

# THE FORMAL NORMAL FORM DEGENERATE SINGULAR POINTS IN THE CASE OF CASE OF FOCUS

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# OBJECTIVE

- ◉ In the work we study the problem for the case  $\lambda \notin \mathbb{R}, r = 2$ . Have a normal form

$$(V_H + x^p \varphi(x) E_H) \cdot (1 + x^q \phi(x)),$$

where  $E_H = (x, 2y), V_H = (y + (1 + \lambda)x^2, -2\lambda x^2),$

Formal normal form takes the form

$$(y + (1 + \lambda)x^2 + x^2 \varphi(x), -2\lambda x^2 + 2xy \varphi(x)) \cdot (1 + \phi(x)),$$

# AIM OF WORK

- The direct study of a formal normal form with the feature type Bogdanov-Takens for a particular case, for the purpose of comparison with the results of Zolondek-Strozhinói.

# PREFACE

- ⊙ Taken in 1974 for the system of equations of the form

$$\dot{x} = y + \dots, \quad \ddot{y} = \dots$$

got a fairly simple formal normal form

$$\dot{x} = y + a(x), \quad \dot{y} = b(x)$$

but this formal normal form admits of further simplification.

- General( for all cases) the formal normal form was obtained in 2015.  
This Form looks very complicated and has the form:

$$(V_H + x^p \varphi(x) E_H) \cdot (1 + x^q \psi(x))$$

# THEOREM

- The field  $V$  (для случая  $\lambda \notin \mathbb{Q}$ ) is formally equivalent to one of the following fields

$$N_{p;q}^{r;\lambda} : (V_H + x^p \varphi(x) E_H) \cdot (1 + x^q \psi(x)), \text{ где}$$
$$r - 1 < p \leq \infty, 0 < q \leq \infty, \text{ и}$$
$$\varphi = \sum_{i \in \mathbb{Z}_+ / I_{1+p}} a_i x^i, \psi = \sum_{i \in \mathbb{Z}_+ / I_{1+q}} b_j x^j,$$

formal power series. Forms  $N_{p;q}^{r;\lambda}$  are the only modulo substitutions

$$(x, y) \longrightarrow (\alpha x, \alpha y), \alpha^{r-1} = 1..$$

# DEFINITION

- Two analytical vector fields  $V, V_0$  in  $(\mathbb{C}^2; 0)$  are formally equivalent, then and only then when there is a formal diffeomorphism  $H \in (\mathbb{C}^2; 0)$

$$H' \cdot V = V_0 \circ H$$

# THE FUNCTIONAL EQUATION OF EQUIVALENCE AND THE SCHEME OF REDUCTION TO SYSTEMS OF LINEAR EQUATIONS

$$H' \cdot V = V_0 \circ H$$

$$H(x,y) = (x + \sum c_{ij} x^i y^j, y + \sum d_{ij} x^i y^j)$$

$$V = (y + (1 + \lambda)x^2 + \sum a_{ij} x^i y^j, -2\lambda x^2 + \sum b_{ij} x^i y^j)$$

$$V_0 = (y + (1 + \lambda)x^2 + \sum \tilde{a}_{ij} x^i y^j, -2\lambda x^2 + \sum \tilde{b}_{ij} x^i y^j)$$

Objective :to find a formal normal form as simple as possible species



- ◉ We substitute the expansions  $H, V, V_0$  in the main equation :

$$\left( \begin{array}{l} (1 + 2c_{20}x + c_{11}y + \dots)(y + (1 + \lambda)x^2 + a_{11}xy + \dots) + \\ + (c_{11}x + 2c_{02}y + c_{21}x^2 + \dots)((-2\lambda)x^2 + b_{11}xy + \dots) \\ (2d_{20}x + d_{11}x + 3d_{30}x^2 + \dots)(y + (y + (1 + \lambda)x^2 + \dots) + \\ + (1 + d_{11}x + 2d_{02}y + d_{21}x^2 + \dots))((-2\lambda)x^2 + b_{11}xy + \dots) \end{array} \right) =$$

$$\left( \begin{array}{l} (y + d_{20}x^2 + \dots) + (1 + \lambda)(x^2 + c_{20}^2x^4 + \dots) \\ -2\lambda(x^2 + c_{20}^2x^4 + \dots) \end{array} \right)$$

	$c_{20}$	$c_{11}$	$c_{02}$	$c_{30}$	$c_{21}$	$c_{12}$	$c_{03}$	$d_{20}$	$d_{11}$	$d_{02}$	$d_{30}$	$d_{21}$	$d_{12}$	$d_{03}$
1(2, 0)								-1						
1(1, 1)	2								-1					
1(0, 2)		1								-1				
1(3, 0)		$-2\lambda$									-1			
1(2, 1)	$2a_{11}$	$b_{20}-$ $-\mu$	$-4\lambda$	3								-1		
1(1, 2)	$2a_{02}$	$a_{11}+$ $+b_{02}$	$2(b_{11}-$ $-\mu)$		2								-1	
1(0, 3)		$a_{02}$	$2b_{02}$			1								-1
2(2, 0)														
2(1, 1)								2						
2(0, 2)									1					
2(3, 0)	$4\lambda$							$2\mu$	$-2\lambda$					

tab1.

# INFERENCE

- After we solve this system of equations the formal normal form takes the form

$$\begin{cases} \dot{x} = y + (1 + \lambda)x^2 \\ \dot{y} = -2\lambda x^2 + \sum \alpha_k x^{3k} + \sum \beta_k y x^{3k} + \sum \gamma_k x^{3k+1} \quad k = 1, \dots \end{cases}$$

We have 3 adjacent degree - 3 monoms from a formal normal form. Thus , our result is consistent with the result of Zolondek-Strozhinoi.

# REFERENCES

1. Formal normal form Zholandeka Strozhinoy  
– PWS Publishing, 1997

*Thank you for attention!*