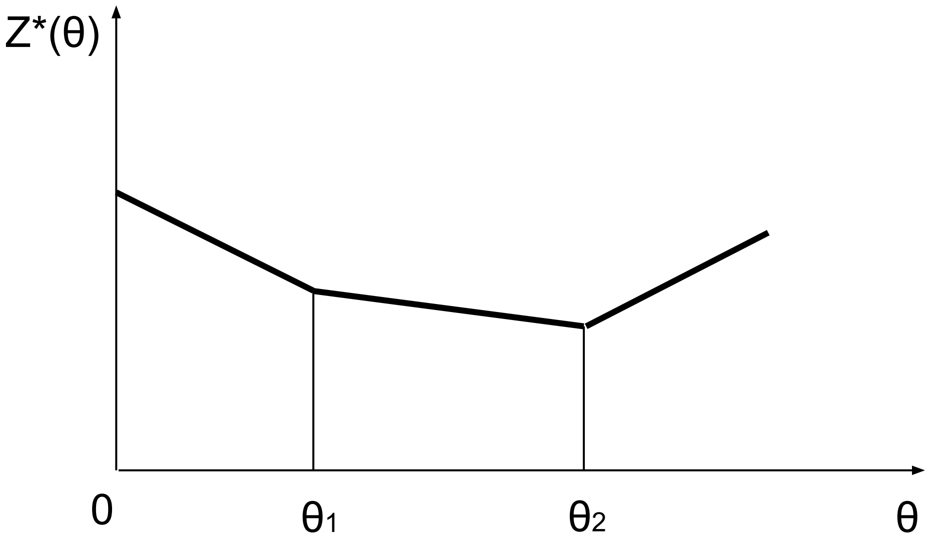
Parametric Linear Programming

Systematic Changes in c_j

- Objective function $Z = \sum_{j=1}^{n} c_j x_j$ is replaced by

$$Z(\theta) = \sum_{j=1}^{n} \left(c_j + \alpha_j \theta\right) x_j$$

- Find the optimal solution as a function of θ



Example: Wyndor Glass Problem

- $Z(\theta) = (3 + 2\theta)\, x_1 + (5 - \theta)\, x_2$

Example: Wyndor Glass Problem

Range of θ	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
	$Z(\theta)$	1	0	0	0	$(9-7\theta)/6$	$(3+2\theta)/3$	$36-2\theta$
$0 \le \theta \le 9/7$	x_3	0	0	0	1	1/3	-1/3	2
	x_2	0	0	1	0	1/2	0	6
	x_1	0	1	0	0	-1/3	1/3	2

Example: Wyndor Glass Problem

Range of θ	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
	$Z(\theta)$	1	0	0	$(-9+7\theta)/2$	0	$(5-\theta)/2$	$27+5\theta$
	x_4	0	0	0	3	1	-1	6
$9/7 \le \theta \le 5$	x_2	0	0	1	-3/2	0	1/2	3
	x_1	0	1	0	1	0	0	4

Example: Wyndor Glass Problem

Range of θ	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
	$Z(\theta)$	1	0	$-5+\theta$	$3+2\theta$	0	0	$12+8\theta$
$\theta \geq 5$	x_4	0	0	2	0	1	0	12
	x_5	0	0	2	-3	0	1	6
	x_1	0	1	0	1	0	0	4

Procedure Summary for Systematic Changes in c_j

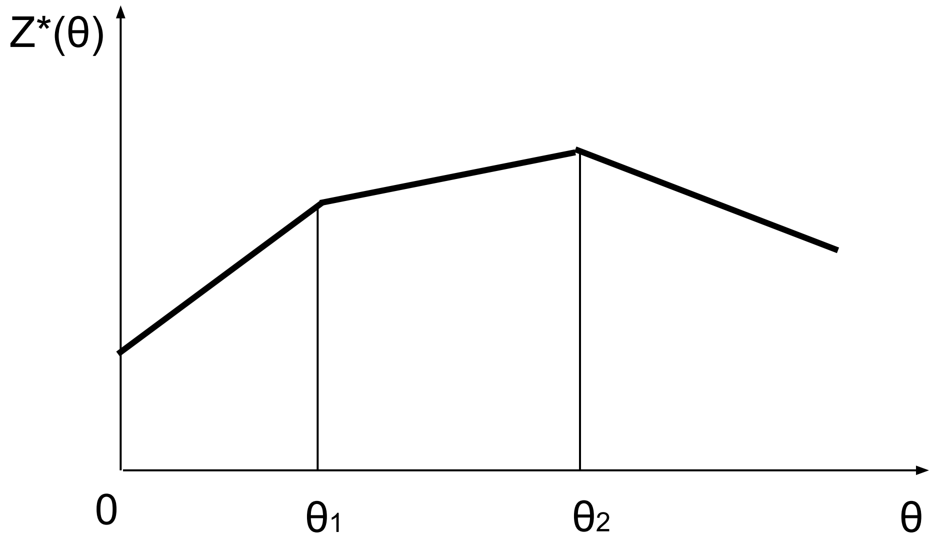
1. Solve the problem with $\theta = 0$ by the simplex method.
2. Use the sensitivity analysis procedure to introduce the $\Delta c_j = \alpha_j \theta$ changes into Eq.(0).
3. Increase θ until one of the **nonbasic** variables has its coefficient in Eq.(0) go negative (or until θ has been increased as far as desired).
4. Use this variable as the entering basic variable for an iteration of the **simplex** method to find the new optimal solution. Return to Step 3.

Systematic Changes in b_i

- Constraints $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ for $i = 1, 2, \ldots, m$ are replaced by

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i + \alpha_i \theta \quad \text{for } i = 1, 2, \ldots, m$$

- Find the optimal solution as a function of θ



Example: Wyndor Glass Problem

- $y_1 \qquad\quad + 3y_3 \geq 3 + 2\theta$

  $2y_2 + 2y_3 \geq 5 - \theta$

Example: Wyndor Glass Problem

Range of θ	Basic Var.	Z	y_1	y_2	y_3	y_4	y_5	RHS
	$Z(\theta)$	1	2	0	0	2	6	$-36+2\theta$
$0 \le \theta \le 9/7$	y_3	0	1/3	0	1	-1/3	0	$(3+2\theta)/3$
	y_2	0	-1/3	1	0	1/3	-1/2	$(9-7\theta)/6$

Example: Wyndor Glass Problem

Range of θ	Basic Var.	Z	y_1	y_2	y_3	y_4	y_5	RHS
	$Z(\theta)$	1	0	6	0	4	3	$-27-5\theta$
$9/7 \le \theta \le 5$	y_3	0	0	1	1	0	-1/2	$(5-\theta)/2$
	y_1	0	1	-3	0	-1	3/2	$(-9+7\theta)/2$

Example: Wyndor Glass Problem

Range of θ	Basic Var.	Z	y_1	y_2	y_3	y_4	y_5	RHS
	$Z(\theta)$	1	0	12	6	4	0	$-12-8\theta$
$\theta \geq 5$	y_5	0	0	-2	-2	0	1	$-5+\theta$
	y_1	0	1	0	3	-1	0	$3+2\theta$

Procedure Summary for Systematic Changes in b_i

1. Solve the problem with $\theta = 0$ by the simplex method.
2. Use the sensitivity analysis procedure to introduce the $\Delta b_i = \alpha_i \theta$ changes to the *right side* column.
3. Increase θ until one of the **basic** variables has its value in the *right side* column go negative (or until θ has been increased as far as desired).
4. Use this variable as the leaving basic variable for an iteration of the **dual simplex** method to find the new optimal solution. Return to Step 3.