1$ ,  $CM = C_1M_1$ .

**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

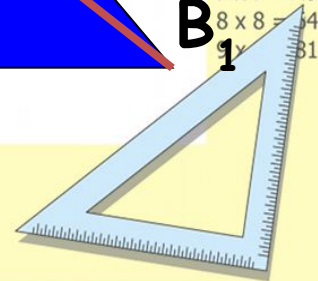
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



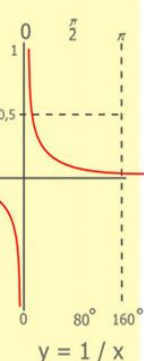
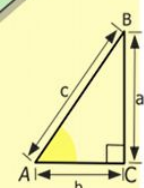
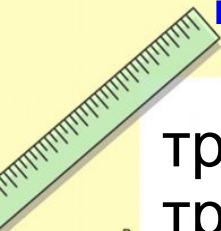
# Доказательство:

Пусть  $O$  и  $O_1$  — точки пересечения медиан данных треугольников. Заметим, что медианы  $OM$  и  $O_1M_1$  треугольников  $\triangle ABO$  и  $\triangle A_1B_1O_1$  равны, так как они составляют одну треть часть соответствующих медиан данных треугольников. Аналогично равны  $AO$  и  $A_1O_1$ ,  $BO$  и  $B_1O_1$ , так как они составляют две третьих соответствующих медиан данных треугольников.

По признаку равенства треугольников, доказанному нами под номером [2](#),  $\triangle ABO = \triangle A_1B_1O_1$ , значит,  $AB = A_1B_1$ .

Аналогично доказывается, что  $BC = B_1C_1$  и  $AC = A_1C_1$ .

Таким образом,  $\triangle ABC$  и  $\triangle A_1B_1C_1$  равны по трем сторонам ( по третьему признаку равенства треугольников ).



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

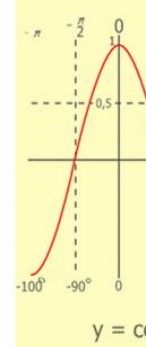
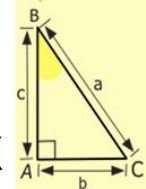
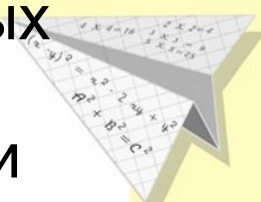
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

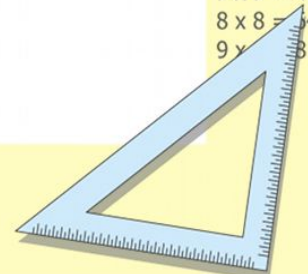


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



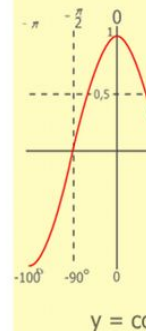
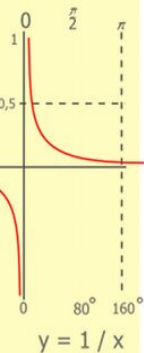
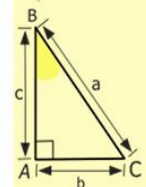
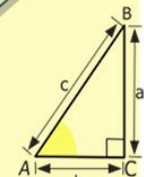
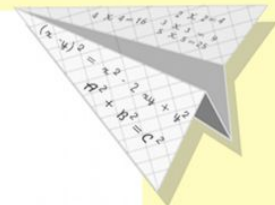
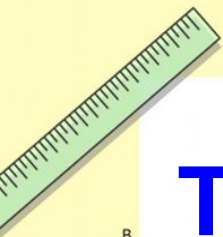
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Теорема

## 9

Два треугольника равны, если три высоты одного треугольника соответственно равны трем высотам другого треугольника.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

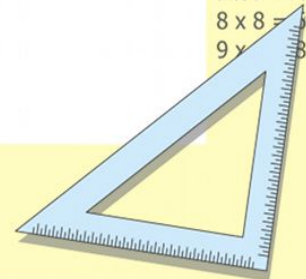
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

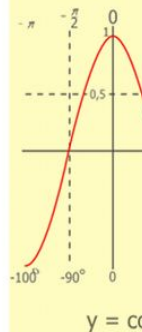
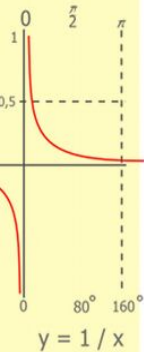
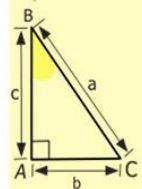
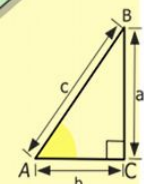
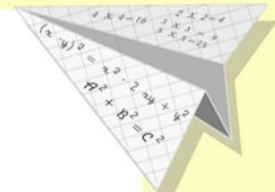
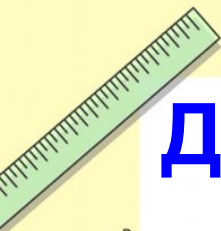
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



**Дано:**  $\triangle ABC$  и  $\triangle A_1B_1C_1$ ,  $AB = A_1B_1$ ,  
 ВЫСОТЫ  $AH = A_1H_1$ ,  $BG = B_1G_1$ ,  $CF = C_1F_1$ .

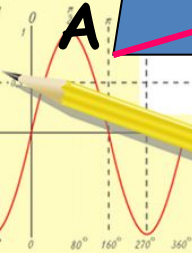
**Доказать:**  $\triangle ABC = \triangle A_1B_1C_1$



$y = 1/x$

$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$y = \cos$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$


$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

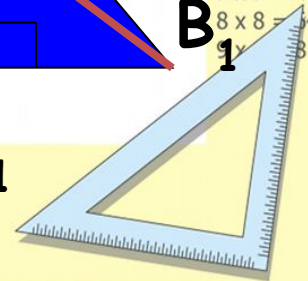
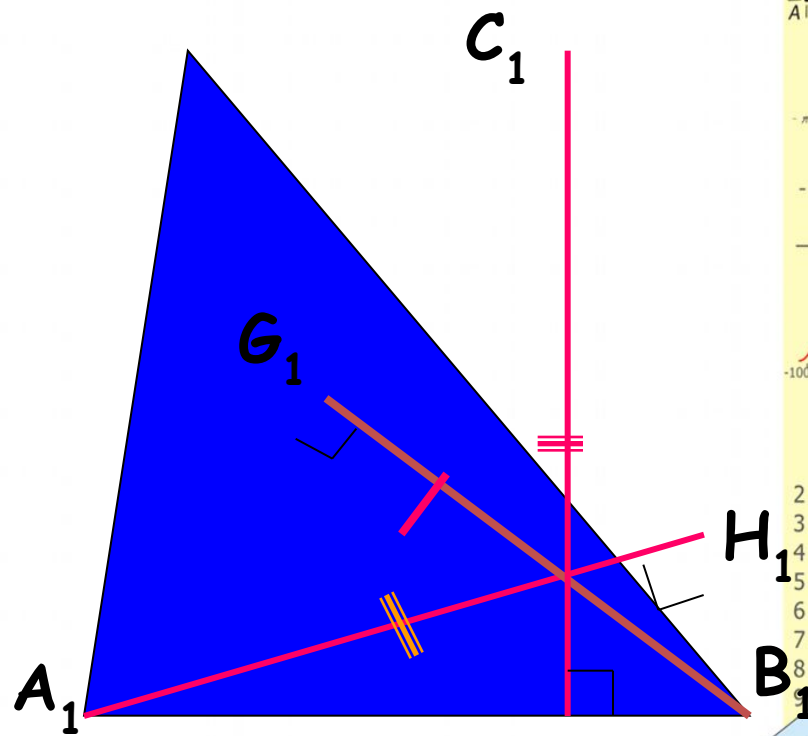
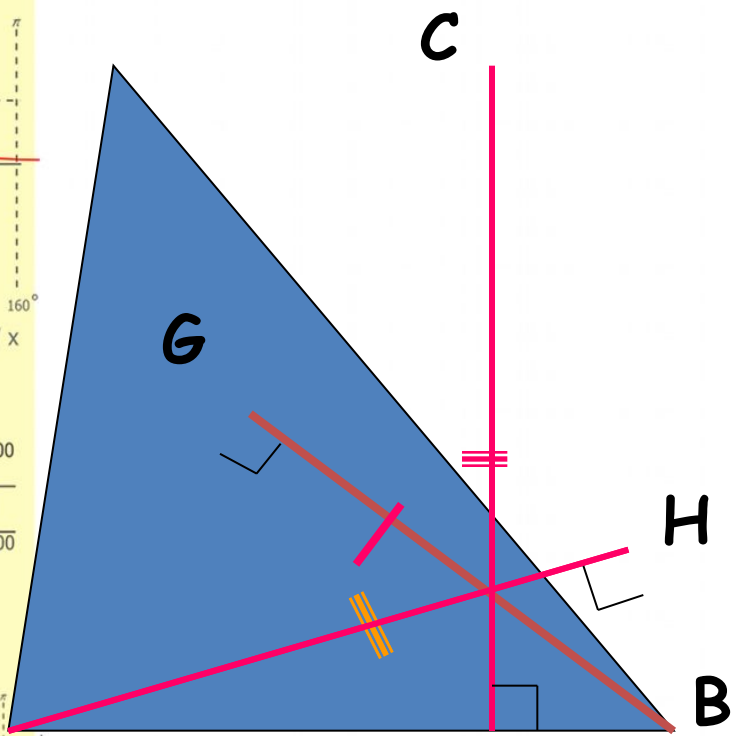
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

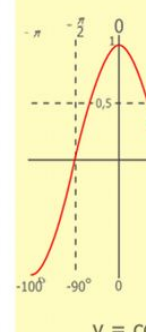
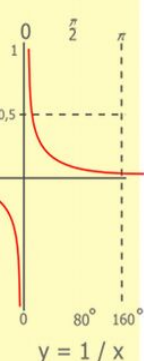
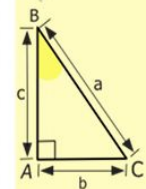
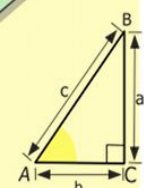
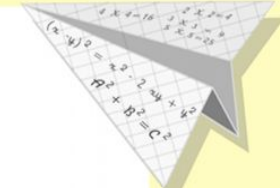
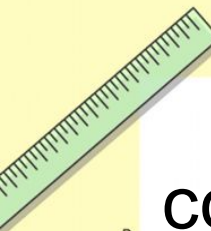


# Доказательство:

Обозначим стороны треугольников соответственно  $a, b, c$  и  $a_1, b_1, c_1$ , а соответствующие высоты  $h_a, h_b, h_c$  и  $h_{1a}, h_{1b}, h_{1c}$ .

Имеют место равенства  $ah_a = bh_b = ch_c$  и  $a_1h_{1a} = b_1h_{1b} = c_1h_{1c}$ . Разделив почленно первые равенства на вторые, получим равенства  $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$

из которых следует, что треугольники  $ABC$  и  $A_1B_1C_1$  подобны. А так как соответствующие высоты этих треугольников равны, то они не только подобны, но и равны.



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

$$\frac{x}{70}$$

