



# **Ikeldilen argumentiň trigonometrik funksiýalary**



# Barlagnama:

1.  $\cos(\alpha - \beta) = ?$

a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$     b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$

ç)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$     d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

2.  $\cos(\alpha + \beta) = ?$

a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$     b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$

ç)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$     d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

3.  $\sin(\alpha + \beta) = ?$

a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$     b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$

ç)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$     d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

4.  $\sin(\alpha - \beta) = ?$

a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$     b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$

ç)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$     d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$



5.  $\text{tg}(\alpha + \beta) = ?$

a)  $\frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \text{tg } \beta}$

b)  $\frac{\text{tg } \alpha + \text{tg } \beta}{1 + \text{tg } \alpha \text{tg } \beta}$

ç)  $\frac{\text{tg } \alpha - \text{tg } \beta}{1 - \text{tg } \alpha \text{tg } \beta}$

d)  $\frac{\text{tg } \alpha - \text{tg } \beta}{1 + \text{tg } \alpha \text{tg } \beta}$

6.  $\text{tg}(\alpha - \beta) = ?$

a)  $\frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \text{tg } \beta}$

b)  $\frac{\text{tg } \alpha + \text{tg } \beta}{1 + \text{tg } \alpha \text{tg } \beta}$

ç)  $\frac{\text{tg } \alpha - \text{tg } \beta}{1 - \text{tg } \alpha \text{tg } \beta}$

d)  $\frac{\text{tg } \alpha - \text{tg } \beta}{1 + \text{tg } \alpha \text{tg } \beta}$

7.  $\text{Ctg}(\alpha - \beta) = ?$

a)  $\frac{\text{ctg } \alpha \text{ctg } \beta + 1}{\text{ctg } \alpha + \text{ctg } \beta}$

b)  $\frac{\text{ctg } \alpha \text{ctg } \beta - 1}{\text{ctg } \alpha - \text{ctg } \beta}$

ç)  $\frac{\text{ctg } \alpha \text{ctg } \beta + 1}{\text{ctg } \alpha - \text{ctg } \beta}$

d)  $\frac{\text{ctg } \alpha \text{ctg } \beta - 1}{\text{ctg } \alpha + \text{ctg } \beta}$

8.  $\text{Ctg}(\alpha + \beta) = ?$

a)  $\frac{\text{ctg } \alpha \text{ctg } \beta + 1}{\text{ctg } \alpha + \text{ctg } \beta}$

b)  $\frac{\text{ctg } \alpha \text{ctg } \beta - 1}{\text{ctg } \alpha - \text{ctg } \beta}$

ç)  $\frac{\text{ctg } \alpha \text{ctg } \beta + 1}{\text{ctg } \alpha - \text{ctg } \beta}$

d)  $\frac{\text{ctg } \alpha \text{ctg } \beta - 1}{\text{ctg } \alpha + \text{ctg } \beta}$

1	2	3	4	5	6	7	8
<b>b</b>	<b>a</b>	<b>ç</b>	<b>d</b>	<b>a</b>	<b>d</b>	<b>ç</b>	<b>d</b>



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

**Goý,  $\beta = \alpha$  bolsun, onda**

$$\begin{aligned} \sin 2\alpha &= \sin(\alpha + \alpha) = \\ &= \sin \alpha \cdot \cos \alpha + \cos \alpha \cdot \sin \alpha = 2\sin \alpha \cos \alpha \end{aligned}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

**Goý,  $\beta = \alpha$  bolsun, onda**

$$\begin{aligned} \cos 2\alpha &= \cos(\alpha + \alpha) = \\ &= \cos \alpha \cos \alpha - \sin \alpha \sin \alpha = \cos^2 \alpha - \sin^2 \alpha \end{aligned}$$

$$\cos^2 \alpha - \sin^2 \alpha = 1 - \sin^2 \alpha - \sin^2 \alpha = 1 - 2\sin^2 \alpha$$

$$\begin{aligned} \cos^2 \alpha - \sin^2 \alpha &= \cos^2 \alpha - (1 - \cos^2 \alpha) = \cos^2 \alpha - 1 + \cos^2 \alpha = \\ &= 2\cos^2 \alpha - 1 \end{aligned}$$





$$tg(\alpha + \beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha \cdot tg\beta}$$

$$\beta = \alpha$$

$$tg(\alpha + \alpha) = \frac{tg\alpha + tg\alpha}{1 - tg\alpha \cdot tg\alpha}$$

$$tg 2\alpha = \frac{2tg\alpha}{1 - tg^2\alpha}$$



Берлен  $\cos x = 0,8$ ;  $0 < x < \frac{\pi}{2}$ . Тапмалы:  $\sin 2x$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\begin{aligned}\sin x &= \pm\sqrt{1 - \cos^2 x} = \pm\sqrt{1 - 0,8^2} = \\ &= \pm\sqrt{1 - 0,64} = \pm\sqrt{0,36} = \pm 0,6\end{aligned}$$

$$\sin x = 0,6$$

$$\sin 2x = 2 \sin x \cdot \cos x = 2 \cdot 0,8 \cdot 0,6 = 0,96$$



Añlatmany ýönekeýleşdiriň:  $\frac{\sin 40^\circ}{\sin 20^\circ}$

$$\frac{\sin 40^\circ}{\sin 20^\circ} = \frac{\sin(2 \cdot 20^\circ)}{\sin 20^\circ} = \frac{2 \sin 20^\circ \cdot \cos 20^\circ}{\sin 20^\circ} = 2 \cos 20^\circ$$





# Temany berkitmek:

103-nji gönükme.

$\cos \alpha = \frac{1}{2}$  we  $0 < \alpha < \frac{\pi}{2}$  bolsa, onda  $\sin 2\alpha$ ,  $\cos 2\alpha$ ,  $\operatorname{tg} 2\alpha$  hasaplaň

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha} = \pm \sqrt{1 - \left(\frac{1}{2}\right)^2} = \pm \sqrt{1 - \frac{1}{4}} = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$0 < \alpha < \frac{\pi}{2}$ , onda  $\sin \alpha = \frac{\sqrt{3}}{2}$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$\operatorname{tg} 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$



## 106-njy gönükme.

$$\begin{aligned} \text{a)} \quad \frac{(\sin\alpha + \cos\alpha)^2}{1 + \sin 2\alpha} &= \frac{\sin^2 \alpha + 2\sin\alpha\cos\alpha + \cos^2 \alpha}{1 + \sin 2\alpha} = \\ &= \frac{1 + \sin 2\alpha}{1 + \sin 2\alpha} = 1 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \frac{\cos 2\alpha}{\sin\alpha + \cos\alpha} &= \frac{\cos^2 \alpha - \sin^2 \alpha}{\sin\alpha + \cos\alpha} = \\ &= \frac{(\cos \alpha + \sin \alpha) \cdot (\cos \alpha - \sin \alpha)}{\sin \alpha + \cos \alpha} = \cos \alpha - \sin \alpha \end{aligned}$$



# Öý işi

- 107-nji gönükme.

