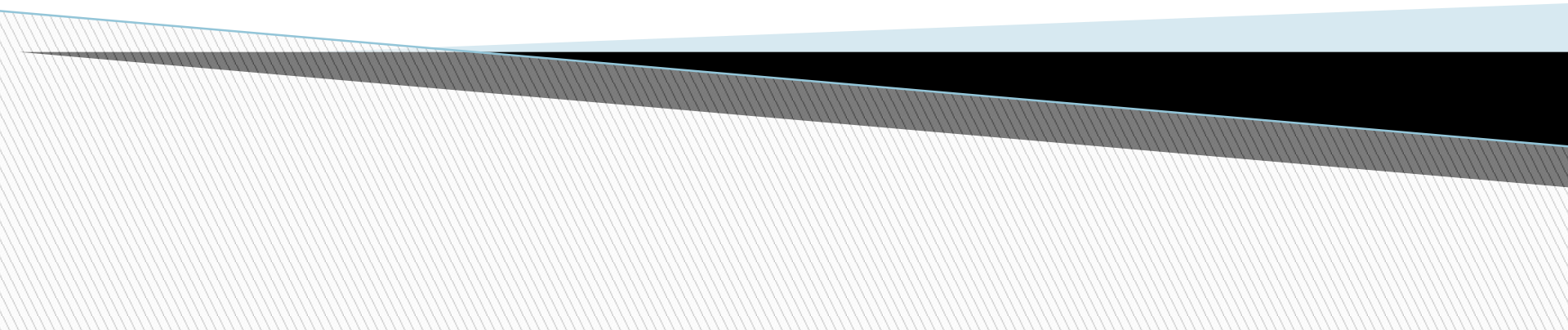


ASYMPTOTES of GRAPHS

Vertical

Horizontal

Slant (Oblique)



Definition of an Asymptote

- An **asymptote** is a straight line - boundary for a graph of $f(x)$.
- $F(x)$ gets **closer and closer to the asymptote** as it approaches either a specific **value a or positive or negative infinity**.
- The functions most likely to have asymptotes are **rational functions**

Vertical Asymptotes

Vertical asymptotes occur when the following condition is met:

- The denominator of the *simplified* rational function is equal to 0.
- The *simplified* rational function may have cancelled factors common to both the numerator and denominator.

Finding Vertical Asymptotes

Example 1

Given the function $f(x) = \frac{2-5x}{2+2x}$

- Let denominator
 $(2+2x) = 0$

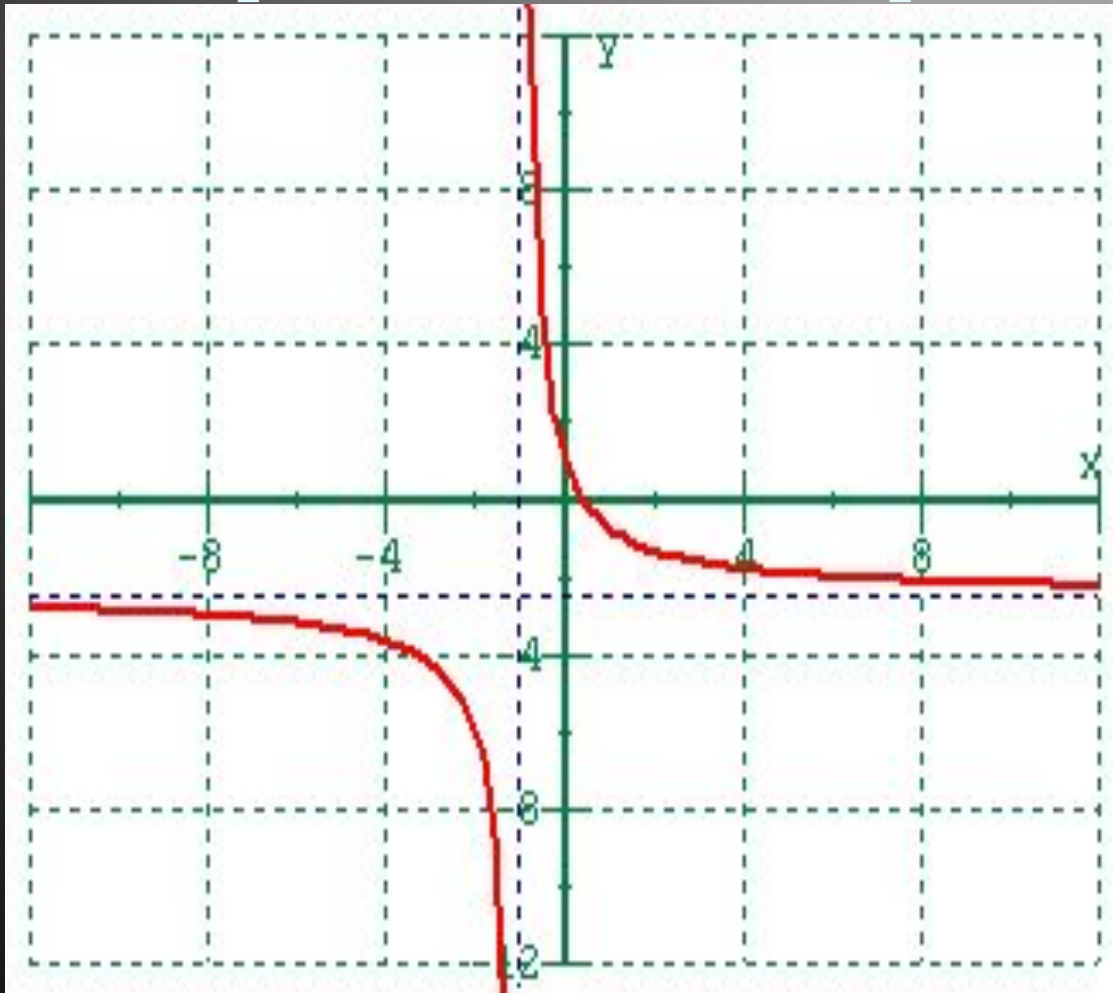
$$2 + 2x = 0$$

$$2(1 + x) = 0$$

$$1 + x = 0$$

Vertical asymptote $x = -1$

Graph of Example 1



The vertical dotted line at $x = -1$ is the vertical asymptote.

Finding Vertical Asymptotes

Example 2

$$f(x) = \frac{2x^2 + 10x + 12}{x^2 - 9}$$

1. Factorise

the **numerator**

$$\frac{2x^2 + 10x + 12}{x^2 - 9} = \frac{\cancel{(x+3)}(2x+4)}{\cancel{(x+3)}(x-3)}$$

and denominator

$$= \frac{2x+4}{x-3}$$

2. Cancel any

Common factors.

$$\frac{2x^2 + 10x + 12}{x^3 - 9} = \frac{(x + 3)(2x + 4)}{(x + 3)(x - 3)}$$
$$= \frac{2x + 4}{x - 3}$$

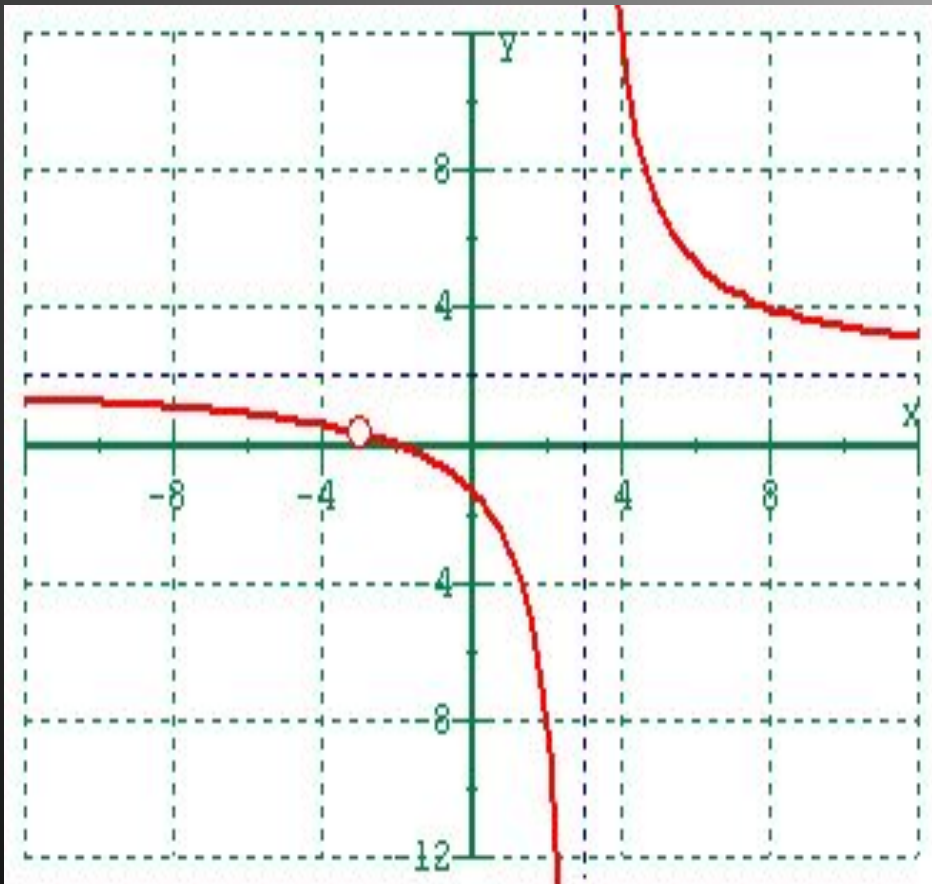
Denominator

$$x - 3 = 0$$

$$x - 3 = 0$$

Vertical Asymptote $\longrightarrow x = 3$

Graph of Example 2



The vertical dotted line at $x = 3$ is the vertical asymptote

Finding Vertical Asymptotes

Example 3

$$g(x) = \frac{x-5}{x^2 - x - 6}$$

$$\frac{x-5}{x^2 - x - 6} = \frac{x-5}{(x+2)(x-3)}$$

Factorise

There are *no common factors to cancel.*

Finding Vertical Asymptotes

Example 3 Con't.

$$\frac{x-5}{x^2-x-6} = \frac{x-5}{(x+2)(x-3)}$$

Denominator = 0

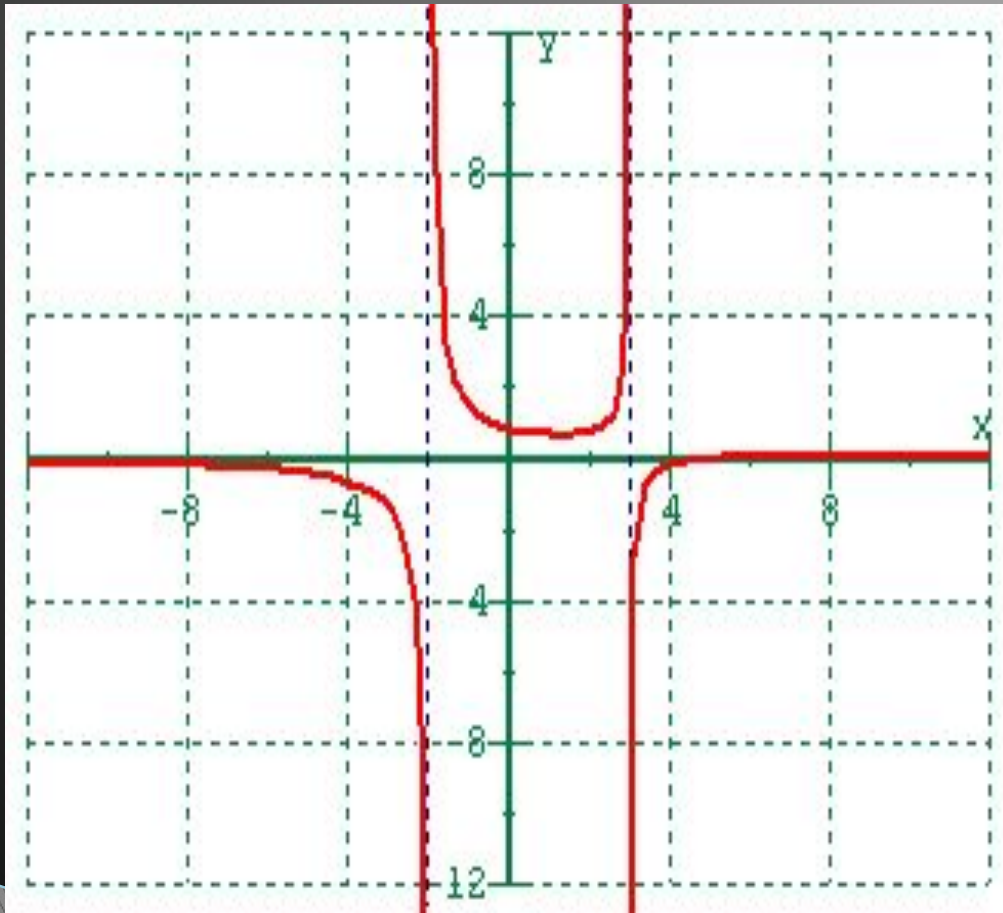
$$x+2=0 \quad x-3=0$$

$$x = -2 \quad x = 3$$

$g(x)$ has **two vertical asymptotes**

$$x = -2 \text{ and } x = 3$$

Graph of Example 3



The two vertical dotted lines at $x = -2$ and $x = 3$ are the vertical asymptotes

Horizontal Asymptotes

Rational Function: $\frac{\text{Numerator (N)}}{\text{Denominator (D)}}$

- 1) **Degree N** < **Degree D** □ Horizontal Asymptote: $y=0$
- 2) **Degree N** = **Degree D** □ Horizontal Asymptote: $y =$ Co-eff. of leading 'x'
- 3) **Degree N** > **Degree D** □ Horizontal Asymptote: $y =$ slant or DNE

Finding Horizontal Asymptotes

Example 4

$$f(x) = \frac{x^2 + 3x - 5}{x^3 - 27} \quad \begin{array}{l} \text{N} \\ \text{D} \end{array}$$

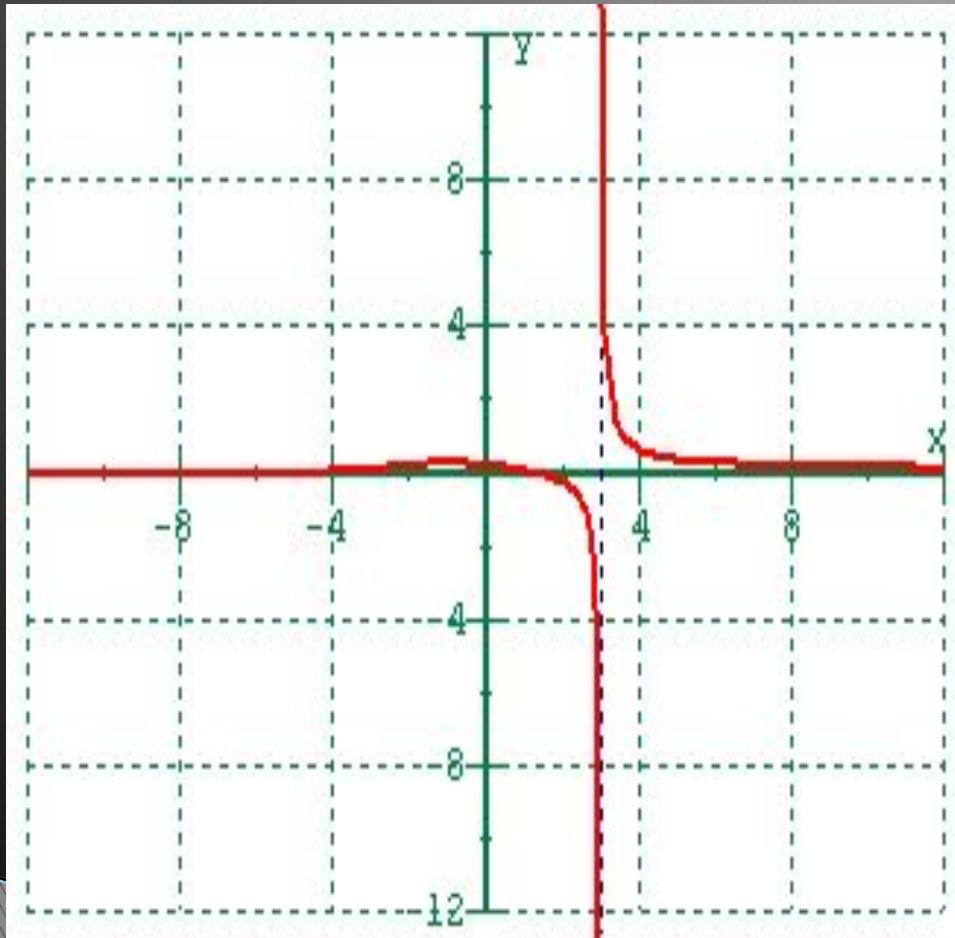
Horizontal asymptote: $y=0$

Degree N < Degree D

($x \rightarrow \infty$ and $x \rightarrow -\infty$)

horizontal line $y = 0$

Graph of Example 4



The horizontal line $y = 0$ is the horizontal asymptote.

Finding Horizontal Asymptotes

Example 5

$$g(x) = \frac{6x^2 - 3x + 5}{5x^2 + 7x - 9}$$

Degree N = Degree D

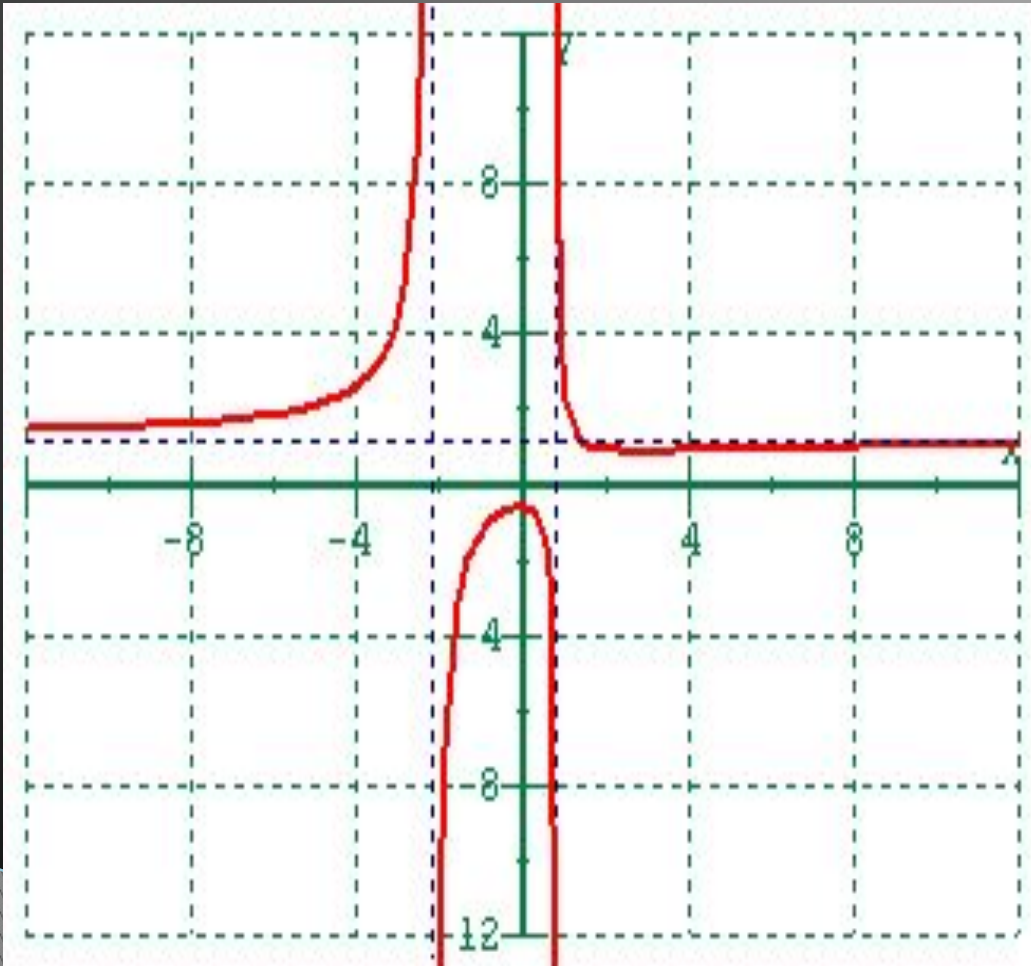
Horizontal asymptote: $y = \frac{6}{5}$.

Note: 6 and 5 are leading coefficients

($x \rightarrow \infty$ and as $x \rightarrow -\infty$)

line $y = \frac{6}{5}$

Graph of Example 5



The horizontal dotted line at $y = \frac{6}{5}$ is the horizontal asymptote.

Finding Horizontal Asymptotes

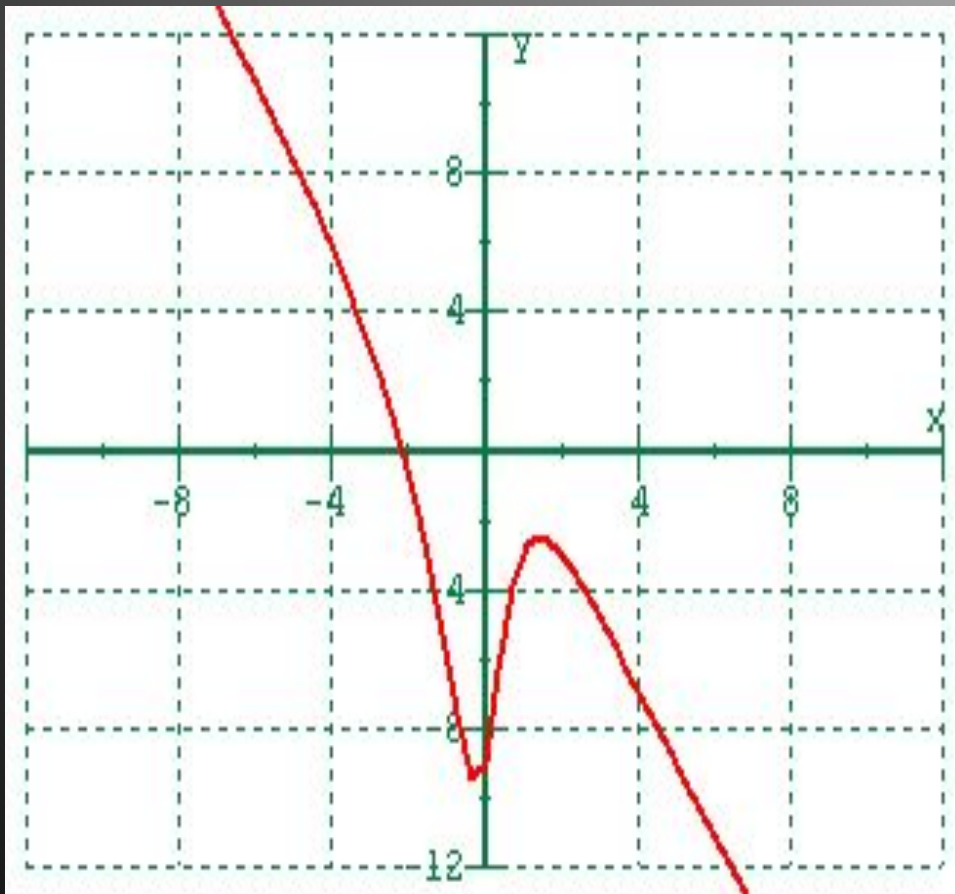
Example 6

$$f(x) = \frac{-2x^3 + 5x - 9}{x^2 + 1}$$

No horizontal asymptote

Degree N > Degree D

Graph of Example 6



Finding a Slant Asymptote

Example 7

$$f(x) = \frac{x^3 + 2x^2 + 5x - 9}{x^2 - x + 1} \quad \begin{array}{l} \text{N} \\ \text{D} \end{array}$$

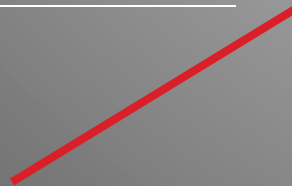
- Slant asymptote
- Degree N is one bigger than Degree D.
- Use long division: divide N by D

Finding a Slant Asymptote

Example 7 Con't.



Use $y=x+3$
Slant Asymptote



Finding a Slant Asymptote

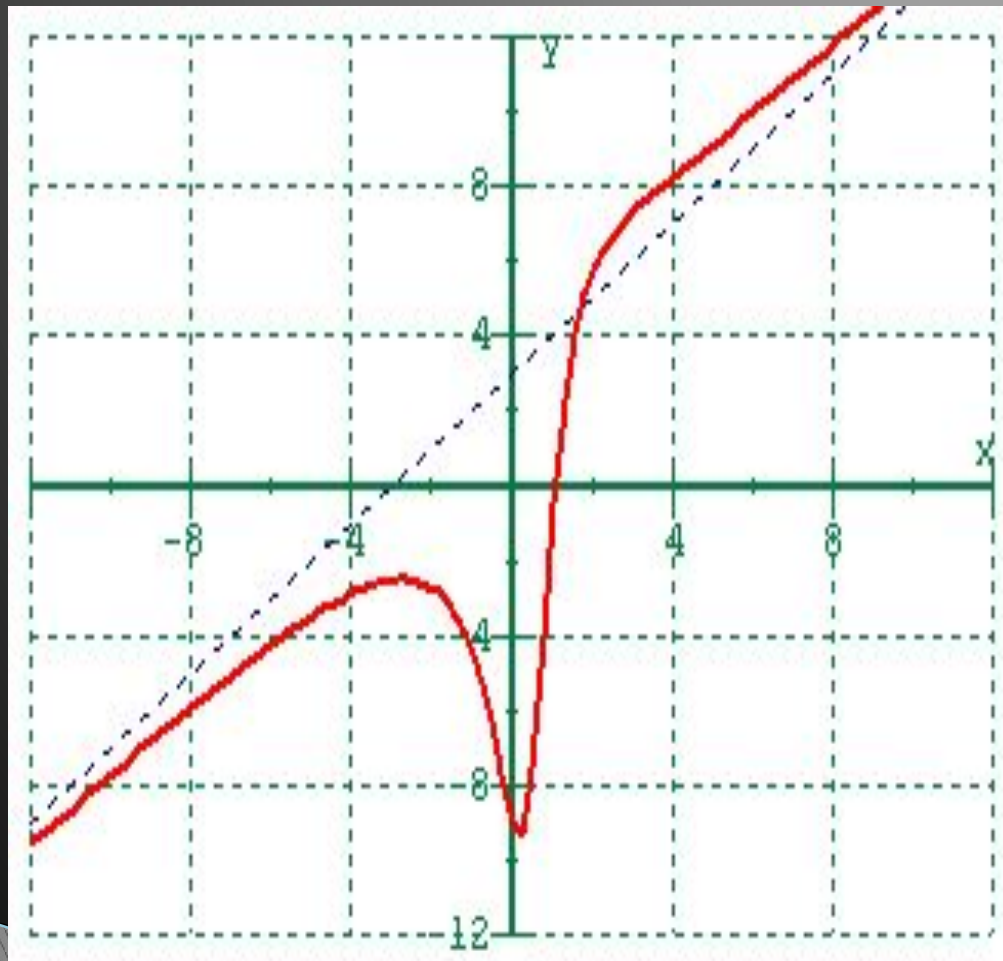
Example 7 Con't.

Ignore the remainder: $7x - 12$

Use the quotient: $x + 3$

The slant asymptote is: $y = x + 3$

Graph of Example 7



The slanted line
 $y = x + 3$ is the slant
asymptote

Problems

Find the vertical asymptotes, horizontal asymptotes and slant asymptotes for each of the following functions.

ANSWERS to Problems:

$$f(x) = \frac{x^2 + 2x - 15}{x^2 + 7x + 10}$$

Vertical: $x = -2$
Horizontal : $y = 1$
Slant: none
Hole: at $x = -5$

$$g(x) = \frac{2x^2 + 5x - 7}{x - 3}$$

Vertical: $x = 3$
Horizontal : none
Slant: $y = 2x + 11$
Hole: none