ASYMPTOTES of GRAPHS

Vertical

Horizontal

Slant (Oblique)

Definition of an Asymptote

- An **asymptote is a straight line** - boundary for a graph of f(x).
- F(x) gets **closer and closer to the asymptote** as it approaches either a specific value *a* or positive or negative infinity.
- The functions most likely to have asymptotes are rational functions

Vertical Asymptotes

Vertical asymptotes occur when the following condition is met:

- The denominator of the *simplified* rational function is equal to 0.
- The *simplified* rational function may have cancelled factors common to both the numerator and denominator.

Finding Vertical Asymptotes

Example 1

Given the function $f(x)=\dfrac{2-5x}{2+2x}$

- Let denominator (2+2x) = 0

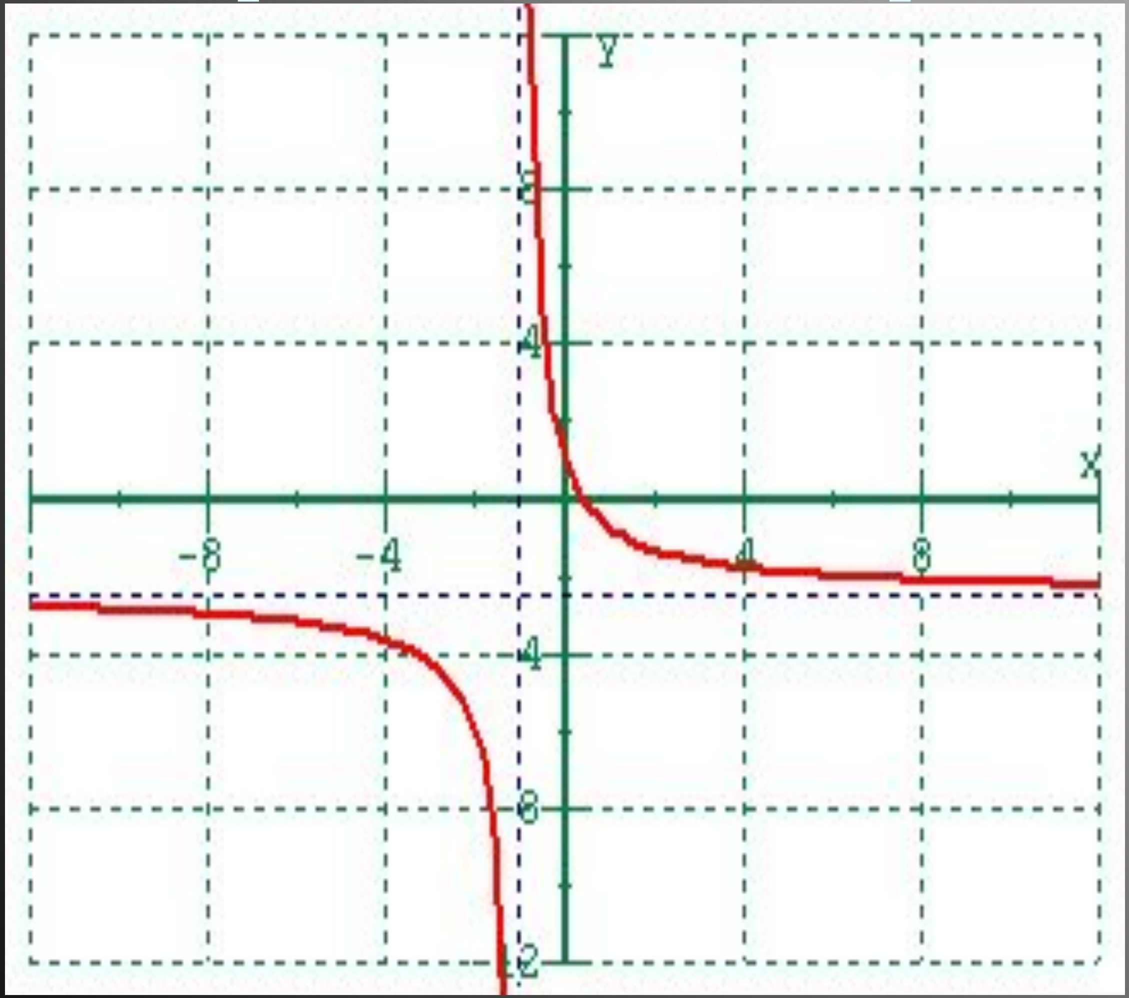
$$2+2x=0$$

$$2(1+x)=0$$

$$1+x=0$$

Vertical asymptote $x=-1$

Graph of Example 1



The vertical dotted line at $x = -1$ is the vertical asymptote.

Finding Vertical Asymptotes
Example 2

$$f(x) = \frac{2x^2 + 10x + 12}{x^2 - 9}$$

1. Factorise

the **numerator**

and denominator

$$\frac{2x^2 + 10x + 12}{x^3 - 9} = \frac{\cancel{(x+3)}(2x+4)}{\cancel{(x+3)}(x-3)}$$

$$= \frac{2x+4}{x-3}$$

2. Cancel any

Common factors.

$$\frac{2x^2+10x+12}{x^3-9}=\frac{(x+3)(2x+4)}{(x+3)(x-3)}$$

$$=\frac{2x+4}{x-3}$$

Denominator

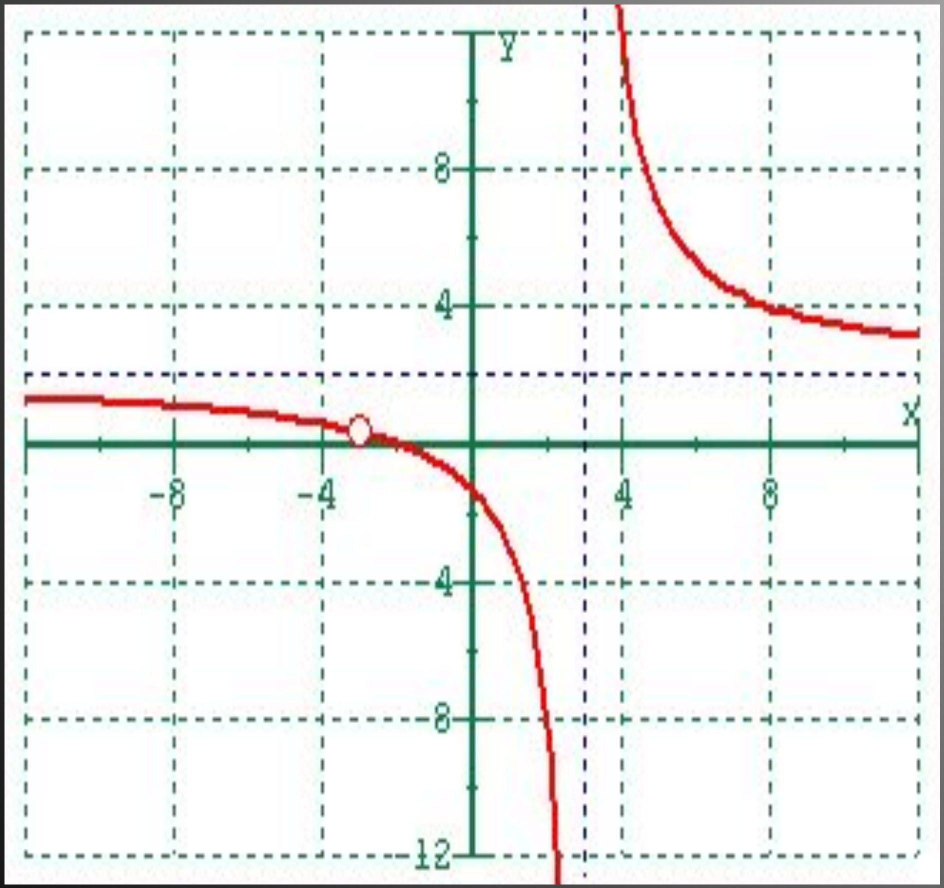
$X - 3 = 0$

$$x-3=0$$

Vertical Asymptote

$$x=3$$

Graph of Example 2



The vertical dotted line at $x = 3$ is the vertical asymptote

Finding Vertical Asymptotes
Example 3

$$g(x) = \frac{x-5}{x^2-x-6}$$

$$\frac{x-5}{x^2-x-6} = \frac{x-5}{(x+2)(x-3)}$$

Factorise

There are no common factors to cancel.

Finding Vertical Asymptotes
Example 3 Con't.

$$\frac{x-5}{x^2-x-6}=\frac{x-5}{(x+2)(x-3)}$$

Denominator = 0

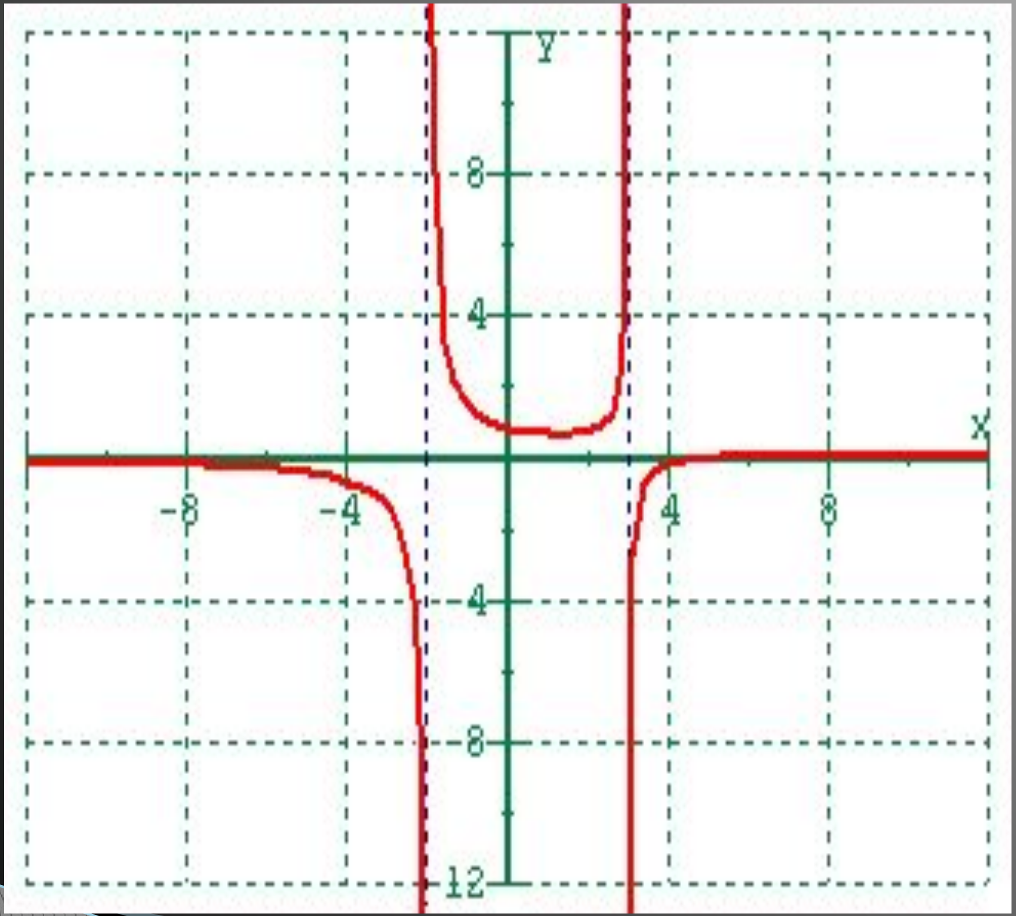
$$x+2=0 \qquad x-3=0$$

$$x=-2 \qquad x=3$$

g(x) has two vertical asymptotes

x = -2 and $x = 3$

Graph of Example 3



The two vertical dotted lines at $x = -2$ and $x = 3$ are the vertical asymptotes

Horizontal Asymptotes

Rational Function: $\dfrac{\text{Numerator (N)}}{\text{Denominator (D)}}$

1) **Degree N < Degree D** Horizontal Asymptote: y=0

2) **Degree N = Degree D** Horizontal Asymptote: y= Co-eff. of leading ‘x’

3) **Degree N > Degree D** Horizontal Asymptote: y = slant or DNE

Finding Horizontal Asymptotes

Example 4

$$f(x) = \frac{x^2 + 3x - 5}{x^3 - 27} \quad \begin{matrix} \text{N} \\ \text{D} \end{matrix}$$

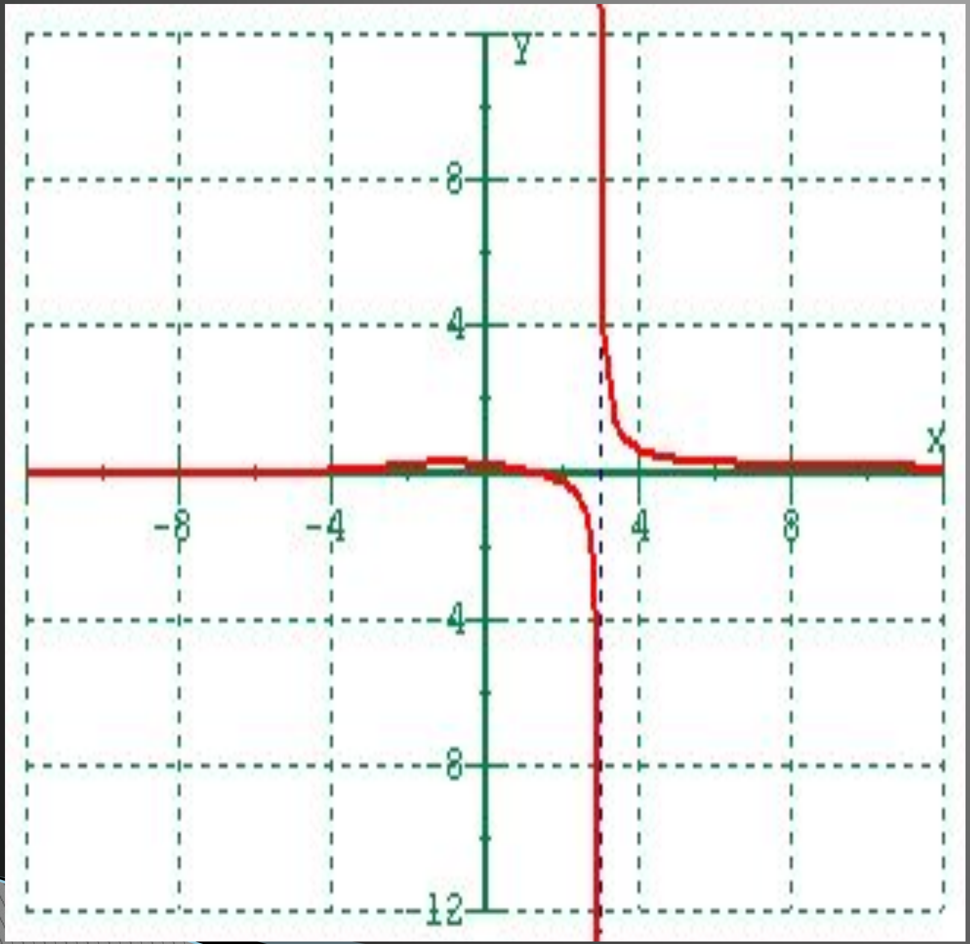
Horizontal asymptote: y=0

Degree N < Degree D

($x \rightarrow \infty$ and $x \rightarrow -\infty$)

horizontal line y = 0

Graph of Example 4



The horizontal line $y = 0$ is the horizontal asymptote.

Finding Horizontal Asymptotes
Example 5

$$g(x) = \frac{6x^2 - 3x + 5}{5x^2 + 7x - 9}$$

Degree N = Degree D

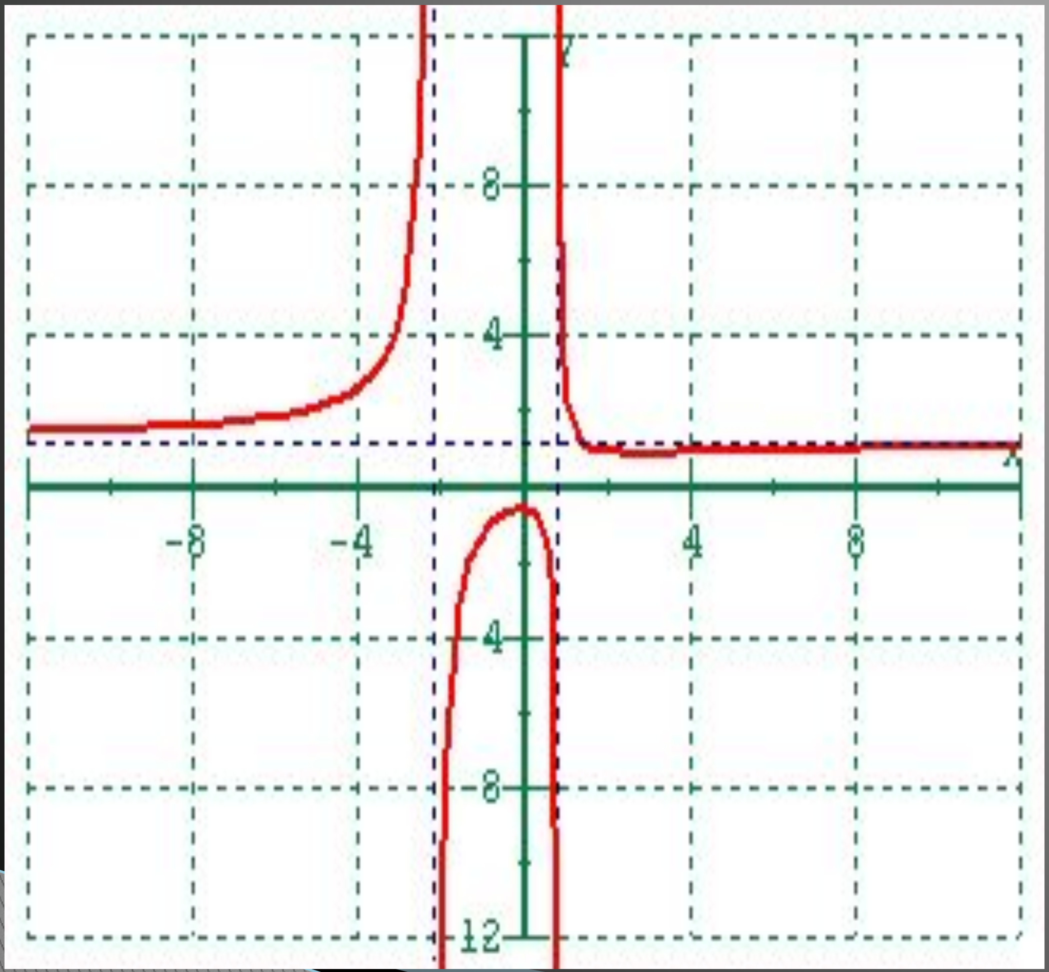
Horizontal asymptote: $y = {}^{6}/_{5}$.

Note: 6 and 5 are leading coefficients

($x \to \infty$ and as $x \to -\infty$)

line $y = {}^{6}/_{5}$

Graph of Example 5



The horizontal dotted line at $y = {}^{6}/_{5}$ is the horizontal asymptote.

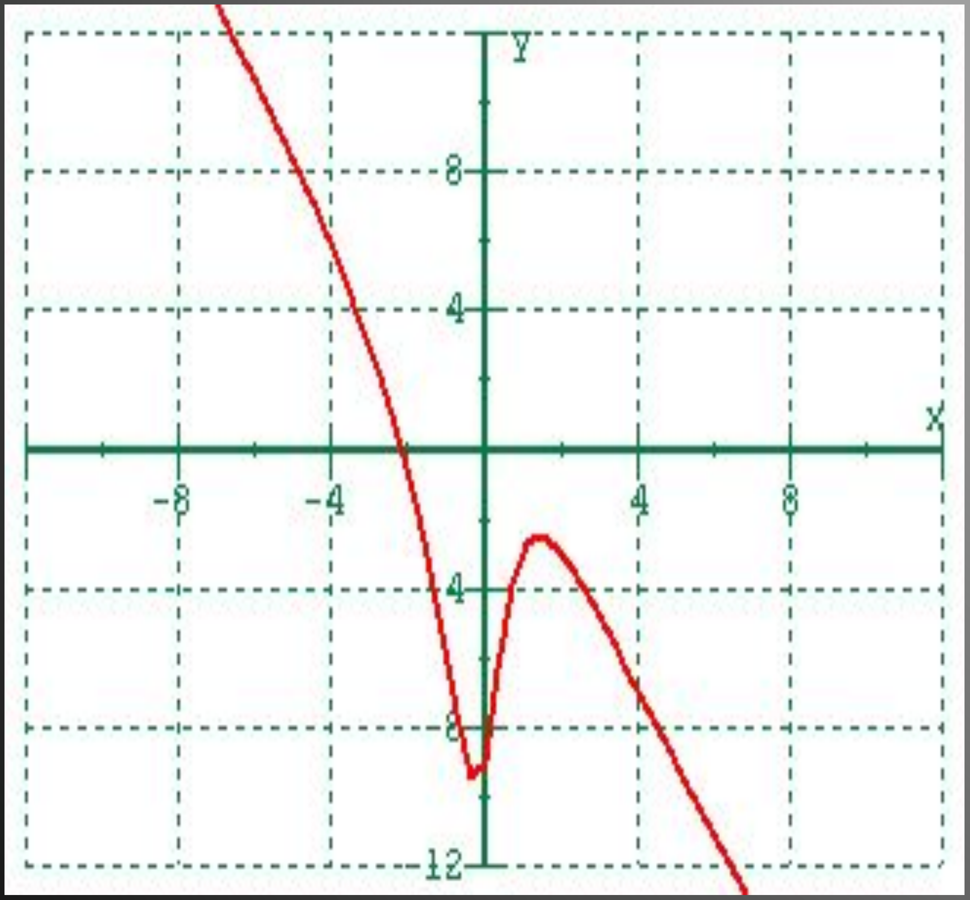
Finding Horizontal Asymptotes
Example 6

$$f(x) = \frac{-2x^3 + 5x - 9}{x^2 + 1}$$

No horizontal asymptote

Degree N > Degree D

Graph of Example 6



Finding a Slant Asymptote
Example 7

$$f(x)=\frac{x^3+2x^2+5x-9}{x^2-x+1}\quad \begin{matrix}\text{N}\\ \text{D}\end{matrix}$$

- Slant asymptote
- Degree N is <u>one</u> bigger than Degree D.
- Use long division: divide N by D

Finding a Slant Asymptote Example 7 Con't.

Use y=x+3
Slant Asymptote

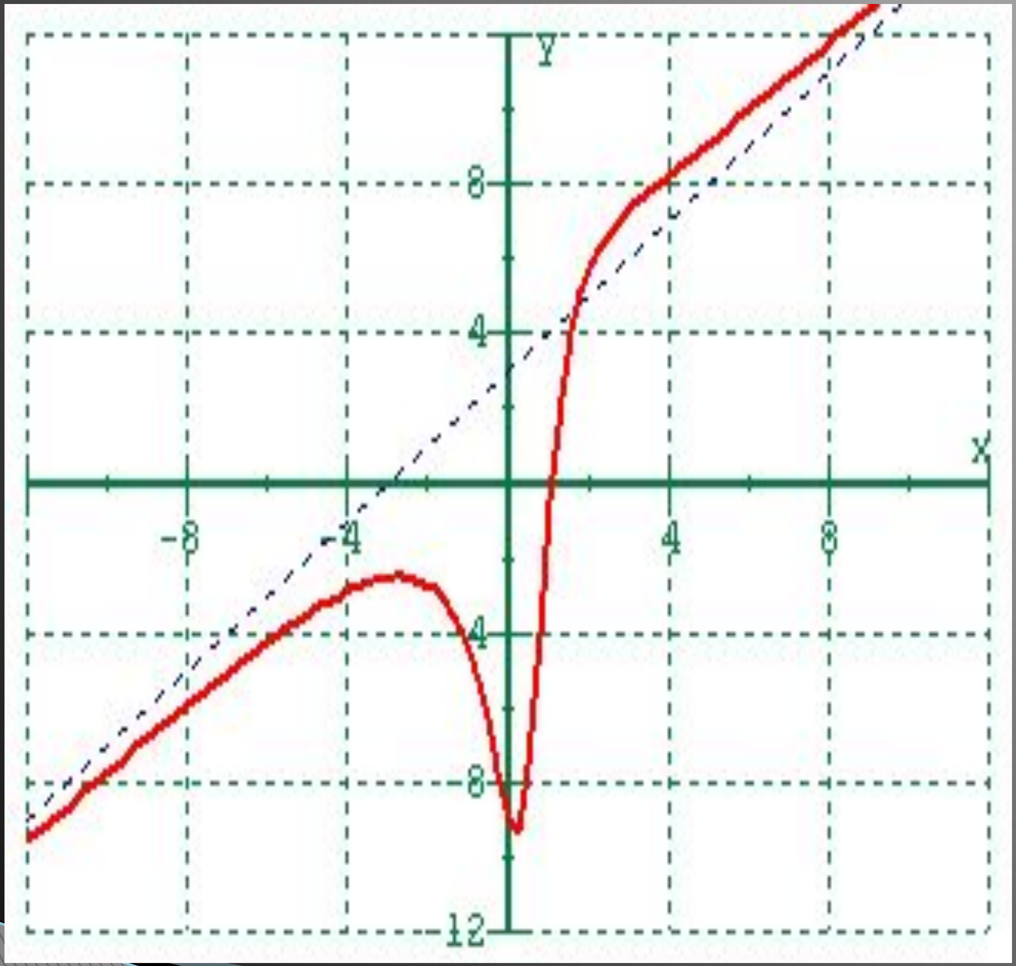
Finding a Slant Asymptote
Example 7 Con't.

Ignore the remainder: $\cancel{7x - 12}$

Use the quotient: $\underline{x + 3}$

The slant asymptote is: $\underline{y = x + 3}$

Graph of Example 7



The slanted line $y = x + 3$ is the slant asymptote

Problems

Find the vertical asymptotes, horizontal asymptotes and slant asymptotes for each of the following functions.

ANSWERS to Problems:

$$f(x)=\frac{x^2+2x-15}{x^2+7x+10}$$

Vertical: $x = -2$
Horizontal : $y = 1$
Slant: none
Hole: at $x = -5$

$$g(x)=\frac{2x^2+5x-7}{x-3}$$

Vertical: $x = 3$
Horizontal : none
Slant: $y = 2x + 11$
Hole: none